University of Basel

Faculty of
Business and Economics

WWZ

August 2022

Product Lotteries and Loss Aversion

WWZ Working Paper 2022/06

Sebastian Schäfers

A publication of the Center of Business and Economics (WWZ), University of Basel.

Universität Basel
Peter Merian-Weg 6
4052 Basel, Switzerland
wwz.unibas.ch

Corresponding Author:
Sebastian Schäfers
Tel. +41 61 207 55 25
sebastian.schaefers@unibas.ch

Product Lotteries and Loss Aversion*

Sebastian Schäfers†

University of Basel, Peter Merian-Weg 6, 4002 Basel, Switzerland

Abstract

Product lotteries are a sales strategy where companies hide features of differentiated products from consumers until the purchase is complete. I identify loss aversion as an important factor explaining the existence of vertical product lotteries. I consider a profit-maximizing monopolist serving loss-averse consumers with rational expectations about the lottery. I find that the optimal strategy consists of offering a premium product with high and deterministic quality and a lottery with stochastic and lower expected quality. When consumers are reasonably loss averse, I show that the profit increase from adding a quality lottery exceeds 10% compared to the case without a lottery.

JEL Classification: D42, D81, D91, L12

Keywords: Product lotteries, Probabilistic selling, Reference-dependent preferences, Loss aversion

*I would like to thank Aaron Edlin, Beat Hintermann, Botond Kőszegi, Frank Krysiak, Marc Möller, Catherine Roux, and John Thanassoulis as well as the participants of the EARIE 2021 in Bergen for helpful comments and suggestions.

†University of Basel. E-mail: sebastian.schaefers@unibas.ch

1 Introduction

Offering product lotteries is a selling strategy where sellers hide product information from customers prior to the purchase and reveal it once the purchase is finalized.[1] In practice, product lotteries can be observed for horizontally and vertically differentiated products. The car rental company Thrifty, for example, offers a WildCard program where customers instead of renting a compact car can opt for a "special car" deal at a slightly higher price.[2] Consumers then either get a compact car or a free upgrade to a more luxurious and more expensive model.[3] The online travel agencies Hotwire or Priceline[4] sometimes combine three hotels with often vastly different retail prices into a lottery.[5]

Horizontal product lotteries have been shown to be screening mechanisms that allow for profitable market segmentation (Thanassoulis, 2004; Anderson and Celik, 2015; Balestrieri et al., 2021). These findings, however, do not extend to the case of vertical product lotteries. For vertically differentiated products, unlike horizontally differentiated products, all consumers share the same preference order. This limits the ability of vertically differentiated offerings to segment customers based on their preferences. Deneckere and McAfee (1996) show that extending a product line to sell damaged goods leads to an overall profit cannibalization. Moreover, Mussa and Rosen (1978) and Myerson (1981) show that an optimal mechanism for one-dimensional quality preferences is always deterministic, implicitly ruling out vertical product lotteries for consumers with "standard" preferences. Taken together, these previous findings seem to rule out profitability of vertical product lotteries. In this article, I show that consumer loss aversion relaxes that result and can explain the use of vertical product lotteries.[6] I therefore identify a novel mechanism of how probabilistic product quality can be part of a firm's optimal strategy. Although there is a substantial body of literature pointing to the empirical relevance of consumer loss aversion (Kahneman et al., 1991; Camerer, 2005), it has not yet been considered in the context of product lotteries.

[1] While probabilistic or opaque selling is the expression used in the marketing literature (e.g., Anderson and Celik (2015); Zhang et al. (2015)), product lottery is the term commonly used in economics (Thanassoulis, 2004; Balestrieri et al., 2021). Both terminologies refer to selling practices where product attributes are purposefully hidden from the consumer. In this article I use the term product lottery following Thanassoulis (2004).

[2] https://www.thrifty.com/OurCars/WildCard.aspx

[3] Similar offers by competitors are the "Special Car" deal on Hotwire and the "Supplier's Choice" deal on Priceline.

[4] https://tinyurl.com/pricebreaker and https://tinyurl.com/hotrate

[5] For more examples of vertical product lotteries, see Zhang et al. (2015) or Zheng et al. (2019): an internet provider, where consumers can choose between one tariff with guaranteed high connection quality or a second tariff where the provider guarantees connection quality to be in an interval between medium and very high speed, surprise grab bags, and run-of-house booking schemes where hotels charge a room fee for minimum guaranteed quality while offering the chance of an upgrade.

