The pricing of volatility risk in the US equity market

Lukas Hitz, Ismail H. Mustafi, Heinz Zimmermann

WWZ (Wirtschaftswissenschaftliches Zentrum), Department of Business and Economics, University of Basel, Switzerland

ARTICLE INFO

JEL classification:
G12
C52
E44

Keywords:
Pricing of volatility risk
Cross sectional asset pricing
Macroeconomic vs. fundamental risk factors
Evaluation of portfolio sort technique

ABSTRACT

We analyze whether the pricing of volatility risk depends on the asset pricing framework applied in the tests, the specified volatility proxies, and the portfolio sorts used for spanning the asset universe. For this purpose, we compare the results using a macroeconomic and fundamental based asset pricing model using three proxies of volatility and uncertainty, using size/value sorted and industry sector portfolios. Our results reveal that the marginal pricing effect of the VIX volatility factor is strong and statistically significant throughout the models and specifications, while the effect of an EGARCH-based volatility factor is mixed, mostly smaller but with the correct sign. In most cases, the EGARCH factor does not impair the pricing effect of the VIX. The portfolio sorts have a substantial impact on the volatility premiums in both model frameworks. The size of the volatility risk premium is more uniform across the models if the industry sector portfolio sort is used. Finally, the size/value portfolio sort generates larger volatility risk premiums for both models.

1. Introduction

Volatility risk is a common source of variation among asset returns and plays an important role in investment decisions. Many investors seek to follow low-volatility or volatility-insensitive strategies in times of market turmoil, or try to enhance investment returns by taking tactical bets on volatility. In contrast, volatility risk is only occasionally included in asset pricing models. While there is little controversy in the empirical literature that aggregate stock market volatility is a priced risk factor in the cross section of portfolio returns, the research results can be hardly compared. Apart from the asset pricing framework, the various studies mainly differ in the specification of the test asset universe and the measurement of volatility. Most papers use a single volatility (or uncertainty) measure without addressing multiple measures. Representative studies and their main findings are briefly discussed in Section 2.

The main contribution of this paper is to analyze the impact of model selection, variable specification, and portfolio sorts on the pricing of aggregate stock market volatility. Almost all papers such as Adrian and Rosenberg (2008), Ang, Hodrick, Xing, and Zhang (2006) or Brooks, Li, and Milne (2009) estimate a variant of the Fama–French (FF) multifactor pricing model using returns from standard portfolio sorts (e.g. book-to-market, size) in addition to volatility-based sorts. This is surprising, because it is well known that the sorting-procedure of portfolio returns has a strong impact on the identified risk premiums; the papers of Ernstberger, Haupt, and Vogler (2011) and Ferson, Sarkissian, and Simin (1999) are revealing. Moreover, the sensitivity of equities to changes in aggregate volatility actually fits better to the logic of macroeconomic factor pricing models than to FF-style models with factors representing spread returns derived from firm fundamentals. Therefore, the results from the widely used Fama–French three factor model (FF3) are contrasted to those of (a slightly modified version of) the macroeconomic Chen–Roll–Ross model (CRR). Both models are estimated using the well-known 25 size/value-sorted Fama–French portfolios and, alternatively, 48 sector portfolios, both covering the US stock market.

In a first step, volatility risk is proxied by the innovations of the VIX volatility index which is the most popular measure used in the literature. Second, in order to get a more precise picture about the nature of volatility risk, we extend our models by adding a short-term, EGARCH-based measure of stock market volatility and a non-stock market related variable measuring monetary policy uncertainty (MPU). This variable is used to disentangle short-term stock market variance from monetary uncertainty. Finally, the robustness of our results is tested by including some additional variables that measure aggregate uncertainty in our model: an index for macroeconomic uncertainty; a proxy for cyclical

Received 21 December 2020; Received in revised form 17 October 2021; Accepted 19 October 2021
Available online 22 November 2021
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aggregate risk aversion; and an index measuring long-run interest rate volatility (MOVE). Robustness is further tested with respect to subperiods, alternative proxies for some of the macroeconomic variables, and alternative estimation procedures.

In a nutshell, the contribution of our paper is threefold: we analyze the pricing of volatility risk (a) in the context of two economically distinct asset pricing frameworks - a “fundamental based” FF model with traded factors, and a version of the macroeconomic CRR model with non-traded risk factors; (b) by spanning the asset universe by 25 size/value-sorted FF portfolios as well as 48 sector portfolios; and (c) using multiple aggregate volatility proxies.

We find that the marginal pricing effect of the VIX volatility factor always has the expected sign and is statistically significant in the FF as well as in the CRR model frameworks. The effect of the EGARCH-based volatility factor is mixed but exhibits the expected sign throughout the models as well. In most cases, the EGARCH factor does not impair the pricing effect of the VIX. If regressions are run by GLS, the EGARCH-premium becomes more important if the size/value-sorted portfolios are used. Monetary policy uncertainty exhibits only a few significant pricing effects, almost all of them in the CRR model using the size/value portfolio sort. Overall, the portfolio sorts have a substantial impact on the volatility premiums in both model frameworks. The volatility risk premiums are more uniform across the models if the asset span is sorted by industry sector portfolios, but size/value-sorted portfolios generate larger volatility risk premiums (in absolute terms) for both models. The cross-sectional estimation technique, OLS vs. GLS, has also an effect on the results. The pricing effects of the VIX- and EGARCH-volatility factors are highly robust against the inclusion of additional uncertainty factors. From these factors, only interest rate volatility (MOVE) exerts a marginal pricing impact in the cross-section of returns.

The rest of the paper is structured as follows: Section 2 gives a brief overview on the literature to which this paper refers. Section 3 describes our models and the test methodology. Our factors and their specification can be found in Section 4, and Section 5 contains the empirical results. Section 6 summarizes our findings.

2. Selective literature review

The major reference for our paper is the study of Ang et al. (2006). The authors analyze the impact of stocks’ sensitivity to shocks in aggregate volatility and the size of idiosyncratic volatility on average returns using the FF3-factor model and extensions. They find that both variables are significantly priced in the cross-section of returns. Portfolio sorts are based on VIX-factor betas and idiosyncratic volatility. The estimated volatility premium is roughly −0.1% per month (see their Table V) and statistically significant across all models. The ex-post spread of volatility betas between the top and bottom quintile portfolios is 8.07 − (−5.06) = 13.13, which explains a sizeable volatility-risk related average return premium of 13.13 × (−0.080) = −1.05% per month between the portfolios.

Subsequent papers extend this research in various ways, in particular with respect to the economic nature of the volatility factors: Adrian and Rosenberg (2008) analyze two aspects of volatility risk, a short-run skewness related component as an indicator of the tightness of financial constraints, and a long-run business cycle related part. Both factors are included in a FF3-factor model and estimated with size and value (BE/ME) sorted portfolios. Both volatility factors are statistically significant and coexist. The average monthly premium of the short-run factor across all portfolios is −0.17%, and −0.23% for the long-run component. They find that the value-growth spread of the portfolios is much more related to the short-term volatility factor than to the long-run component. Cremers, Halling, and Weinbaum (2015) test for the pricing of aggregate jump and volatility risk as orthogonal factors in the cross-section of stock returns. By estimating a FF3-factor model based on volatility-beta and jump-beta sorted portfolios, they find that both risk factors exhibit the expected sign, are both statistically significant and do not subsume each other. For a one standard deviation shock, jump risk exhibits a slightly larger impact than volatility risk. Brooks et al. (2009) find strong pricing effects of asymmetric GARCH-volatility in the cross-section of 100 size and value (BE/ME) sorted portfolios using various pricing models, the CAPM, FF3-factor and CRR-style macroeconomic factor model. They do however not provide a systematic comparison of the valuation framework or alternative portfolio sorts. They attribute their findings to either a missing risk factor in traditional models or from the test portfolios’ differing sensitivities to idiosyncratic risk. This hypothesis is tested in a follow-up paper in Miffre, Brooks, and Li (2013). A different theoretical framework than in the other papers is used by Campbell, Giglio, Poll, and Turley (2018): They estimate an intertemporal, Merton–Campbell-style asset pricing model with low-frequency risk factors (discount rate, cash flow, and volatility “news”) to the cross-section of size, value and market risk (beta) sorted portfolios. They find that persistent shocks in equity volatility, tied to the default spread, are priced in the cross section of stock returns.

Not only volatility, but uncertainty in a broader sense is tested by Brogaard and Detzel (2015). The authors find that a widely used indicator for US economic policy uncertainty (EPU) has substantial explanatory power for the cross section of 25 size–momentum portfolio returns using the FF3-factor model augmented by momentum and liquidity effects. They also find that after controlling for two stock market volatility factors (historical and implied volatility), only EPU exhibits a significant risk premium.

A different approach is taken by González-Urteaga and Rubio (2017): instead of treating volatility as latent (non-traded) risk factor with a risk premium indirectly estimated from mimicking portfolios (i.e. slopes of Fama–MacBeth regressions), the authors correctly claim that volatility risk premiums can be observed from option markets. They are able to jointly test a return and volatility risk premiums in the cross section of variance-risk-beta and market-beta sorted portfolios. They find that different risk factors drive the stochastic discount function of return and volatility premiums.

The focus of these papers (including our own) should not be confused with studies that include asset specific volatility as a characteristic – not a volatility-beta – for explaining cross-sectional anomalies or as predictor for future (abnormal) returns. For example, Lewellen (2015) finds that the Fama–MacBeth regression framework provides an effective way to combine many firm characteristics, including volatility, into a composite forecast of future returns. Jordan and Riley (2015) find that past volatility of mutual fund returns loses its predictive power for future abnormal returns if an LVH factor (spread returns of low minus high volatility portfolios) is added to a FF4-factors model. However, these papers do not study the role of volatility as a risk factor for stock returns. Similarly, several papers analyze the explanatory power of idiosyncratic volatility on the cross-section of returns, for example Bali and Cakici (2008). This is one of the few papers, which highlights the critical role of data frequency, the weighting scheme used to compute test portfolio returns, and the breakpoints utilized to sort stocks into quintile portfolios – among other issues – in determining the existence and significance of the tested cross-sectional relationship.

Overall, with very few exceptions, most of the papers in the existing literature estimate Fama–French based factor models for analyzing volatility risk, use volatility-based or standard firm-characteristics based portfolio sorts, and test single volatility measures without addressing the pricing effects of multiple measures. In this paper, we compare the pricing of volatility risk in a Fama–French type model with traded factors with a macroeconomic CRR model with non-traded risk factors; we test the impact of two distinct, not-volatility based portfolio sorts; and finally, we analyze the pricing effect of alternative proxies measuring uncertainty.

