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On shape optimization with parabolic state equation

Helmut Harbrecht

(joint work with Johannes Tausch)

Shape optimization is a well established mathematical and computational tool in case of an elliptic state equation, see, e.g., [1] and the references therein. In contrast, in case of a parabolic state equation, the literature on shape optimization is quite limited.

Theoretical results on shape optimization with parabolic state equation can be found, for instance, in [3, 2, 7, 8, 9] and the references therein. Nonetheless, the development of efficient numerical methods for shape optimization problems with parabolic state equation is still in its beginning stages, especially for three-dimensional geometries.

We aimed at the development of such efficient methods in [4, 5, 6], where the focus was on shape identification problems for the heat equation. Namely, it is intended to determine inclusions or voids from measurements of the temperature and the heat flux at the outer boundary of the domain under consideration. The particular shape identification problem is reformulated as a shape optimization problem. Then, the Hadamard representation of the shape gradient is computed by means of the adjoint method. By identifying the sought boundary with its parametrization, a gradient based nonlinear Ritz-Galerkin scheme can be applied to discretize the shape optimization problem.

Since the adjoint equation is reversal in time, space-time discretization schemes are quite attractive for the determination of the states and their adjoints. To that end, we cast the parabolic boundary value problems into parabolic boundary integral equations. These boundary integral equations are discretized by using a Nyström discretization in space. For the time discretization, appropriate singularity corrected trapezoidal quadrature rules are applied to handle the singularities of the heat kernel and the solution. A space-time fast multipole method for the rapid evaluation of thermal potentials, developed in [10, 11], is employed to solve the discretized boundary integral equations.

Numerical experiments are carried out to demonstrate the feasibility and scope of the present approach. In particular, we are able to reconstruct unknown shapes in three dimensions on a laptop in less than half an hour computation time even though up to 1200 design parameters and about 120 000 boundary elements have been used for the discretization of the shape optimization problem.

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Iterative solution of optimality systems in optimal control

Roland Herzog

(joint work with Kirk M. Soodhalter)

In this talk we address the iterative solution of self-adjoint saddle-point systems in Hilbert spaces, as well as the construction of preconditioners. Suppose that V and Q are two (real) Hilbert spaces and V^* and Q^* are their duals. Suppose further that $A \in \mathcal{L}(V, V^*)$, $B \in \mathcal{L}(V, Q^*)$ and $C \in \mathcal{L}(Q, Q^*)$ are bounded linear operators. We consider self-adjoint saddle-point systems, which are of the form

$$\begin{bmatrix} A & B^\star \\ B & -C \end{bmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \quad \text{with } A = A^\star,\ C = C^\star.$$

These systems arise as optimality conditions in optimal control problems, but also, for instance, in mixed finite element discretizations of numerous problems. In the context of partial differential equations, and especially time-dependent PDEs, these systems are usually of large scale after discretization, which calls for the use of iterative solvers.

In the first part of the talk, we discuss the role of block-diagonal preconditioners $P = \mathrm{blkdiag}(P_V, P_Q)$ composed of self-adjoint, positive definite blocks $P_V : V \to V^*$ and $P_Q : Q \to Q^*$. On the one hand, the obvious purpose of preconditioning is to render the spectrum of the preconditioned operator favorable so that iterative solvers, such as MINRES [4], exhibit sufficiently fast convergence. It is also well