[6] This makes vertical product lotteries a profit-maximizing strategy in the Mussa and Rosen (1978) model.

Other attempts to explain the existence of product lotteries include the opportunity for firms to increase their flexibility or optimize their inventories. Although these aspects are certainly important, in practice they often explain the existence of product lotteries inadequately. This is for example the case when firms offer a lottery long time in advance and announce the result of the lottery immediately after the purchase (Fay and Xie, 2008, 2010; Huang and Yu, 2014). More recently, marketing researchers have therefore pointed to the importance of behavioral factors in understanding product lotteries (Fay and Xie, 2008, 2010; Huang and Yu, 2014; Zheng et al., 2019). Zheng et al. (2019) argue that product lotteries could increase the salience of product attributes in the lottery and thereby shape the consumer's quality perception. Such an argument, however, essentially contradicts the assumption of product quality as an observable product attribute and is thus only of limited relevance to understand vertical product lotteries.

I consider a monopolist who serves a continuum of loss averse consumers with unit demand. Accounting for consumers' loss aversion has been argued to be more consistent with observed behavior in modest stakes scenarios than risk aversion (Rabin, 2013). Following Kőszegi and Rabin (2006), I call the utility that consumers derive from owning the good and money "consumption utility". In addition, I introduce separate gain-loss utility, both in the money and in the product dimension. I employ a simple lottery structure with a low-quality base option and the chance for an upgrade to a high-quality alternative. The reference point against which consumers evaluate their outcomes is endogenously formed by their rational expectations about the lottery outcomes.[7] Furthermore, I derive the monopolist's optimal quality-schedule depending on the degree of consumer loss aversion and identify a set of conditions under which product lotteries are part of the monopolist's optimal strategy.

For moderate levels of loss aversion, I find that it is optimal for the monopolist to expand his product line to include a product lottery that has lower expected quality than the premium product. The intuition behind this result is driven by two main insights: On the one hand, offering a lottery increases the total market share of the monopolist by serving an additional segment on the low market end. On the other hand, the expected profits and losses of the lottery act as a screening mechanism that dissuade customers of the premium product from switching to the cheaper segment. This is because loss averse consumers with a high valuation for quality incur higher expected losses from random outcomes than consumers with a low valuation for

[7] Crawford and Meng (2011), Marzilli Ericson and Fuster (2011), Gill and Prowse (2012), Banerji and Gupta (2014), and Karle et al. (2015) provide evidence for the notion that reference points are driven by consumers' expectations over outcomes. For papers discussing the limitations of expectation-based reference points, see for example Gneezy et al. (2017).

quality. Thus, the probabilistic outcomes reduce the share of high valuation consumers switching from the premium product to the cheaper alternative, allowing for an overall profit increase.

My contribution to the literature is twofold. First, I provide a novel theoretical explanation for the existence of vertical product lotteries. In addition, this work serves as a complement to Deneckere and McAfee (1996), by outlining how monopolistic firms can sell damaged goods without cannibalizing their own profit. I also show that the profit increases from offering a lottery can be substantial. Even under conservative assumptions, product lotteries may increase profits by more than 10% compared to the case where the monopolist offers only deterministic product quality. Second, I find that offering a lottery with lower expected product quality can be optimal, even if the highest product quality has the best benefit-to-cost ratio. In contrast to the more restrictive conditions of Mussa and Rosen (1978) and Myerson (1981), I show that probabilistic quality choices for a single quality dimension can, in fact, be optimal.

2 Literature

In their seminal work, Mussa and Rosen (1978) discuss a monopolist's optimal choices of product quality and defined properties of an optimal quality-price schedule. Moreover, Myerson (1981) shows that an optimal selling mechanism is always deterministic in the case of one-dimensional agent preferences. Based on that, Riley and Zeckhauser (1983) extend this discussion to the question of whether probabilistic quality choices would be part of an optimal firm strategy. For the case where product quality can be expressed as a one-dimensional variable they show that there is always a clear first-best quality choice. They argue that any deviation - be it probabilistic or deterministic - would be unprofitable.