3. Models and methodology

In this section, we give an overview of the models and the estimation methodology used in our empirical analysis.
3.1. Asset pricing models

The objective of this paper is to test whether expected short- and long-run stock market volatility as well as a proxy for monetary policy uncertainty are priced risk factors in the cross-section equity returns. We test these factors in the context of two distinct asset pricing models: the macroeconomic factor model by Chen, Roll, and Ross (1986) and the firm characteristic factor model by Fama and French (1993). We mostly refer to the two papers, respectively to the two models, by CRR and FF.

In a first step, we test the pricing of uncertainty risk as an extension of the CRR model. For this purpose, we first try to recreate the CRR model with the new data and a new estimation period (i.e. 1990–2018). The basic model contains the following factors: industrial production growth, change in expected inflation and unexpected inflation, credit risk premium and the term structure of interest rates. The construction of the macroeconomic factors is shown in Section 4.1. The findings of Chen et al. (1986) indicate that all these macro variables are priced risk factors. Shanken and Weinstein (2006) however found no such evidence when replicating the paper with the same data and the same estimation period (1953–1983). Ferson (2006) suspects that these differences of findings arise due to the usage of different test portfolios.

Therefore, we test our results with alternative test portfolios to examine whether the pricing results differ with respect to the portfolio sorts.

In a second step, we test our factors as an extension to the FF model. This model is widely used in the empirical asset pricing literature whether the pricing results differ with respect to the portfolio sorts. Shanken and Weinstein (2006) however find no such evidence when replicating the paper with the same data and the same estimation period (1953–1983). Ferson (2006) suspects that these differences of findings arise due to the usage of different test portfolios. FMB emphasize that the sampling errors for the estimates should be derived from the (empirical) distribution of the estimates. Therefore, the standard deviation of the cross-sectional regression estimates are used by calculating

\[ \sigma^2(\hat{\alpha}) = \frac{1}{T} \sum_{t=1}^{T} (\hat{\alpha}_t - \hat{\alpha})^2. \]

In order to test whether the different asset pricing models still inherit pricing errors we follow FMB by applying their joint test of alphas:

\[ \text{cov}(\hat{\alpha}) = \frac{1}{T} \sum_{t=1}^{T} (\hat{\alpha}_t - \hat{\alpha})(\hat{\alpha}_t - \hat{\alpha})', \]

using

\[ \text{acos}(\hat{\alpha})^{-1} \hat{\alpha}' \sim \chi^2_{N-K}. \]

The joint test of alphas follows a \( \chi^2 \)-distribution. In other words, this test examines whether the factors are able to explain the portfolio equity returns.

To test for robustness we also estimate the risk premiums using generalized least squares (GLS) in Section 5.3. Lewellen, Nagel, and Shanken (2010) state that using GLS instead of OLS forces the risk premiums on a factor, or in the case of non-return factors on a factor's mimicking portfolio, to be equal to its expected return.

3.2. Test methodology

In order to calculate the risk premium of the factors described in Sections 4.1 and 4.2, we apply the two-step methodology of Fama and MacBeth (1973), abbreviated in this paper by FMB: the first step (Eq. (1)) is a time-series regression of portfolio excess returns on the FF factors and the macroeconomic factors in the CRR framework respectively:

\[ R_{jt} = a_j + \sum_{k=1}^{K} \beta_{jk} f_{jt} + \varepsilon_{jt}. \]

This step gives the estimated factor exposures or loadings of the test assets to each factor. FMB use rolling regressions over a 5-year interval. However, today most authors (including ourselves) follow the approach to estimate a single beta over the entire sample period.

The second step is a cross-sectional regression,

\[ \hat{R}_{jt} = a_{jt} + \sum_{k=1}^{K} \hat{\beta}_{jk} \lambda_{jt}, \]

whereby at each time period \( t \), the portfolio excess returns are regressed on the betas estimated in the first step. This results in a time series of risk premium in each point \( t \), hereby called \( \lambda_t \). After the second step, FMB proceed by taking the average of the estimated \( \lambda_t \)

\[ \lambda_t = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_{jt}, \]

in order to obtain the market price of risk of the respective factor. As we test the factors on portfolio returns, i.e. 25 size- and valuesorted portfolios and 48 industry sector portfolios, we control for errors-in-variable (EIV). By taking the average of the time varying \( a_t \),

\[ \hat{a}_t = \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{jt}, \]

the average pricing errors are calculated.

In order to test whether the estimated risk premiums of the factors are significantly different from zero we calculate the standard errors by Newey and West (1987). To check if the pricing errors are significantly different from zero we calculate the sample errors. FMB emphasize that the sampling errors for the estimates should be derived from the (empirical) distribution of the estimates. Therefore, the standard deviation of the cross-sectional regression estimates are used by calculating

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3.3. Test portfolios

The portfolios used in the pricing tests are based on U.S. stock returns sorted by size and value, and by industry sectors, but not by volatility. Sorting should not be based on the factors to be tested. We therefore avoid the critique by Ernstberger et al. (2011), Ferson et al. (1999) and others showing that this implies a bias of the pricing results towards the characteristic used in the portfolio sort. Furthermore, we expand both test portfolio sorts with the traded factors of the model by Carhart (1997) and Fama and French (1993, 2015), as Lewellen et al. (2010) suggest.

The 48 sector portfolios are constructed by assigning each NYSE, AMEX, and NASDAQ stock to an industry portfolio at the end of June in year \( t \). This assignment is based on the stocks SIC code at that time. Kenneth French’ data library uses Compustat SIC codes for the fiscal year ending in calendar year \( t + 1 \). However, when Compustat SIC codes are not available, they use CRSP SIC codes for June of year \( t \). The relevant returns are calculated from July of \( t \) to June of \( t + 1 \).

The returns for the 25 size- and value-sorted portfolios are downloaded from Kenneth French’ data library. The portfolios are constructed at the end of each June. They represent a double sort of 5 portfolios sorted on size, i.e. market equity (ME), and 5 portfolios sorted
on value, i.e. the ratio of book equity to market equity (BE/ME). The size breakpoints for year $t$ are the quintiles of the NYSE market equity at the end of June of $t$. BE/ME for June of year $t$ is the book equity for the recent fiscal year end in $t - 1$ divided by ME for December of $t - 1$.

Again, the BE/ME breakpoints are quintiles of the NYSE. According to the data library of Kenneth French the portfolios for July of year $t$ to June of $t + 1$ include all NYSE, AMEX, and NASDAQ stocks for which they have market equity data for December of $t - 1$ and June of $t$, and (positive) book equity data for $t - 1$.

4. Factor construction and descriptive statistics

In this section, we first characterize the variables included in the macroeconomic factor model following Chen et al. (1986). While the factors capture the same economic effects, the construction however differs in certain cases. The factors from Fama and French (1993) need no description because the data are well documented in the empirical literature and on Ken French’s website. Second, we characterize the factors related to uncertainty, which are the main focus of this paper: the VIX volatility index, our exponential GARCH model estimate, and the monetary uncertainty factor by Baker, Bloom, and Davis (2016). For our robustness tests we include three additional uncertainty factors: the macroeconomic uncertainty index as developed by Jurado, Ludvigson, and Ng (2015), the variance risk premium and the MOVE index. These factors are also briefly discussed in Section 4.2. Third, we show some descriptive statistics of our pricing factors.

4.1. Macroeconomic factors in the spirit of Chen et al. (1986)

The following variables are used in the specification of the CRR macroeconomic factor model:

**Industrial production (MP, YP)**

In order to construct the macroeconomic factor for industrial production we calculate the monthly and yearly growth rate of the U.S. Industrial Production series. The data is obtained from Datastream. Monthly industrial production growth rates (MP) are computed by

$$MP_t = \log(PI_t) - \log(PI_{t-1}).$$ (8)

Notice that IP is a flow variable compiled in the middle of the month. Hence, it reflects last month’s industrial production and lags behind the rest of the factors. Furthermore, CRR point out that the monthly growth rate in industrial production is noisy enough and can therefore be interpreted as innovation of the economy.

To construct the yearly industrial production growth rate (YP) we follow CRR and take the difference in the logarithm with a lag of 12 as

$$YP_t = \log(IP_t) - \log(IP_{t-12})$$ (9)

displays. CRR use the yearly growth rate in order to capture the risk of stock market reactions on long-run changes in industrial production. The time series of MP exhibits an annual seasonal component. However, in our robustness tests we use a seasonally adjusted industrial production factor (MP-SA) to analyze the impact of seasonal adjustment on the price of risk. YP does not contain a seasonal component by construction.

**Inflation factors (UI, DEI)**

We follow CRR and construct two time series in order to capture the potential risk premium from inflation risk. In essence, we derive an unanticipated inflation factor (UI) and a factor that displays the change in expected inflation (DEI). We calculate the inflation factors in two ways. In our major (benchmark) major model we use survey data, whereas in our robustness test we use Treasury Inflation Protected Securities (TIPS) to calculate the expected inflation.

The variable UI is derived as the difference between the actual inflation in $t$ and the expected inflation in $t - 1$ following

$$UI_t = I_t - \mathbb{E}[I_{t-1}].$$ (10)

The actual inflation $I_t$ is derived as the first difference in the logarithm of the Consumer Price Index (CPI). Expected inflation is however calculated differently than in CRR and their reference in Fama and Gibbons (1984) because their approach leads to unstable results. Our estimate is based on the monthly Surveys of Consumers by the University of Michigan, which releases survey data on expected consumer price changes for the next 12 months. Since we need monthly expectations in our analysis, we transform the yearly into monthly expectations.

For the construction of the DEI factor, i.e. the first differences of the monthly survey data, we fit an AR(1) model to the initial survey data as the partial ACF cuts off after lag 1. We then use the innovations from the AR(1) process as our DEI factor.

In our robustness test, we alternatively use a market-based proxy from TIPS by calculating the yield difference as the difference between the treasury bond yield and the Treasury Inflation Protected Securities (TIPS) yield:

$$\mathbb{E}[I_t] = Treasury\ Bond\ yield_t - TIPS\ yield_t.$$ (11)

A TIPS is issued by the U.S. Treasury and provides protection against inflation. While getting interest payments twice a year over 5, 10 and 30 years, the principal of TIPS is linked to the inflation rate as measured by the CPI. We take 5-year maturities of TIPS and U.S. Treasury bond yields respectively. Since the interest payments are made every six months until maturity, we assume that the short-term as well as the long-term expectations are reflected in the TIPS (and the treasury bond). CRR on the other hand get monthly expectations by their approach. Therefore, we have to convert the annualized inflation expectations to monthly data. The DEI-proxy is then computed by

$$DEI_t = \mathbb{E}[I_{t+1} | t] - \mathbb{E}[I_t | t-1].$$ (12)

The TIPS method is not used in the major (benchmark) model because the data is only available from 1997 and does not cover our whole time period which starts in 1990. According to CRR, DEI may capture additional information not present in UI; specifically, “this would occur whenever inflation forecasts are influenced by economic factors other than past forecasting errors” (p. 388).