Rochet (1987), as well as Rochet and Choné (1998) and others (McAfee and McMillan, 1988; McAfee et al., 1989) derive conditions under which lotteries with horizontally differentiated products may be profit increasing. Thanassoulis (2004) and others (Manelli and Vincent, 2007; Pavlov, 2011; Hart and Reny, 2015; Hart and Nisan, 2019) extend this notion and show that lotteries can be used as screening mechanisms for multiproduct monopolists in order to effectively price-discriminate. Hart and Nisan (2019) provide numerical evidence, suggesting that the benefits of lotteries in auctions can be substantial. Balestrieri et al. (2021) apply the concept of product lotteries specifically to the hospitality sector. They show that product lotteries are part of an optimal monopolistic strategy based on a Hotelling model of horizontally differentiated goods. These findings, however, apply to horizontal but not to vertical product lotteries. In the horizontal case, lotteries are sold at a discount to those consumers with similar valuations for all product alternatives. Consumers with a clear preference for one of the available alternatives have no incentive to opt into the lottery. For vertically differentiated products, however, all consumers have a clear preference for the higher product quality. Deneckere and McAfee (1996) point out that increasing a product's attractiveness in the low market end may generate additional sales but may at the same time reduce overall profits due to cannibalization. In line with this, Johnson and Myatt (2018) argue that Extending the result of Mussa and Rosen (1978), Bhargava and Choudhary (2001) prove that in the case of vertically differentiated goods, the monopolist sells only the most expensive product if it has the best benefit-to-cost ratio.

Marketing scholars have explained the existence of vertical product lotteries with general observations such as production cost savings and increased flexibility (Fay and Xie, 2008; Jerath et al., 2010; Balestrieri et al., 2021). These considerations cannot fully explain the phenomenon for two reasons. Marginal cost savings are negligible in markets where goods are produced assemble-to-order and firms gain only little in flexibility when they specify the product after the purchase. In addition, marketing scholars also recognize that the assumption of perfectly rational and patient consumers does not help to explain the use of product lotteries (Fay and Xie, 2008, 2010; Huang and Yu, 2014; Anderson and Celik, 2015; Zhang et al., 2015; Anderson and Celik, 2020). They point out that the current understanding of probabilistic selling strategies is incomplete and call for a more extensive analysis, considering also behavioral aspects.

Advancements in behavioral economics spearheaded by Kahneman and Tversky (1979, 1992) have led to the conclusion that observed consumer behavior often systematically differs from "rational" consumer behavior, particularly in the context of loss aversion. Kőszegi and Rabin (2006) extend the concept of loss aversion by including endogenous reference points. Based on their seminal work, consumer loss aversion has been shown to have important consequences for optimal firm strategies. Heidhues and Kőszegi (2008) and Carbajal and Ely (2016) integrate consumer loss aversion into the firm problem and formalize how consumers' expectations may affect a firm's pricing decisions. While these works provide important insights into how loss aversion changes the firm problem, they treat consumer expectation as (partly) exogenous.

Heidhues and Kőszegi (2014) as well as Rosato (2016) include consumer expectation-building as an endogenous process into their models and show that it can be optimal for firms to expose consumers to risk instead of shielding them from it. Risk can be introduced for example by announcing a distribution of future price discounts (Heidhues and Kőszegi, 2014) or by limiting the availability of a discounted product (Rosato, 2016). In both cases, the anticipated discount leads to consumers raising their expectations and consequently accepting an ex-ante unfavorable offer to avoid the "pain" of walking away empty-handed. Both models, however, rely on the fact that the element impacting consumers' expectations is not available at the time of purchase. Thus, they are not able to explain the existence of product lotteries. Moreover, their models incorporate uncertainty in the price dimension and thus, do not allow for the analysis of uncertainty in the product quality dimension.

Beccuti and Möller (2018) show that randomization between delivery and non-delivery is an optimal monopolistic strategy in the dynamic sale of a perishable good, if sellers are noticeably more patient than buyers. Screening by different patience levels, however, only applies to the case where the monopolist offers temporally differentiated products, but not to the case of product lotteries where the monopolist offers quality differentiated products at the same point in time.

Rochet and Thanassoulis (2019) study optimal price dynamics when sellers have the ability to commit to a price path. They interpret the discount factor in the inter-temporal pricing problem as a probability of delivery, thereby reducing a product's value in line with the model of Deneckere and McAfee (1996). Moreover, they show that a monopolist can maximize profit by delaying the cross-sell to complete a bundle, if the consumers' valuations do not satisfy the Spence-Mirrlees sorting conditions. Unlike their model, however, I consider neither product bundles nor temporal differentiation in the present model.

More recently, Zheng et al. (2019) offer a behavioral explanation by arguing that product lotteries may increase the salience of product attributes in the lottery and thus shape the consumer's quality perception. Such an argument, however, essentially contradicts the assumption of product quality as an observable product attribute and is thus only of limited relevance to understand vertical product lotteries. Consequently, the question whether and if so under which circumstances vertical product lotteries are profitable for firms when consumers observe product quality directly has not been answered.

3 Model

The model section is split into two parts. First, I setup the model where a monopolist offers one good in different quality levels. Second, I show that vertically differentiated product lotteries are part of the monopolist's profit maximizing strategy when consumers are sufficiently loss averse. I will first derive general conditions under which my findings hold and then characterize a complete closed-form solution for the case where the monopolist can increase product quality at no additional cost, which I argue is the most restrictive case.

3.1 Setup

3.1.1 Monopolist

I consider a profit maximizing and risk-neutral monopolist who produces an indivisible good of quality $\beta \in [0, 1]$ facing consumers with unit demand. Note that in this model, a quality level of 0 does not mean that the consumer receives no product. Instead, a quality level of 0 would correspond to the normalized utility value that a consumer would obtain by consuming an outside option that is available to all at a price that is also normalized to 0.

The firm chooses price p, quality level β and faces a demand $q(p, \beta)$. The cost function is given by $C(\beta, q)$ with (i) $C_\beta(\beta) = c \cdot q$ with $c \in [0, 1)$ and (ii) $C_{\beta\beta}(\beta) = 0$. Assumption (i) ensures that for any chosen β the reservation price in the market is always higher than the cost of quality. Assumption (ii) additionally ensures that marginal cost of quality is constant. In a market with only a single quality level the monopolist therefore always supplies a premium segment with $\beta = 1$. A quality-price schedule with a single element is consistent with the model of Mussa and Rosen (1978) for a single quality dimension and constant marginal cost. Due to the linearly increasing cost function and the advantage of higher product quality, this outcome corresponds to the case in the model of Johnson and Myatt (2018) where price discrimination is cost-driven[8]. If, instead, price discrimination was elasticity-driven, the optimal menu would be more complex but may still allow for the profitable integration of product lotteries.

3.1.2 Consumers

Consumers are a continuum with mass 1 and have unit demand for the product offered by the monopolist. The consumers' individual valuations for (riskless) quality follow distribution $F(v)$

[8] See Johnson and Myatt (2003) or Anderson and Dana Jr (2009) for additional discussion on the conditions of profitable price discrimination

over the interval $[0, 1]$ with uniform density and there are no income effects. All consumers have the same outside option, with both its utility and price normalized to 0. If the quality of the offered product exceeds that of the outside option and $\beta > 0$, consumers exhibit individually different levels of appetite for quality according to v_i. Consumers experience utility from two sources. First, consumption utility determined by their individual valuation for quality. Second, gain-loss utility in the price and product dimension where they compare the outcome against their reference points (Kőszegi and Rabin, 2006). Following the notation of Heidhues and Kőszegi (2014), I write consumption utility as $k_i^v = \beta \cdot v_i$ and $k^p = -p$. The individual rational reference points are given by $k_{i,r}^v = E_i[k_i^v]$ and $k_r^p = E[k^p]$, which corresponds to any consumer's rational expectations about expected utility in quality and money dimension. A consumer's gross utility can be written as:

$$u_i = u_i(k_i^v | k_{i,r}^v) + u_i(k^p | k_r^p) = k_i^v + \mu(k_i^v - k_{i,r}^v) + k^p + \mu(k_r^p - k^p) \tag{1}$$

The function μ is a two-piece linear function with $\mu = \eta$ for gains and $\mu = \lambda \cdot \eta$ for losses. The coefficient η can be interpreted as the relative importance a consumer places on gain-loss utility relative to consumption utility, whereas λ is the coefficient of loss aversion. For $\lambda = 1$, gains and losses are weighted equally and a consumer with such a loss aversion coefficient would be considered as loss-neutral. I assume that gain-loss utility is linear around the reference point k_r^j with $j = p, v$ and that all consumers are identical in their degree of loss aversion as well as their valuation for η. That implies that the slope for gains is η and the slope for losses is $\lambda \cdot \eta$ with $\lambda \geq 1$, for all consumers.