**Risk premia (UPR)**

We follow CRR and incorporate a factor to capture the effect of unanticipated changes in credit risk premia, specified by

$$UPR_t = CBR_t - LGB_t.$$ (13)

representing the difference between corporate bond returns (CBR) and the long-term government bond return (LGB). The test by Dickey and Fuller (1979) (ADF) implies that the UPR series is not stationary. CRR argue that UPR should capture the unanticipated change in the degree of risk aversion while revealing the level of risk implicit in the market’s pricing of stocks. We take the first difference of the variable as our risk factor in all our tests.

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5 See footnote 4.

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We use U.S. government zero coupon bonds with a maturity of 10 years in order to approximate LGB. This time series is obtained from Datastream. Thereby, we deviate from the government bond portfolio that is used in the CRR analysis. In order to approximate the corporate bond return, we use the Merrill Lynch U.S. corporate bond index yield with a maturity of 7–10 years containing only BBB and lower rated bonds.

**Term structure (UTS)**

The slope of the term structure is proxied by the difference between a long-term and a short-term interest rate,

\[
UTS_t = LGB_t - TB1_{t-1}.
\]  

(14)

We measure UTS as the difference between the long-term government zero bond with a maturity of 10 years (LGB) and the 1-month treasury bill (TB1). In order to get a stationary time series we take first differences of UTS as our risk factor in all our pricing tests. Therefore, UTS captures the unanticipated excess returns on long-term government bonds.

**TED spread (TED)**

The TED is an indicator of global illiquidity in the financial system and is defined by

\[
TED_t = LIBOR3_t - TB3_t,
\]  

(15)

The TED is calculated by subtracting the 3-month treasury bill (TB3) from the 3-month LIBOR rate (LIBOR3). According to the ADF test, this time series is not stationary. Therefore, we take first differences. The TED spread is a measure of the credit risk of the global banking system and hence a measure of the liquidity of the financial system. The change of the TED spread is high during turbulent economic times. A positive shock indicates higher default probability in the interbank market and vice versa.

**Consumption (CG)**

Following CRR, we incorporate a consumption factor which captures the effects of real consumption growth on stock returns. As in the CRR model, we exclude durables, however we include service flows in our consumption factor. We use the consumption expenditures data from the Bureau of Economic Analysis (NIPA tables) which we obtained from Datastream. In contrast to CRR we compute the real per-capita growth rates following the methodology of Kroencke (2017). We adjust the nominal values for inflation to get real magnitudes using the personal consumption expenditure index (PCE). In a further step we divide the real consumption series by the monthly population estimates of the U.S. to get per-capita magnitudes. The consumption data is seasonally adjusted.

**Oil prices (OG)**

As in CRR, the macroeconomic risk of commodity-markets is proxied by the first differences in the logarithm of oil prices (OG)

\[
OG_t = \log(WTI_t) - \log(WTI_{t-1}).
\]  

(16)

By doing so we test whether sensitivities of stocks relative to oil prices are rewarded by a risk premium. Therefore, we use the WTI petroleum series obtained from Datastream.

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**Stock market index (VW)**

CRR point out that most macroeconomic series, through the applied smoothing and averaging procedures, might not be able to capture all the relevant information which is available to market participants. Stock prices, however, react very strongly to public information. CRR argue that market returns will be very noisy relative to innovations in macroeconomic factors and should therefore improve their results in terms of a stronger relationship between portfolio returns and changes in macroeconomic factors. To capture this relative pricing influence we incorporate the total return of the value weighted NYSE/NYSEMKT/Nasdaq/Arca market index (VW). VW is obtained from CRSP and “should reflect both the real information in the industrial production series and the nominal influence of the inflation variable” (Chen et al., 1986, p. 390).

**4.2. Factors related to uncertainty**

The main objective of this paper is to incorporate different volatility or uncertainty factors in macroeconomic and firm-characteristic asset pricing models and test whether these factors are priced and are able to reduce the pricing errors. The first factor is the VIX, an index representing implied volatility of equity index options. Second, we estimate an exponential generalized conditional volatility model (EGARCH) originating from Nelson (1991). The third factor is the monetary uncertainty index by Baker et al. (2015). The three factors are displayed in Fig. 1. We estimate the EGARCH factor for each subperiod of the analysis separately. Additionally, we include the macroeconomic uncertainty index by Jurado et al. (2015), the variance risk premium, i.e. the difference between expected and realized variance, and finally the MOVE Index into our robustness tests. The three robustness factors are displayed in Fig. 2.

**Implied volatility index (VIX)**

The VIX is a market-based estimate of the expected volatility of the S&P500 index and is calculated from at-the-money index options traded at the Chicago Board Options Exchange (CBOE). It is calculated from real-time, midpoint data of the bid/ask spreads of S&P500 index options (SPX) which are interpolated such that the index represents the implied volatility for the subsequent 30 days. This data series is obtained from Bloomberg and is available since January 1990.

**Exponential GARCH (EG)**

There is a lot of empirical evidence that GARCH models perform well in characterizing daily stock return processes. We use daily differences of implied conditional volatilities as a short-term volatility factor. Our GARCH model is estimated from daily returns of the CRSP value weighted total return index using three specifications: GARCH(1,1) as suggested by Bollerslev (1986), EGARCH(1,1) by Nelson (1991) and GJR-GARCH(1,1) by Glosten, Jagannathan, and Runkle (1993). Based on the Akaike (1974) and Schwarz (1978) information criterions (AIC, BIC) of the fitted model in Table 1, the EGARCH(1,1) model

\[
\log(\sigma_t^2) = \omega + \alpha(|z_{t-1} - \mathbb{E}(z_{t-1})|) + \gamma z_{t-1} + \beta \log(\sigma_{t-1}^2),
\]  

(17)

fits the return process best. The parameter \(z_t\) is defined as \(z_t \equiv \varepsilon_t \sigma_t^{-1}\) and denotes the standardized innovations. A characteristic of the model is that it allows for asymmetry in the relationship between return and volatility. Consistent with the literature we estimate \(\gamma < 0\) which implies that negative shocks have a stronger impact on future volatility than positive shocks. In the context of equity returns, this effect is known as “leverage” effect although, as pointed out by Bollerslev, data obtained from FRED St. Louis.

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8 Recent evidence of this largely overlooked claim is provided by Kroencke (2017) in the context of recovering non-smoothed consumption data in tests of the C–CAPM.

9 The details of the computation can be found at http://www.cboe.com/vix.
et al. (2006).

Long-run volatility (30 days) and the change in the EGARCH volatility forecasts. We interpret the change in the VIX as a change in expected leverage of firms.

Russell, and Watson (2010), it has little to do with the actual financial leverage of firms.

Following Ang et al. (2006), the two pricing factors—VIX and EGARCH—are specified as first differences of end-month volatility forecasts. We interpret the change in the VIX as a change in expected long-run volatility (30 days) and the change in the EGARCH volatility as a change in expected short-run volatility (1 day). Another approach would be to sum up and average the monthly differences over each month as done by Adrian and Rosenberg (2008). However, we choose to use the first approach. No matter which approach is selected, there is an obvious loss in information when using monthly volatility forecasts in fitting asset pricing models.

The two volatility factors as well as their differences are displayed in Fig. 1. The implied volatility has a mean of 5.57% p.m., while the mean EGARCH volatility is 4.34% p.m. Compared to the mean, the median of both volatilities are roughly 0.5% p.m. lower which points to the extreme observations in 1998 (Russian crisis), 2001 (Dot-Com crisis), 2008 (Global Financial crisis) and 2011 (Euro crisis); see Fig. 1. The standard deviation of the volatility are roughly the same at 2.2% p.m. The smallest value of the VIX is 2.75% p.m. and for the EGARCH model at 1.57% p.m. while the maximum value for the VIX and EGARCH is at 17% p.m.

**Monetary policy uncertainty (MPU)**

Extensive research suggests that volatility risk of the stock market is related to monetary policy shocks. In order to disentangle monetary policy uncertainty from stock market volatility as captured by the VIX and our EGARCH factor, an additional risk factor is included in our extended model which proxies uncertainty about monetary policy (MPU).

Among the large body of research, the papers by Gali and Gambetti (2015) and Rigobon and Sack (2003) are particularly suggestive. Unlike previous studies that analyze the impact of monetary shocks on stock market returns, Rigobon and Sack (2003) examine the response of FED’s monetary policy to stock market shocks. They find that a 5% rise (fall) in the S&P500 index makes a 25 basis point tightening (easing) more likely by 50%. Gali and Gambetti (2015) test the reaction of bubbly stock prices in response to an exogenous tightening of monetary policy and an associated increase in interest rates. When allowing for an endogenous, contemporaneous response of interest rates to stock prices their findings confirm the previous result in the sense that a 10% rise in stock prices triggers a 20 basis points rise in the federal funds rate. These results strongly suggest that stock market volatility should be complemented by a simultaneous factor measuring monetary policy uncertainty.

As discussed in Section 2, Brogaard and Detzel (2015) find that economic policy uncertainty (EPU) has a significant risk premium. Therefore, we follow their approach and test whether the respective sub index, the monetary policy uncertainty index by Baker et al. (2016) shows similar results in the cross section of stock returns. An alternative choice for capturing the effect of monetary uncertainty would be in a market-based measure such as implied volatility of 3-months Eurodollar options. However, there are unsolved issues and non-documented, apparent errors associated with this time series. Therefore, we use the monetary policy uncertainty index from Baker et al. (2016) as our proxy. This index is built by counting newspaper articles that satisfy the E, P, U, M criteria. This means that articles are flagged that contain the following keywords:

**Table 1**

<table>
<thead>
<tr>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.074</td>
<td>1.088</td>
<td>−0.363</td>
<td>11.819</td>
</tr>
</tbody>
</table>

This table reports the summary statistics of the daily total return series and parameter estimations of the GARCH(1,1), EGARCH(1,1) and the GJR(1,1) model. Furthermore statistics for the goodness of the model fit are provided as well as statistics for the residual analysis. The used data is obtained from CRSP. The daily series is used from January 1990 to November 2018.