3.1.3 Monopolistic choice

The monopolist knows the valuation distribution of the consumers but is unable to discriminate individual consumers. The monopolist's problem is to offer optimal menu items $m_k(l_k, p_k)$, each consisting of a lottery $l_k = \left(\bar{\beta}_k, \underline{\beta}_k, a_k\right)$ with a stochastic quality distribution and an associated price p_k. I focus on the case where the monopolist determines only two menu items, i.e., $k = 1, 2$, as it illustrates the underlying mechanism sufficiently, even though the model could be extended to higher values of k.

For each menu item m_k, the monopolist defines an optimal lottery by choosing a high outcome $\bar{\beta}_k$ occurring with probability a_k, and a low outcome $\underline{\beta}_k$ occurring with probability $(1 - a_k)$, where $\bar{\beta}_k \geq \underline{\beta}_k$. Moreover, the monopolist chooses an optimal and deterministic price p_k for each lottery.

Characterizing any lottery l_k by a high outcome, a low outcome, and their corresponding entry probabilities is sufficient because a consumer's gain-loss utility is linear around the reference point. The reason for that is that a consumer's utility with a two-piece linear gain-loss function and a linear willingness to pay for quality is fully characterized by the first moments of the lottery's stepwise distribution: expected outcome $E[l_k]$ of the quality lottery l_k, as well as expected losses $E\left[l_k | l_k < E[l_k]\right]$ and expected gains $E\left[l_k | l_k \geq E[l_k]\right]$. Additionally, I define $b_k = \bar{\beta}_k - \underline{\beta}_k \geq 0$ as the quality differential between the high-quality and the low-quality outcome of lottery l_k.

In general, representing product quality as a generic lottery allows one to model both deterministic and probabilistic outcomes. This is because the choice of deterministic product quality can be represented as a degenerate lottery, i.e. a lottery in which all lottery outcomes are identical. If the monopolist chooses a degenerate lottery with only one outcome, i.e. $\bar{\beta}_k = \underline{\beta}_k$, consumers perceive price and product quality as deterministic. Otherwise, if $\bar{\beta}_k > \underline{\beta}_k$, consumers perceive prices as deterministic but product quality as probabilistic. Moreover, because all consumers have the same degree of loss aversion, all consumers have the same preference relation over lotteries l_1 and l_2. I therefore impose that for all consumers we have $l_1 \succeq l_2$ with $E[l_1] \geq E[l_2]$ as a necessary condition and $p_1 \geq p_2$. Thus, I call l_1 the premium lottery and l_2 the cheap lottery (compare Figure (1)).

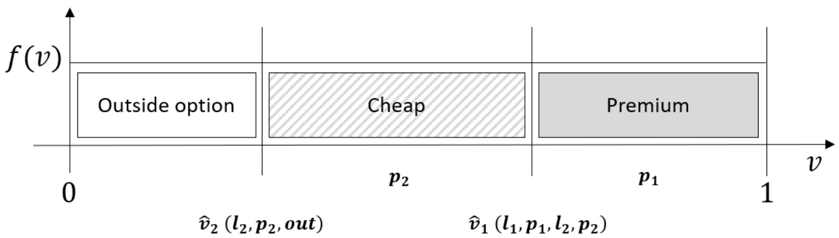


Figure 1
Schematic structure of the market where consumers opt either for the premium lottery l_1 at price p_1, the cheap lottery l_2 at price p_2, or their outside option with utility normalized to 0. The consumers are ordered along according to their valuation for quality and have the same coefficient of loss aversion. The consumer $\hat{v}_2$ is indifferent between the lottery l_2 and the outside option denoted by out. The consumer $\hat{v}_1$ is indifferent between the lottery l_1 and the lottery l_2.