---

6 This is similar to the summation of the daily differences as done in Ang et al. (2006).

10 This data is, in principle, available from the Chicago Mercantile Exchange (CME).
economic indicators uncertainty is derived from the aggregated forecast errors of a set of become more or less variable or disperse per se, but rather whether decision making is not whether particular economic indicators have uncertainty that builds on the idea that “what matters for economic The authors provide a new, compound measure of macroeconomic uncertainty (MU) index developed by Jurado et al. (2015). Macroeconomic uncertainty (MU) ''Factiva'' uncertainty than an article in a Hawaiian newspaper called “Big Island for example, that a news piece on Bloomberg TV about the economic crisis 2007. is worth noticing that unlike the VIX and EGARCH volatility measures, the index; the time series, levels and log differences, are shown in 1. It We incorporate MPU by taking the first logarithmic differences of the MPU index because the data is updated until recently and makes it possible to estimate our models with recent data. We incorporate MPU by taking the first logarithmic differences of the index; the time series, levels and log differences, are shown in 1. It is worth noticing that unlike the VIX and EGARCH volatility measures, the peaks of the MPU index are not only apparent in market turmoil and recessions, but persist e.g. in the aftermath of the Global Financial crisis 2007. A disadvantage of the MPU index is that this is a simple count of newspaper articles. Hence, the article’s sentiment is not included. Moreover, this leaves out all other media channels such as television, radio, business information systems, and social media. One could argue, for example, that a news piece on Bloomberg TV about the economic policy uncertainty in the United States is more important to measure uncertainty than an article in a Hawaiian newspaper called “Big Island Weekly”. Furthermore, it is worth mentioning that business journals are heavily underweighted in this index compared to “regular” news journals. Therefore it might be advantageous to use the news database “Factiva”12 as this database has more business journals included.

Macroeconomic uncertainty (MU)

For testing the robustness of our extended model against the inclusion of additional uncertainty factors, we first include the macroeconomic uncertainty (MU) index developed by Jurado et al. (2015). The authors provide a new, compound measure of macroeconomic uncertainty that builds on the idea that “what matters for economic decision making is not whether particular economic indicators have become more or less variable or disperse per se, but rather whether the economy has become more or less predictable” (p. 1178). Thus, uncertainty is derived from the aggregated forecast errors of a set of economic indicators \( y_j \in Y_j = \{ y_{1j}, \ldots, y_{N_{ij}} \} \). For the jth variable, the \( h \)-period ahead uncertainty is defined by

\[
U_{j,h}(h) \equiv \sqrt{\mathbb{E}(y_{j+h} - \mathbb{E}[y_{j+h} | I_t])^2 | I_t}, \tag{18}
\]

and represents the conditional volatility of the purely unforecastable component of the future value of the series.\(^{13}\) \( \mathbb{E} ( \cdot | I_t) \) stands for the information \( I \) given in \( t \). In order to construct an index of macroeconomic uncertainty, the authors aggregate the \( j \) individual prediction errors at each date using aggregation weights \( w_j \):

\[
U_{j,h}^j(h) \equiv \text{plim}_{N_{ij} \to \infty} \sum_{j=1}^{N_{ij}} w_j U_{j,h}^j(h) \equiv E_q[U_{j,h}^j(h)]. \tag{19}
\]

The individual uncertainties are weighted equally. Throughout the forecasting procedure the factors are estimated using a static principal component analysis (PCA). The authors emphasize two features of their approach (Eqs. (18) and (19)). First, the importance to differentiate between the uncertainty in a series \( y_j \) and its conditional volatility after removing the forecastable component \( \mathbb{E}[y_{j+h} | I_t] \). Second, the distinction of macroeconomic uncertainty versus the uncertainty reflected in single series \( y_j \), because “uncertainty-based theories of the business-cycle typically require the existence of common (often counter cyclical) variation in uncertainty across large numbers of series” (Jurado et al., 2015, p. 1179).

Their macroeconomic uncertainty index (MU) is composed from a data set which contains 132 macroeconomic time series such as: real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, capacity utilization measures, price indexes, bond and stock market indexes, and foreign exchange measures.\(^{14}\) We perform an ADF unit root test and use first differences of the macroeconomic uncertainty index as proxy for the innovations of the series.

Variance risk premium (VRP)

The second uncertainty factor included in our robustness tests is a stock market related variance measure referred to as “variance risk premium” by Bekaert and Hoerova (2014) which is defined by

\[
V_{VRP} = V_{IX}^2 - RV^{(22)}_{t+1}, \tag{20}
\]

where the realized variance \( RV^{(22)} \) is equal to \( \sum_{t=1}^{T} RV_t \). While the VIX factor proxies the expected volatility, the variance risk premium measures the unconditional unexpected variance in the stock market. This factor makes it possible to disentangle expected from unexpected changes in volatility risk. Alternatively, the VRP is often interpreted as a proxy for the economy’s risk aversion as it is the difference between risk neutral probability measure (VIX) the actual “physical” probability measure and indicates whether the economy is in a bad state with high marginal utility of money; see Bekaert and Hoerova (2014). The data can be retrieved from Heber, Asger, Shephard, and Sheppard (2009).\(^{15}\) Note that this data is only available from January 2000, hence not over the full sample period.

ICE BofA US bond market option volatility estimate index (MOVE)

Lastly, in order to discriminate stock market volatility, monetary policy uncertainty and macroeconomic uncertainty from expected (implied) long run interest rate volatility risk, we use the ICE BofA US Bond Market Option Volatility Estimate Index (MOVE). The MOVE index is a well-recognized indicator of US interest rate volatility and is widely used in practice; it is often referred to as the “VIX for Bonds”. The MOVE measures the implied yield volatility of a basket of one-month over-the-counter options on 2-year, 5-year, 10-year and 30-year Treasuries. The MOVE is provided by the Intercontinental Exchange

\(^{13}\) Jurado et al. (2015) calculate their indices for three forecasting horizons, 1 month, 3 months and twelve months. In this paper, we use their 1 month period.\(^{14}\) The full list of variables is displayed in the online appendix of Jurado et al. (2015) https://www.sydneyludvigson.com/data-and-appendixes.\(^{15}\) https://realized.oxford-man.ox.ac.uk/data.
EGARCH (EG) with the market factor, i.e. – reveals that the correlations between the uncertainty factors VIX and macroeconomic factors and the uncertainty factors in Panel B. Panel A and the uncertainty factors in Panel A, and the correlations between the residual factors are rather small. Again, only factors which are available for the full sample period are displayed in Table 2.

This table reports the mean, median, minimal- and maximal values, standard deviation, skewness and kurtosis for the uncertainty factors, VIX, EGARCH (EG), monetary policy uncertainty (MPU), macroeconomic uncertainty (MU), MOVE index (MOVE), the Fama–French factors, excess market return (RP), size (SM), value (VM), momentum (UM), profitability (RMW), investment (CMA) and the macroeconomic CRR factors, industrial production (MP), oil price (OG), consumption (CG), TED spread (TED) and value weighted equity return (WV). Construction of the Factors can be found in Sections 4.1 and 4.2.

### 5. Empirical results

In this section, the empirical results are presented. We first discuss our major models which include the basic FF- and CRR-models expanded by the VIX volatility factor (Section 5.1). Next, these models are expanded by the newly specified uncertainty factors, i.e. EGARCH volatility (EG) and the monetary policy uncertainty index (MPU) (Section 5.2). Finally, we test the robustness of our findings with an alternative estimation approach using GLS, with additional control factors, additional uncertainty factors and for different subperiods in Section 5.3. With the exception of the pricing test for different subperiods, all pricing results are displayed for the 48 industry sector- and 25 size/value sorted portfolios.

#### 5.1 Major models

We start by running the basic model by FF and CRR and expand both models by the VIX volatility factor. These are our “major” models in the subsequent discussion. Table 4 displays the estimation results of the basic models in columns (1), (3), (5), and (7), while the results of the VIX-expanded major models can be found in columns (2), (4), (6), and (8).

When looking at the results in Table 4, columns (1) to (4), we see that the risk premium of the market factor is always statistically significant; the estimated market risk premiums are between 0.6% to 0.75% per month regardless of the used test portfolios and inclusion of the VIX. On the one hand, the risk premium of this traded factor is close to the time series average shown in Table 2. On the other hand, the risk premiums of the size- and value factors are highly sensitive to the VIX. On the one hand, the risk premium of this traded factor is close to the time series average when using the 25 size/value sorted portfolios, the risk premiums are vastly different when running the tests with the 48 industry sector portfolios. However, the market price of risk
of the size- and value factor are not consistently significantly different from zero.

From Eqs. (5) to (8) in Table 4 we observe that the risk premiums of the factors of the basic CRR model are not consistently priced. Again, the size and significance of the risk premiums change with respect to the tested portfolios. For example, industrial production (MP) has a significant negative risk premium in Eq. (5) but not in the basic model when testing with the 25 size/value sorted portfolios in Eq. (7). We conclude from these results that the inclusion of the VIX factor improves fundamental-based as well as macroeconomic-based asset pricing models. This is consistent with observed investor behavior: in their attempt to hedge against adverse changes in investment opportunities in the spirit of Merton (1973), risk averse investors overweight stocks in their portfolios which exhibit high returns in high volatility states. Models with volatility as a systematic risk factor can found in Campbell (1993, 1996) or Chen (2002). This hedging pattern is reinforced by the observation of Campbell and Hentschel (1992) and others that periods of high volatility coincide with stock market downturns. Therefore, assets with a positive sensitivity to market volatility risk provide a hedge against market downside risk (Bakshi & Kapadia, 2003). Investors are willing to hold these stocks at low or, as our results reveal, negative risk premiums. Since most stocks exhibit negative volatility betas, their expected return on volatility risk is mostly positive.

5.2. Extended models

In this section, the major models of the previous section (i.e. the basic FF and CRR models plus VIX) are expanded by two additional factors of economic uncertainty apart from the VIX, abbreviated by EG (EGARCH-based short-run volatility) and MPU (monetary policy uncertainty). The estimation results in Table 5 indicate that the market factor is consistently and significantly priced but also insensitive with respect to the tested portfolios and the added uncertainty factors as well. As in the previous section, the size- and value factors are not consistently priced and highly sensitive to the choice of the test portfolios.

Adding the change in expected short-run volatility (EG) to our model reveals a negative price of risk in all model specifications but it is only statistically significant if the 25 size/value sorted test portfolios

<table>
<thead>
<tr>
<th>A. Fama/French Factors</th>
<th>RPM</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
<th>RMW</th>
<th>CMA</th>
<th>VIX</th>
<th>EG</th>
<th>MPU</th>
<th>MU</th>
<th>MOVE</th>
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<th>UI</th>
<th>UPR</th>
<th>UTS</th>
<th>OG</th>
<th>CG</th>
<th>TED</th>
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<td>0.38</td>
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<td>0.17</td>
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</table>
As in the previous Section 5.1 the VIX has a consistent negative market price of risk but also insensitive to additional uncertainty factors and the used test portfolios. The pricing errors are displayed. The test statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with \( p < 0.10, \) ** \( p < 0.05 \) and *** \( p < 0.01 \). The risk premiums are displayed in percent. The sample period is defined from January 1990 to November 2018 using monthly data.

Table 4

<table>
<thead>
<tr>
<th>Fama-French 3 Factor Model</th>
<th>48 industry sectors + 6</th>
<th>25 size/value + 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{	ext{MPU}} )</td>
<td>0.746***</td>
<td>0.722***</td>
</tr>
<tr>
<td>( \lambda_{	ext{MPU}} )</td>
<td>[0.143]</td>
<td>[0.019]</td>
</tr>
<tr>
<td>( \lambda_{	ext{VIX}} )</td>
<td>-0.347**</td>
<td>-0.450**</td>
</tr>
<tr>
<td>( \lambda_{	ext{MPU}} )</td>
<td>[-2.300]</td>
<td>[-2.741]</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>FF3 factor model expanded by the VIX.</th>
</tr>
</thead>
<tbody>
<tr>
<td>48 industry sectors +6</td>
</tr>
<tr>
<td>25 size/value +6</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums of the FF3 factor- and CRR model with the addition of the VIX (a description of the variables can be found in the caption of Table 2). Eqs. (1), (3), (5), and (7) serve as the benchmark pricing of the basic FF3 factor- and CRR model for 25 size/value- and 48 industry sector portfolios \(+6\) traded factors respectively. Eqs. (2) to (6) and (8) to (12) display the models with the separate and combined extensions including the VIX. Furthermore for each pricing test the respective test of joint \( \alpha \)'s and its \( p \)-value are displayed. The \( r \)-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with \( p < 0.10, \) ** \( p < 0.05 \) and *** \( p < 0.01 \). The risk premiums are displayed in percent. The sample period is defined from January 1990 to November 2018 using monthly data.

Table 4

<table>
<thead>
<tr>
<th>FF3 factor- and CRR model expanded by the MPU-index.</th>
</tr>
</thead>
<tbody>
<tr>
<td>48 industry sectors +6</td>
</tr>
<tr>
<td>25 size/value +6</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums of the FF3 factor model with the addition of the MPU-index (a description of the variables can be found in the caption of Table 2). Eqs. (1), (3), (5), and (7) serve as the benchmark pricing of the basic FF3 factor- and CRR model for 25 size/value- and 48 industry sector portfolios \(+6\) traded factors respectively. Eqs. (2) to (6) and (8) to (12) display the models with the separate and combined extensions including the MPU-index. Furthermore for each pricing test the respective test of joint \( \alpha \)'s and its \( p \)-value are displayed. The \( r \)-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with \( p < 0.10, \) ** \( p < 0.05 \) and *** \( p < 0.01 \). The risk premiums are displayed in percent. The sample period is defined from January 1990 to November 2018 using monthly data.

are used. Here, the EG factor exhibits a negative risk premium between 0.51% and 0.56% p.m. Eqs. (11) and (12) reveal that the short-run (EG) and long-run (VIX) volatility factors are both significantly priced with the magnitude of approximately \(-0.5\) p.m. if simultaneously included in the model. This is an important finding and indicates that investors perceive expected short-run- and long-run volatility as separate risk factors.

With the exception of Eq. (9) in Table 5, changes in monetary policy uncertainty as measured by the changes of the MPU-index are not significantly priced. As in the previous Section 5.1 the VIX has a consistent negative market price of risk but also insensitive to additional uncertainty factors and the used test portfolios. The pricing errors do not drop considerably when adding EG and MPU. The test values of the joint alpha test are considerably lower in the pricing tests for the 48 industry sector portfolios compared to the 25 size/value sorted portfolios.

In Table 6 the marginal explanatory effects of the two uncertainty factors (EG and VIX) are tested in the macroeconomic-based CRR model setting. When testing the 48 industry sector portfolios the basic CRR risk factors are not consistently priced. However, in Eq. (3) industrial production (MP) and term structure (UTS) inherit a negative risk premium when no volatility factor (EG and VIX) is included. The test with the 25 size/value sorted portfolios shows rather different results. Even though the CRR-factors are still not consistently priced, we see more cases with significantly priced factors. For example, in Eq. (8) through (10) the inflation factors, change in expected inflation and unexpected inflation (DEI and UI) exhibit a significantly priced negative premium. The risk factor UPR is significantly priced in Eqs. (11) and

---

16 Unsurprisingly the risk premium is rather high as the changes of the MPU-index are large, therefore the estimated betas are small in regression step one and hence the lambdas are accordingly large.
Premiums are highly sensitive to the used test portfolios. This finding is line with many other empirical studies. In contrast to this general finding, however, the short-run- and long-run volatility factors show quite a stable pricing pattern. Still, throughout all equations, the pricing errors in Table 6 are different from zero at all conventional significance levels. The factor risks seem to be priced much better by the extended CRR models.

Overall, the results in Table 6 indicate that the estimated risk premiums are highly sensitive to the used test portfolios. This finding is in line with many other empirical studies. In contrast to this general finding, however, the short-run- and long-run volatility factors show quite a stable pricing pattern. Still, throughout all equations, the pricing errors in Table 6 are different from zero at all conventional significance levels. The factor risks seem to be priced much better by the extended CRR models.

Regarding the volatility patterns suggested at the end of Section 5.1, the results in Tables 5 and 6 imply the following insights: First, the long-run volatility factor VIX is in line with many other empirical studies. In contrast to this general finding, however, the short-run- and long-run volatility factors show quite a stable pricing pattern. Still, throughout all equations, the pricing errors in Table 6 are different from zero at all conventional significance levels. The factor risks seem to be priced much better by the extended CRR models.
variables can be found in the caption of Table 2). Eqs. (1) and (7) serve as the benchmark pricing of the basic CRR model with the risk of a mimicking factor portfolio.

Lastly, we also test the robustness of our findings with an alternative proxy for expected inflation by using a market-based proxy instead of survey data. A natural choice are TIPS (Treasury Inflation Protected Securities). However, as TIPS data is only available since the late 1990s, we have to restrict our estimation period from August 1998 to November 2018 with a single breakpoint at the Global Financial crisis (see Table 14). All calculations involving sub-periods (except the robustness test with the additional uncertainty factors) are only performed with the 48 industry sector portfolios.

### Estimation with GLS

Tables 7 and 8 display estimation results based on GLS regressions for the extended FF- and CRR-models. GLS is suggested as an alternative to OLS by Lewellen et al. (2010) for different reasons. In our context, GLS forces the traded factors, which are included in our asset universe, to be priced perfectly which is not the case for cross-sectional OLS regressions. This is a natural implication of no-arbitrage pricing. Moreover, the interpretation of the estimated GLS regression parameter of a non-traded factor can be easily interpreted as the market price of risk of a mimicking factor portfolio.

For the FF specification, the results (see Table 7) reveal that the risk premium of the market factor is smaller than in the OLS model when using the 48 industry sector portfolios, but remain highly significant as opposed to both remaining factors. Interestingly, the estimated risk premia are much closer to the values estimated from the 25 size/value sorted portfolios. The VIX-factor remains statistically significant when included individually, but its significance vanishes in Eqs. (11) and (12) when tested jointly with the EG factor using the 25 size/value sort. Notably, in contrast to the OLS regressions, the short run EGARCH volatility factor reveals a negative, statistically significant risk premium across all estimations amounting at some –0.38% p.m. In contrast to the OLS regressions, the magnitude of the VIX premium is reversed with regard to the portfolio sort: for the size/value sort, the premium falls to approximately –0.2% p.m. while the premium using the industry sector sorts increases to –0.46% p.m.

For the sake of comparability, we also run the CRR models with GLS (see Table 8). The general observation is that the number of significant pricing effects for the original CRR factors increases when applying GLS, but the results still depend on the underlying portfolio...
As mentioned in Section 5.1 the industrial production factor (MP) and unexpected inflation (UI) are significantly priced in the 48 industry sector sort: while industrial production (MP) and unexpected inflation (UI) are significantly priced in the 48 industry sector sort, almost all CRR-factors (with the exception of UTS) are priced significantly using 25 size/value portfolios. The pricing of the VIX- and EGARCH volatility factors in the CRR-model is consistent with the FF-model in terms of statistical significance and the size of the premium. The market price of short-term volatility risk, however, differs across the estimated equations. The pricing effect of monetary policy uncertainty (MPU) remains mixed with respect to the models and portfolio sorts, and mostly insignificant as in the OLS case.

The overall conclusion is that the VIX and EG volatility factors capture different economic effects and should therefore be included as two separate factors as well as simultaneously in asset pricing models.

**Alternative specification of variables**

Table 9 displays the risk premiums for alternative specifications of the FF and CRR model framework using the 48 industry sector portfolios. The FF 3-factor model is expanded by the momentum factor by Carhart (1997) and the profitability- and investment- factor by Fama and French (2015). Overall, the additional factors do not have considerable impact on the risk premiums of the volatility factors. The sign of both volatility factors remains negative and only the VIX premium is significant. The three additional factors are not significantly priced and do not affect the risk premium of the factors of the basic model.

The CRR model is estimated with the following alternative, respectively, additional factors: the year-on-year growth rate of industrial production (YP), the growth rate of oil prices (OG) and consumption (CG), the TED spread and the value weighted equity return (VW). As mentioned in Section 5.1 the industrial production factor (MP) requires some comments. In contrast to Chen et al. (1986) and Shanken and Weinstein (2006) our estimates show a negative sign for the MP factor, which contradicts intuition and the findings of two referenced papers as well. However, our MP-variable is not seasonally adjusted. From the additional factors included in the CRR model, the value and size factors are no longer significant in any equation. In Eqs. (1), (2) and (4) the value factor, which contradicts intuition and the findings of two referenced papers as well. However, our MP-variable is not seasonally adjusted.

### Table 9

<table>
<thead>
<tr>
<th>Fama-French 3 factor model</th>
<th>Chen-Roll-Ross model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extended Robustness</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>λ_{RF}</td>
<td>0.722***</td>
</tr>
<tr>
<td></td>
<td>0.762***</td>
</tr>
<tr>
<td></td>
<td>0.689***</td>
</tr>
<tr>
<td></td>
<td>0.724***</td>
</tr>
<tr>
<td>λ_{MP}</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>-0.373</td>
</tr>
<tr>
<td></td>
<td>-0.354</td>
</tr>
<tr>
<td></td>
<td>-0.358</td>
</tr>
<tr>
<td></td>
<td>-0.445</td>
</tr>
<tr>
<td></td>
<td>-0.405</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums of the FF3 factor- and CRR model with the VIX and EGARCH (EG) (a description of the variables can be found in the caption of Table 2) while controlling for additional factors. Eqs. (1) and (5) serve as the benchmark pricing of the basic FF3 factor- and CRR model with the incorporation of the VIX and EG for 48 industry sector portfolios (+ 6 traded factors). Eqs. (2) to (4) control for the additional factors, momentum (UMD), profitability (RMW) and investment (CMA). Eqs. (6) to (11) control for the additional factors, i.e. the seasonally adjusted monthly growth rate of industrial production (MP-SA), yearly growth rate of industrial production (YP), oil price (OG), consumption (CG), TED spread (TED), value weighted equity return (VW). Furthermore for each pricing test the respective test of joint a’s and its p-value are displayed. The t-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with * (+0.05), ** (+0.01) and *** (+0.001). The risk premiums are displayed in percent. The sample period is defined from January 1990 to November 2018 using monthly data.