Thus, the monopolist's maximization problem is given the sum of the market shares for each menu position weighted by price minus cost of production:

$$\max_{p_k, \bar{\beta}_k, \underline{\beta}_k, a_k} \quad \sum_{k=1}^{2} \Pi_k(m_k; c, \lambda, \eta) \tag{2}$$

In my further analysis I will show that a monopolist always offers a premium product with $\beta_1 = 1$, and under certain conditions an additional non-degenerate product lottery l_2.

3.2 Analysis of results

3.2.1 Deterministic benchmark

I first derive an optimal quality-price schedule following the framework of Mussa and Rosen (1978), where all quality choices and prices are deterministic. I will use this result as a benchmark to determine under which conditions product lotteries can be optimal.

Proposition 1. *In the absence of consumer loss aversion, the optimal schedule for the monopolist consists of a single quality-price pair with* $\beta_1^* = 1$ *and* $p_1 = \frac{1+c}{2}$*. Additionally, the monopolist's profit decreases in the number of unique quality levels.*

Proof. For a full description of the proof, please refer to the Appendix sections 6.2 and 6.3.1

The monopolist's problem is to decide how many and which levels of qualities he wants to offer. First, consider the case where the firm offers only a single quality level β_1. Maximizing the profit function with respect to the price gives $p_1^*(\beta_1) = \frac{\beta_1+c}{2}$. It is easy to see that in this case it would be optimal to maximize quality, i.e. $\beta_1^* = 1$. This is because the marginal willingness-to-pay for quality increases at a strictly higher rate than the marginal cost of production for quality and therefore:

$$\Pi\left(p_1^*, \beta_1 = 1\right) = \beta_1 \frac{(1-c)^2}{4} = \frac{(1-c)^2}{4} \tag{3}$$

The case where the firm offers multiple quality levels can be solved through backwards induction. Profit in the segment with the highest quality increases with β_1, regardless of the choice for β_2. Thus, we know that the profit-maximizing quality choice in the first segment is $\beta_1^* = 1$, whereas offering a second quality level β_2 in the remaining market. The problem simplifies to finding the optimal β_2 after solving for optimal prices p_2^* and p_1^* and plugging them back into the firm's choice. This gives the profit as a function of the quality in the second segment:

$$\Pi_{1,2}(\beta_1^* = 1, \beta_2) = \frac{(1-c)^2(2-\beta_2)^2}{4(4-3\beta_2)} \tag{4}$$

Expression (4) is the monopolistic profit as a function of the two distinct quality levels β_1 and β_2, with their optimal (interior) prices p_1 and p_2. Compared to the case with only a single segment, offering a second distinct quality level increases the total number of supplied consumers but reduces the number of consumers purchasing the premium version. The overall decrease in profit shows that there is no interior optimum for β_2 and the firm's optimal choices are then $\beta_2^* = 0$ or $\beta_2^* = 1$. Both outcomes have the same interpretation as for $\beta_2^* = 0$ no consumer will purchase the second segment and only the first segment is relevant, whereas for $\beta_2^* = 1$ the second segment is identical to the first one and thus irrelevant to the firm problem. This result follows from the assumption that the marginal cost per quality is linear and therefore the optimal quality level is located in a corner of the solution space. The finding that the single optimal choice of quality corresponds to its highest available value seems restrictive at first. Its application in reality, however, can be explained relatively well. Consider the case where a monopolist's optimal menu contains a finite number of items $n > 1$. Think further about all consumers choosing the product with the lowest quality offered. This market segment cannot be further subdivided without violating the prior optimality assumption. Thus, the present problem with a single optimal quality level $\beta_1 = 1$ corresponds to the sub-problem focusing on the lowest market segment of a more general menu structure. The results are illustrated in Figure 2.

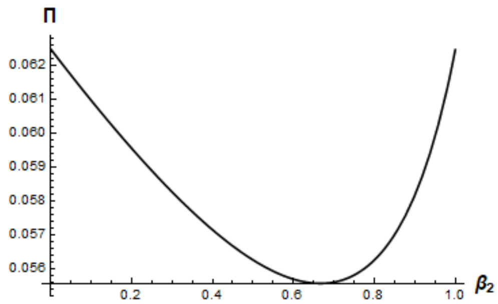


Figure 2
Firm profit $\Pi(\cdot)$ *as a function of* β_2 *with* $\beta_1 = 1$ *and marginal cost of quality* $c = 1/2$

Increasing the number of unique quality levels strictly decreases the monopolist's profit, if each level is chosen by a positive share of consumers. I prove this by backwards induction. Consider the case where the monopolist chooses an optimal quality level β_i in the low market end. Because β_i is for the low market end, it must be that $\beta_i < \beta_{-i} \leq 1$. The monopolist's problem therefore reduces to choosing an optimal $\beta_i \in [0, \beta_{-i}]$. Intuitively, this problem is identical to the firm problem given in equation (4), scaled to a smaller interval. Analogously, the optimal choice is then $\beta_i^* = 0$.

The underlying intuition is this: The monopolist incurs net losses by offering multiple quality levels because without any screening mechanisms, consumers self-select into less profitable product segments. This result illustrates the commitment problem the monopolist faces in the deterministic case and is due to the inability to screen for their individual reservation values. It is similar to the text book example of a monopolist selling a durable good to (sufficiently) patient consumers. Furthermore, previous works also come to similar conclusions, when analyzing optimal quality schedules in a monopolistic setting (Mussa and Rosen, 1978; Bhargava and Choudhary, 2001).

3.2.2 Conditions for optimality of lotteries

From the previous section it becomes clear that the monopolist's optimal choice of quality in any deterministic segment beyond the first is $\beta_i^* = 0 \quad \forall i \neq 1$. In other words, the optimal strategy consists of a single unique quality in the deterministic case. I now show that the result of a single optimal quality level breaks down when the firm has the possibility to offer a quality lottery with multiple quality outcomes in each segment. Even though this result can be generalized to higher number of quality levels, I restrict my analysis to the case where the firm supplies to two distinct and non-empty market segments in order to keep the analytical complexity low.

Proposition 2. *Consider the monopolist's problem to find an optimal quality-price schedule by offering either deterministic quality or a lottery with two quality levels in both market segments.*

1. *The monopolist's optimal strategy includes serving a premium segment with a degenerate lottery l_1, i.e., deterministic quality of $\beta_1^* = 1$*
2. *For any cost function $C(\cdot)$ there exists a lower threshold of consumer loss aversion $\hat{\lambda}^{low}$ above which offering a product lottery with expected quality $\mathbb{E}[l_2]$, where $0 < \mathbb{E}[l_2] < \beta_1^*$, increases profits.*
3. *The value for the lower threshold of consumer loss aversion $\hat{\lambda}^{low}$ is strictly decreasing in the marginal cost of quality c.*

Proof. For a full description of the proof, please refer to the Appendix sections 6.2 and 6.3.2

Consumers can be ordered according to their taste for quality v_i. Let $\hat{v}_1$ be the consumer indifferent between the premium product and the product lottery with lower expected quality. Additionally, let $\hat{v}_2$ be the consumer indifferent between the product lottery and the outside option. By design, it must be that $\hat{v}_1 \geq \hat{v}_2$ and thus, the market shares of the premium segment and the lottery are $1 - \hat{v}_1$ and $\hat{v}_1 - \hat{v}_2$ respectively.

The first part of my proposition states that product quality in the premium segment is maximized without any randomness. I already showed that this is true for the deterministic case, outlined in proposition 1. To show that this is also true when the monopolist can offer stochastic product quality, two insights are important: First, any consumers profit contribution is higher in the premium segment than in the lower market segment. This is because profit is strictly increasing in (expected) product quality (compare expression in (3)) and strictly decreasing in the level of randomness due to loss aversion.

Second, adjusting quality or price in the premium segment affects the indifferent consumer $\hat{v}_1$ but not the indifferent consumer at $\hat{v}_2$ (compare (1)). The reason is that the indifferent consumer's decision at $\hat{v}_2$ is driven only by the comparison between the lottery and the outside option. Thus, for any given value of $\hat{v}_2$, profit is maximized by setting $\beta_1 = 1$. This can be proven by contradiction. Suppose the premium segment would include stochastic product quality with positive variance and a high lottery $\bar{\beta}_1$. The monopolist could then keep $\hat{v}_1$ constant by setting $\mathbb{E}[l_1] = \bar{\beta}_1$ while simultaneously increasing p_1, leading to higher profits. This contradicts the initial assumption of $\bar{\beta}_1$ with positive variance being an optimal choice and proves the first part of my proposition.