**sort:**

- while industrial production (MP) and unexpected inflation (UI) are significantly priced in the 48 industry sector sort, almost all CRR-factors (with the exception of UTS) are priced significantly using 25 size/value portfolios. The pricing of the VIX- and EGARCH volatility factors in the CRR-model is consistent with the FF-model in terms of statistical significance and the size of the premium. The market price of short-term volatility risk, however, differs across the estimated equations. The pricing effect of monetary policy uncertainty (MPU) remains mixed with respect to the models and portfolio sorts, and mostly insignificant as in the OLS case.

The overall conclusion is that the VIX and EG volatility factors capture different economic effects and should therefore be included as two separate factors as well as simultaneously in asset pricing models.

**Alternative specification of variables**

Table 9 displays the risk premiums for alternative specifications of the FF and CRR model framework using the 48 industry sector portfolios. The FF 3-factor model is expanded by the momentum factor by Carhart (1997) and the profitability- and investment- factor by Fama and French (2015). Overall, the additional factors do not have considerable impact on the risk premiums of the volatility factors. The sign of both volatility factors remains negative and only the VIX premium is significant. The three additional factors are not significantly priced and do not affect the risk premium of the factors of the basic model.

The CRR model is estimated with the following alternative, respectively, additional factors: the year-on-year growth rate of industrial production (YP), the growth rate of oil prices (OG) and consumption (CG), the TED spread and the value weighted equity return (VW). As mentioned in Section 5.1 the industrial production factor (MP) requires some comments. In contrast to Chen et al. (1986) and Shanken and Weinstein (2006) our estimates show a negative sign for the MP factor, which contradicts intuition and the findings of two referenced papers as well. However, our MP-variable is not seasonally adjusted. From the additional factors included in the CRR model, the value and size factors are no longer significant in any equation. In Eqs. (1), (2) and (4) the value factor, which contradicts intuition and the findings of two referenced papers as well. However, our MP-variable is not seasonally adjusted. If we replace the factor by a seasonally adjusted proxy (MP-SA), the estimated premium in Eq. (6) still exhibits a negative sign, remains insignificant and is smaller in absolute magnitude.

From the additional factors included in the CRR model, the value weighted equity factor VW is significantly priced and has a similar value as the market factor in the FF model (see Eqs. (1) to (4)). Interestingly, the VIX exhibits an even higher premium as in Eqs. (5) to (10). Apart from the VW factor, none of the additional factors is significantly priced and they do not influence the volatility factors. UI is the only factor which occasionally gets a significant premium with the additional factors. Otherwise, the newly added factors are neither significantly priced nor do they have any substantial effect on the size or significance of the existing premiums in the extended CRR model.

In both asset pricing frameworks the pricing errors are significantly different from zero. However, they are considerably smaller in the macroeconomic CRR framework compared to FF.

Testing the same models using the 25 size/value portfolios leads to some major differences in both model frameworks. The results are displayed in Table 10. In the FF model, the value- and size factors are no longer significant in any equation. In Eqs. (1), (2) and (4) the short-run- and long-run volatility factors are priced simultaneously with
Table 10
Fama-French 3 factor model: Robustness, 25 size/value sorted portfolios.

<table>
<thead>
<tr>
<th></th>
<th>Extended</th>
<th>Robustness</th>
<th></th>
<th>Extended</th>
<th>Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>( \lambda_{MP} )</td>
<td>0.596**</td>
<td>0.628**</td>
<td>0.607**</td>
<td>0.616**</td>
<td>( \lambda_{MP} )</td>
</tr>
<tr>
<td></td>
<td>[2.486]</td>
<td>[2.637]</td>
<td>[2.539]</td>
<td>[2.576]</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{SMB} )</td>
<td>0.172</td>
<td>0.153</td>
<td>0.216</td>
<td>0.151</td>
<td>( \lambda_{SMB} )</td>
</tr>
<tr>
<td></td>
<td>[1.046]</td>
<td>[0.934]</td>
<td>[1.316]</td>
<td>[0.925]</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{HML} )</td>
<td>0.118</td>
<td>0.148</td>
<td>0.08</td>
<td>0.101</td>
<td>( \lambda_{HML} )</td>
</tr>
<tr>
<td></td>
<td>[0.622]</td>
<td>[0.778]</td>
<td>[0.427]</td>
<td>[0.534]</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{UMD} )</td>
<td>0.387</td>
<td>0.089</td>
<td>0.008</td>
<td>0.101</td>
<td>( \lambda_{UMD} )</td>
</tr>
<tr>
<td></td>
<td>[1.473]</td>
<td>[0.098]</td>
<td>[0.030]</td>
<td>[0.086]</td>
<td></td>
</tr>
<tr>
<td>( \lambda_{RMW} )</td>
<td>0.28</td>
<td>0.12</td>
<td>0.19</td>
<td>0.151</td>
<td>( \lambda_{RMW} )</td>
</tr>
<tr>
<td></td>
<td>[1.624]</td>
<td>[1.259]</td>
<td>[1.582]</td>
<td>[1.435]</td>
<td></td>
</tr>
</tbody>
</table>

α-test: 477.068 468.585 466.512 472.56  p-value: 0 0 0 0 0

The table reports the monthly risk premiums of the F3 factor- and CRR model with the VIX and EGARCH (EG) (a description of the variables can be found in the caption of Table 2) while controlling for additional factors. Eqs. (1) and (5) serve as the benchmark pricing of the basic F3 factor- and CRR model with the incorporation of the VIX and EG for 25 size/value sorted portfolios (+ 6 traded factors). Eqs. (2) to (4) control for the additional factors, momentum (UMD), profitability (RMW) and investment (CMA). Eqs. (6) to (11) control for the additional factors, i.e. the seasonally adjusted monthly growth rate of industrial production (MP - SA), yearly growth rate of industrial production (YP), oil price (OG), consumption (CG), TED spread (TED), value weighted equity return (VW). Furthermore for each pricing test the respective test of joint α’s and its p-value are displayed. The t-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with *<p<0.10, **<p<0.05 and ***<p<0.01. The risk premiums are displayed in percent. The sample period is defined from January 1990 to November 2018 using monthly data.

Table 11
Fama-French 3 factor model: Subperiods, 48 industry sector portfolios.

Panel A

<table>
<thead>
<tr>
<th></th>
<th>01/1990-11/2018</th>
<th>01/1990-08/1998</th>
<th>09/1998-09/2008</th>
<th>10/2008-11/2018</th>
<th>( \lambda_{MP} )</th>
<th>( \lambda_{SMB} )</th>
<th>( \lambda_{HML} )</th>
<th>( \lambda_{VIX} )</th>
<th>( \lambda_{EG} )</th>
<th>α-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.722**</td>
<td>0.784**</td>
<td>0.255</td>
<td>0.999**</td>
<td>-0.333*</td>
<td>-0.621*</td>
<td>-0.043</td>
<td>-0.406</td>
<td>-0.356**</td>
<td>212.71</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>[3.021]</td>
<td>[1.689]</td>
<td>[0.614]</td>
<td>[2.447]</td>
<td>[-1.689]</td>
<td>[-0.631]</td>
<td>[-0.096]</td>
<td>[-1.500]</td>
<td>[-1.593]</td>
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<td></td>
</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th></th>
<th>01/1990-11/2018</th>
<th>01/1990-08/1998</th>
<th>09/1998-09/2008</th>
<th>10/2008-11/2018</th>
<th>( \lambda_{MP} )</th>
<th>( \lambda_{VIX} )</th>
<th>( \lambda_{EG} )</th>
<th>α-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.56</td>
<td>-0.419</td>
<td>0.673</td>
<td>-0.133</td>
<td>0.057</td>
<td>0.098</td>
<td>0.015</td>
<td>-0.106</td>
<td>-0.359**</td>
</tr>
<tr>
<td></td>
<td>[1.128]</td>
<td>[-0.934]</td>
<td>[0.012]</td>
<td>[-0.470]</td>
<td>[-1.429]</td>
<td>[-0.096]</td>
<td>[-0.046]</td>
<td>[-0.045]</td>
<td>[-0.169]</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums regarding the 48 industry sector portfolios (+ 6 traded factors) for all estimation periods. Panel A reports the risk premium for the basic F3 factor model with the incorporation of the VIX and EG. Panel B reports the risk premium for the CRR model with the incorporation of the VIX and EG (a description of the variables can be found in the caption of Table 2). Furthermore for each pricing test the respective test of joint α’s and its p-value are displayed. The t-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with *<p<0.10, **<p<0.05 and ***<p<0.01. The risk premiums are displayed in percent.
a magnitude of approximately 0.5% p.m. Again, the pricing results are highly sensitive to the used test portfolios, with the exception of VIX and RPM. Eqs. (5) to (11) reveal a similar picture; especially the factors of the basic model and the additional control factors are priced more often compared to Table 9. The factor UPR has a significant risk premium in almost all equations. Furthermore, the inflation factors are significantly priced in Eqs. (6), (7) and (10) and the robustness factors OG, CG and TED are now priced as well. Interestingly the seasonally adjusted industrial production factor (MP-SA) now has the expected positive market price of risk as in Chen et al. (1986) and Shanken and Weinstein (2006). The premium of the value weighted equity return (VW) is still significant but slightly smaller than before. The VIX premium is also significant with a risk premium of approximately 0.2% to 0.4% p.m. larger than in the previous table. As noticed earlier, the pricing errors are considerably higher when using the 25 size/value portfolios.

We finally use a market-based proxy, TIPS, for constructing the inflation risk factor. Since TIPS data is only available after 1998, the results cannot be directly compared with those in the preceding tables. Therefore, they are presented and discussed separately at the end of this section.

Revisiting the volatility patterns suggested at the end of Section 5.1, the results in Tables 9 and 10 lead to the following insights: Again, the portfolio sort has a substantial impact on both model frameworks and not just the FF model, which supports our findings in the previous section. Second, the results support our observation that the volatility factors are more uniformly priced in the tests using the 48 industry sector portfolio sorts. Our third hypothesis, that the volatility risk premiums are larger (in absolute terms) when sorting by size- and value, is also supported by the results in this section.

Subperiods

The third part of our robustness tests includes the estimation of the risk premiums in different subperiods. The results for the extended FF- and CRR models are displayed in Table 11 using the 48 industry sector portfolios as test assets. Panel A of Table 11 contains the results using the extended FF model. They reveal that the market factor, which is highly significant in the full period, is not significantly priced in the second subperiod (09/1998–09/2008). Moreover, the size factor is only priced in the first subperiod (and even has a positive sign in the second subperiod). The short-run and long-run volatility factors keep their negative sign through all periods. The VIX factor has a significant risk premium in the first- and third subperiod and is largest in magnitude in the first subperiod. Notice, in the second subperiod (09/1998–09/2008), the EG factor contains a significant negative risk premium.

For the extended CRR model (see Panel B), the results for the subperiods are similar to those of the full period. The industrial production factor (MP) has a positive premium in the second subperiod, but is not significantly priced. The inflation factors (DEI and UI) are significant in the last subperiod. With the exception of the second subperiod the VIX is significantly priced carrying the expected negative risk premium. The magnitude of the risk premium of the VIX (in absolute terms) is marginally higher compared to the full period. Interestingly, in the first subperiod EG and VIX are simultaneously and significantly priced. It is striking that the pricing error in the first subperiod in both model frameworks has an extremely high value. In the second and third subperiod the CRR- and FF pricing errors are smaller than those in the full period. Overall, the results are unstable in both model frameworks. This is also true for the volatility factor VIX whose behavior was stable over most model specifications. Nevertheless, with the exception of the market factor, the long-run volatility factor (VIX) is the most consistently priced factor.

Additional uncertainty factors

Next we take a look whether additional uncertainty factors, i.e. macroeconomic uncertainty (MU), the variance risk premium (VRP) and implied interest rate volatility (MOVE) have a marginal pricing impact in our extended model. Since the variance risk premium is only applicable from January 2000 we estimated all the models in Tables 12 and 13 from December 1999.18

The results in Tables 12 and 13 show that, although the estimation time period is different and additional uncertainty factors are included in the regression, the VIX remains significantly priced with the expected sign in all model specifications and portfolio sorts. Again, the EGARCH factor is not priced in all specifications, however, it exhibits a significant negative risk premium in the 25 size/value portfolio sort using the FF-model. Furthermore, the EGARCH factor becomes significant when adding the MOVE index in Eq. (5) in Table 12 and in Eqs. (9), (10) in Table 13.

Moving on to the new uncertainty factors, the results reveal that the coefficients of macroeconomic uncertainty (MU) and the variance risk premium (VRP) exhibit the expected sign in most cases, but are not significantly priced regardless of the sort and the model framework. In contrast, the MOVE index is significantly priced in all model specifications which indicates that changes in expected interest rate volatility is a priced risk factor after controlling for long- and short-run stock market volatility risk. The magnitude of the MOVE risk premium is larger when sorting by size and value. The factors of the basic CRR model remain inconsistently priced and highly dependent on the portfolio sort and model specification. Interestingly, the market factor is only significantly priced at the 10% level when sector portfolios are used. In the case of the size- and value sorts the market premium is no longer significant. The pricing errors remain significantly different from zero in all model specifications of Tables 12 and 13.

The important finding from adding uncertainty factors is that the estimation results of the major model remain highly robust; this is in particular true for the VIX- and EG-factors. Interestingly, the addition

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18 As the VRP is calculated for month $t$ using the realized variance from $t+1$, the estimation period ranges from December 1999 to November 2018.
interest rate volatility as proxied by the MOVE-index exerts a significant impact. From the three added uncertainty factors, only implied volatility of the MOVE factor makes the short-run volatility factor (EG) more significant. The sample period is defined from December 1999 to November 2018 using monthly data.

Table 12
FF3 factor- and CRR model: Additional uncertainty factors, 48 industry sector portfolios.

<table>
<thead>
<tr>
<th>Fama-French 3 factor model</th>
<th>Chen-Roll-Ross model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extended</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>( \lambda_{RF} )</td>
<td>0.528*</td>
</tr>
<tr>
<td></td>
<td>[1.722]</td>
</tr>
<tr>
<td>( \lambda_{MB} )</td>
<td>-0.082</td>
</tr>
<tr>
<td></td>
<td>[-0.306]</td>
</tr>
<tr>
<td>( \lambda_{HML} )</td>
<td>0.267</td>
</tr>
<tr>
<td></td>
<td>[0.013]</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums of the FF3 factor- and CRR model with the VIX and EGARCH (EG) (a description of the variables can be found in the caption of Table 2) while controlling for additional uncertainty factors. Eqs. (1) and (6) serve as the benchmark pricing of the basic FF3 factor- and CRR model with the incorporation of the VIX and EG for 48 industry sector portfolios (+ 6 traded factors). Eqs. (2) to (5) and (7) to (10) respectively control for the additional uncertainty factors, macroeconomic uncertainty (MU), variance risk premium (VRP) and the MOVE Index (MOVE). Furthermore for each pricing test the respective test of joint \( \alpha \)'s and its \( p \)-value are displayed. The t-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with * for \( p < 0.10 \), ** for \( p < 0.05 \) and *** for \( p < 0.01 \). The risk premiums are displayed in percent. The sample period is defined from December 1999 to November 2018 using monthly data.

Table 13
FF3 factor- and CRR model: Additional uncertainty factors, 25 size/value sorted portfolios.

<table>
<thead>
<tr>
<th>Fama-French 3 factor model</th>
<th>Chen-Roll-Ross model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extended</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>( \lambda_{RF} )</td>
<td>0.413</td>
</tr>
<tr>
<td></td>
<td>[1.317]</td>
</tr>
<tr>
<td>( \lambda_{MB} )</td>
<td>0.404**</td>
</tr>
<tr>
<td></td>
<td>[0.207]</td>
</tr>
<tr>
<td>( \lambda_{HML} )</td>
<td>0.185</td>
</tr>
<tr>
<td></td>
<td>[0.079]</td>
</tr>
<tr>
<td>( \lambda_{MU} )</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>[0.008]</td>
</tr>
<tr>
<td>( \lambda_{EG} )</td>
<td>0.185</td>
</tr>
<tr>
<td></td>
<td>[0.059]</td>
</tr>
</tbody>
</table>

The table reports the monthly risk premiums of the FF3 factor- and CRR model with the VIX and EGARCH (EG) (a description of the variables can be found in the caption of Table 2) while controlling for additional uncertainty factors. Eqs. (1) and (6) serve as the benchmark pricing of the basic FF3 factor- and CRR model with the incorporation of the VIX and EG for 25 size/value sorted portfolios (+ 6 traded factors). Eqs. (2) to (5) and (7) to (10) respectively control for the additional uncertainty factors, macroeconomic uncertainty (MU), variance risk premium (VRP) and the MOVE Index (MOVE). Furthermore for each pricing test the respective test of joint \( \alpha \)'s and its \( p \)-value are displayed. The t-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with * for \( p < 0.10 \), ** for \( p < 0.05 \) and *** for \( p < 0.01 \). The risk premiums are displayed in percent. The sample period is defined from December 1999 to November 2018 using monthly data.

of the MOVE factor makes the short-run volatility factor (EG) more significant. From the three added uncertainty factors, only implied interest rate volatility as proxied by the MOVE-index exerts a significant cross-sectional impact on returns. In view of the observation in Megarithis, Vlastakis, and Triantafyllou (2021) that the MU index has better forecasting properties for the stock market volatility than implied volatility as proxied by the VIX, this result may be surprising at first glance. However, a superior predictor need not represent a superior
premiums are displayed in percent. The risk premium, i.e. the unexpected part of the VIX factor. Finally, our results only indicate that the marginal pricing impact is not significant after the VIX is included in the model. This seems also to be the case for the variance risk premium, i.e. the unexpected part of the VIX factor.

**TIPS-based inflation proxy**

The final part of our robustness tests is related to the inflation factors in the CRR model. The construction of the TIPS-based factors is described in Section 4.1. Because the TIPS data is not available until the late 1990s, the results in Table 14 are based on the restricted time period from September 1998 to November 2018 and are subdivided in two subperiods only. Again, for brevity, we only display the results using the 48 industry sector portfolios.

### Table 14

| CRR model (with TIPS): Subperiods, 48 industry sector portfolios. |
|---|---|---|---|---|---|---|---|
| Panel A | | | | | | | |
| $\lambda_{MP}$ | $\lambda_{DEI}$ | $\lambda_{VI}$ | $\lambda_{PR}$ | $\lambda_{UTS}$ | $\lambda_{VIX}$ | $\lambda_{EG}$ | $t$-test | $p$-value |
| 09/1998–11/2018 | $-0.125$ | $0.053$ | $0.154^{**}$ | $-0.015$ | $-0.057$ | $-0.352^{**}$ | $0.065$ | 95.792 | 0 |
| 09/1998–09/2008 | $0.673$ | $0.102$ | $0.069$ | $-0.046$ | $0.045$ | $0.158$ | $-0.292$ | 148.457 | 0 |
| 10/2008–11/2018 | $-0.133$ | $0.120^{*}$ | $0.172^{**}$ | $0.045$ | $-0.085^{*}$ | $-0.413^{**}$ | $-0.24$ | 148.437 | 0 |

The table reports the monthly risk premiums regarding the 48 industry sector portfolios (+ 6 traded factors) from September 1998 to November 2018 and the two respective subperiods. Panel A reports the risk premium for the basic CRR model with the incorporation of the VIX and EG. Panel B reports the risk premium for the CRR model with the incorporation of the TIPS-based inflation factors DEI and UI. Interestingly, the unexpected inflation factor (UI–TIPS) is calculated with treasury inflation protected securities (TIPS). Furthermore, for each pricing test the respective test of joint $\alpha$'s and its $p$-value are displayed. The $t$-statistics by Newey and West (1987) are shown in square brackets and statistical significance is indicated with $^*$ and $^{**}$ p < 0.01 and $^{***}$ p < 0.001.

The final part of our robustness tests is related to the inflation factors in the CRR model. The construction of the TIPS-based factors is described in Section 4.1. Because the TIPS data is not available until the late 1990s, the results in Table 14 are based on the restricted time period from September 1998 to November 2018 and are subdivided in two subperiods only. Again, for brevity, we only display the results using the 48 industry sector portfolios.

Panel A in Table 14 shows the risk premiums for the same model specification as in Table 11, while Panel B contains the results using the TIPS-based inflation factors DEI and UI. Interestingly, the unexpected inflation factor (UI) has essentially the same risk premium in both models, and this over the full time period and the second subperiod (approximately 0.15% to 0.17% p.m.). The coefficients are statistically significant in both models. The change of expected inflation (DEI–TIPS) on the other hand provides a quite different picture. Unlike DEI in the original model, the risk premium on DEI-TIPS is considerably smaller and not significant in the second subperiod. Furthermore, the switch of inflation factors has no impact on the pricing of the short-run- and long-run volatility factors. In both Panels, the VIX exhibits a significant negative risk premium in the full period and the second subperiod. The pricing errors do not substantially decrease with the re-specification of the inflation factors, and they remain significantly different from zero.

### 5.4. Practical implications: Industry sector return effects

Apart from the aspect of statistical significance, it is of practical interest to what extent the inclusion of a systematic volatility factor affects expected excess returns. In order to shed some light on the net effect of the addition of the VIX on expected returns, and whether there is a possible substitution effect with respect to expected returns, we choose the major model equations (1), (2), (5) and (6) (see Table 4) and calculate the average expected return for each of the 48 (+ 6) industry sector portfolios.

Fig. 7 reveals that the effect is rather sensitive to the respective sectors and therefore not uniformly across sector portfolios. When comparing the two model frameworks, it is notable that the effect is predominantly visible in the CRR model (Panel A), where the addition of the VIX seems to generate a noticeable additional expected return in a majority of portfolios compared to basic model without the VIX. Picking out the market portfolio (Mkt in Fig. 7), the addition of the VIX generates an additional expected return of approximately 1.2% p.a. However, in the FF3 model (Panel B), the VIX addition does not lead to clear results. While still the majority of sectors have an additional expected return with the inclusion of the VIX the differences are by far not as noticeable in magnitude and in our opinion rather negligible, whereby the market portfolio even has a slightly smaller expected return when adding the VIX. A potential reason for this discrepancy might be that the market factor already captures a large part of the variations.
in the cross section of portfolio returns. From the VIX sensitivities in Fig. 3 it is apparent that the VIX-betas are not as uniformly negative across portfolios in the FF3 model as in the CRR. Moreover, we can see from Fig. 3 that the relationship is stronger between the VIX-betas and average expected returns in the CRR model which is indicated by the steeper least square regression line (subplot C versus subplot A in Fig. 3).

In the following we take a closer look at specific industries. In the CRR model framework sectors such as ‘agriculture’, ‘food’, ‘soda’, ‘beer’, ‘smoke’, ‘health’, ‘medical equipment’, ‘drugs’, ‘steel’ ‘chips’, ‘lab equipment’, ‘retail’, ‘oil’, ‘telecom’ and ‘financials’ carry the largest part of additional expected return. However, this is not consistently the case when looking at the FF3 model framework. In this framework ‘agriculture’, ‘soda’, ‘beer’, ‘smoke’, ‘steel’ carry larger parts of the
additional expected return as well, however, the largest contributing sectors are 'gold', 'mines' and 'boxes', where the first two have exactly the opposite sign in the CRR model. Moreover, the betas of the specific sectors are not consistently higher in the same sectors across both model frameworks. However, VIX factor loadings across all sectors are significantly higher in the CRR model with an average VIX beta of −1.94 compared to the FF3 model where the average VIX factor loading is smaller by a factor 10. In the CRR model the highest VIX factor loadings can be found in the sectors 'steel', 'chips' and 'financials' with betas smaller than −3. In contrast in the FF3 model the highest VIX factor loadings can be observed in the sectors 'gold', 'mines', 'steel', 'soda' and 'agriculture' with betas smaller than −0.6.
Fig. 7. Expected returns on 48 industry sector portfolios.
Panel A displays the expected excess return ($\hat{R} = \hat{\lambda} \hat{\beta}$) on all 48 industry sector portfolios (+6 traded factors) estimated using the CRR and the CRR with the addition of the VIX. Panel B displays the expected excess return ($\hat{R} = \hat{\lambda} \hat{\beta}$) on all 48 industry sector portfolios (+6 traded factors) estimated using the FF3 and the FF3 with the addition of the VIX. The bars are indicating the difference between the expected excess returns through the addition of the VIX. The sample period is defined from January 1990 to November 2018 using monthly data.

Table 15
Factor premiums in the CRR model.

<table>
<thead>
<tr>
<th>Factor specific risk premiums</th>
<th>Health</th>
<th>Steel</th>
<th>Autos</th>
<th>Gold</th>
<th>Banks</th>
<th>Mkt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$CRR$</td>
<td>$CRR + VIX$</td>
<td>$CRR$</td>
<td>$CRR + VIX$</td>
<td>$CRR$</td>
<td>$CRR + VIX$</td>
</tr>
<tr>
<td>$MP$</td>
<td>0.09</td>
<td>0.04</td>
<td>0.06</td>
<td>0.03</td>
<td>0.29</td>
<td>0.13</td>
</tr>
<tr>
<td>$DEI$</td>
<td>-0.05</td>
<td>0.10</td>
<td>0.23</td>
<td>-0.15</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>$UI$</td>
<td>0.02</td>
<td>0.03</td>
<td>-0.04</td>
<td>0.15</td>
<td>0.13</td>
<td>-0.09</td>
</tr>
<tr>
<td>$URP$</td>
<td>0.28</td>
<td>-0.04</td>
<td>0.67</td>
<td>-0.13</td>
<td>0.72</td>
<td>-0.19</td>
</tr>
<tr>
<td>$UTS$</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>$VIX$</td>
<td>0.57</td>
<td>1.16</td>
<td>0.81</td>
<td>0.28</td>
<td>0.28</td>
<td>0.76</td>
</tr>
</tbody>
</table>

The table reports the expected excess returns ($\hat{\beta} \cdot \hat{\lambda}$) of each risk factor in the basic CRR model with and without the VIX for the 6 sectors: Health, Steel, Autos, Gold, Banks, and the market portfolio. $\sum$ shows the total expected return for each equation in the respective sector. The estimation period ranges from January 1990 to November 2018. The risk premiums are displayed in percent.

Some additional insight is provided by the expected return decomposition displayed in Table 15 which highlights the contribution of the various factors in the basic model, with and without the VIX factor, for 5 selected industry sectors (‘health’, ‘steel’, ‘auto’, ‘gold’, and ‘banks’) as well as the market portfolio. The key observation across almost all industries is actually reflected in the market portfolio (last two columns): The credit risk premium (UPR) is virtually fully substituted (if not overcompensated) by the volatility premium. While such an effect is economically plausible in principle, its magnitude is surprising. Moreover, the volatility premium dominates all other macroeconomic premiums. Across the industries, the size of the volatility premium is fairly different. For example, the size is small in the gold mining and processing sector (0.28% p.m.) implying that these stocks are regarded as a good hedge against volatility risk requiring a small risk premium. The risk premium is above average for the automobile and the steel sectors (0.81% and 1.16%) which appear very vulnerable to an increase in uncertainty risk. Notice that the net effect of the inclusion of volatility risk can result in a lower expected risk premium for some sectors (e.g. ‘gold’, ‘autos’).

Megaritis et al. (2021) also analyze industry effects of their volatility factors. Interestingly, the largest marginal effect of the VIX on the $R^2$ is in the financial sector. The sector marked ‘Fin’ in our industry classification is a rather diverse group of firms not classified as banks or insurance companies. Therefore, the results for the “bank” sector are more revealing (see second last pair of columns in Table 15); again the relative size of the VIX-premium is substantial compared to the remaining CRR factor premiums, but its size is average when compared to the other sectors and the market portfolio.

Overall, it is apparent that the inclusion of a VIX factor might yield substantially larger expected returns for an investor using a macroeconomic factor model, and a lower premium in some sectors (e.g. autos, gold, real estate). In any case, apart from the statistical significance
of the risk factors, it is worth examining their impact on the size and structure of expected returns.

6. Conclusion

Is stock market uncertainty priced in the cross-section of stock returns? There is quite a substantial body of studies showing, based on various volatility proxies and portfolio sorts, that more sensitive stocks exhibit a larger premium. The question addressed in this paper is whether this conclusion is sensitive to the asset pricing framework used in the tests. For this purpose, we compare the results using a macroeconomic and fundamental based asset pricing model (abbreviated by CRR and FF). We use three proxies measuring uncertainty: the standard (long-run) VIX-factor, a (short-run) EGARCH-based factor, and a proxy measuring monetary policy uncertainty. Finally, it is well known that asset pricing tests are very sensitive to the structure of the underlying asset universe. We apply two distinct portfolio sorts in our empirical tests, a fundamental based sort and an industry sector based. Moreover, robustness is tested using alternative specifications of models and variables.

In such a diverse setting, we cannot expect homogeneous results across all estimated equations. However, the question is which pricing results are robust for different model specifications and which are not. The findings can be summarized briefly as follows: The marginal pricing effect of the VIX volatility factor is strong and statistically significant, and has always the expected negative sign. In comparison, the marginal effect of the EGARCH-based volatility factor is mixed but exhibits the expected sign throughout the models. The inclusion of the EGARCH factor does not impair the pricing effect of the VIX. The size of the VIX-premium is between 0.3% and approximately 0.5% p.m., with values above 0.5% for the CRR model using the size/value sorted portfolios. The EGARCH-volatility premium is smaller and more volatile across the models; the typical value is between 0.2% and 0.3% p.m., with most values above 0.5% for the FF model using the size/value portfolio sort. If regressions are run by GLS, the EGARCH-premium becomes more important using the size/value sorted portfolios. Monetary policy uncertainty exhibits only a few significant pricing effects, almost all of them in the CRR model using the size/value portfolio sort.

The following pricing patterns can be observed across the models and specifications: First, our portfolio sorts have a substantial impact on the volatility premiums in both, the CRR and FF model frameworks. Second, the size of the volatility risk premiums are more uniform across the models if the 48 industry sector portfolio sort is used. Third, the size/value portfolio sort generates larger volatility risk premiums (in absolute terms) for both models. While the first two general implications regarding the volatility factors remain the same if GLS is applied, the third observation is reversed and the EGARCH factor is significantly priced in more cases.

The pricing effects of the VIX- and EGARCH-volatility factors are highly robust against the inclusion of additional uncertainty factors such as an index for macroeconomic uncertainty, a proxy for cyclical aggregate risk aversion, and an index measuring long-run interest rate volatility (MOVE). From these variables, only the last factor (MOVE) exerts a marginal pricing impact in the cross-section of returns. At the same time, the MOVE factor reinforces the short-run volatility factor (EG).

The overall insight from this paper is that volatility as measured by conventional proxies (VIX, EGARCH) matters as a systematic and priced risk factor irrespective of the pricing framework, test portfolios and major subperiods. This indicates that volatility risk captures, in addition to other well-known fundamental or macroeconomic factors, a major risk-related source of variation between expected returns. The results also suggest that the performance of volatility-based investment strategies must be evaluated with benchmarks reflecting exposure to aggregate volatility risk in order not to misinterpret the size and attribution of excess returns. This is particularly true for benchmarks which are based on macroeconomic factors.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References