Regarding the second part of my proposition, consider that we now know that the optimal quality in the premium segment for the case of lotteries remains the same $\beta_1 = 1$ with or without a lottery. We can therefore focus on the question under which conditions a lottery is profit maximizing compared to the outcome of proposition 1. As outlined in section 3.1.3 expected gains, losses, and value of the lottery in the second segment are fully captured by a low outcome $\underline{\beta_2}$, the quality differential $b = \bar{\beta}_2 - \underline{\beta}_2$, and the entry probability for the low outcome $P(\underline{\beta}_2) = 1 - a_2$. I then maximize the firms profit over two segments and solve by backwards induction for optimal p_1 and p_2 as functions of all remaining lottery parameters. Note that a lottery with the two outcomes $\underline{\beta_2} = 0$ and $\bar{\beta}_2 = \underline{\beta_2} + b_2 = 0$ is identical to the monopolist's optimal choice $\beta_2 = 0$ in the deterministic case of proposition 1.

I analyze the marginal profit with respect to b, evaluated at the former optimum of $l_2 = (0, 0, a_2)$ to check whether a lottery increases profit. If the solution from Proposition 1 ($\beta_1^* = 1$, $\beta_2^* = 0$) is still the optimal choice, the partial derivative of the profit function has to be negative. Thus, we differentiate equation (34) with respect to b_2 and set $\beta_1^* = 1$, $\underline{\beta}_2^* = 0$, and fix a_2 as infinitesimally larger than 0 (compare 6.3.2). The resulting expression (compare Expression 35 in Appendix) is evaluated at $b_2 = 0$ and has the following form:

$$\frac{\partial \Pi_{1,2}(p_1; a_2, b_2)}{\partial b_2}|_{\beta_1^*=1, \underline{\beta}_2=0, b_2=0} =$$

$$\frac{a_2(1 + c(-1 + (1 - a_2)\eta(1 - \lambda))) + (1 - a_2)\eta(1 - \lambda)}{16(1 + (1 - a_2)\eta(1 - \lambda))} \cdot$$

$$\frac{(-1 + c + 3c \cdot (1 - a_2)\eta(1 - \lambda) - \eta(1 - a_2 - \lambda + a_2\lambda))}{1}$$

Inequality (5) investigates under which parametric conditions a product lottery is profit maximizing.

$$\frac{\partial \Pi_{1,2}(\cdot)}{\partial b_2}\Big|_{b_2=0} > 0. \tag{5}$$

The solution shows that including a product lottery does, in fact, increase firm profit compared to the deterministic benchmark, depending on the marginal cost of quality c:

1. For $c = 0$, marginal profit increases in b_2 for any level of loss aversion exceeding the lower threshold

$$\underbrace{1 + \frac{1}{\eta}}_{\hat{\lambda}^{low}} \quad < \quad \lambda. \tag{6}$$

2. For $c \in (0, 1)$, marginal profit increases in b_2 if consumer loss aversion lies between the lower threshold λ_1^{low} and the upper threshold λ_1^{high}

$$\underbrace{1 + \frac{1 - c}{\eta(1 + c)}}_{\lambda_1^{low}} \quad < \quad \lambda \quad < \quad \underbrace{1 + \frac{1}{\eta}}_{\lambda_1^{high}}. \tag{7}$$

3. For $c \in (0, \frac{1}{3})$, there is an additional lower bound for loss aversion λ_2^{low}, where $\lambda_2^{low} > \lambda_1^{high}$. For this cost range, introducing randomness increases profits if either the coefficient of loss aversion lies in the range given by inequality (7) or it exceeds the limit of λ_2^{low}

$$\underbrace{1 + \frac{(1 - c)}{(1 - 3c)\eta}}_{\lambda_2^{low}} < \lambda. \tag{8}$$

Figure 3 shows the range of degree for consumer loss aversion λ under which a lottery is profit increasing.

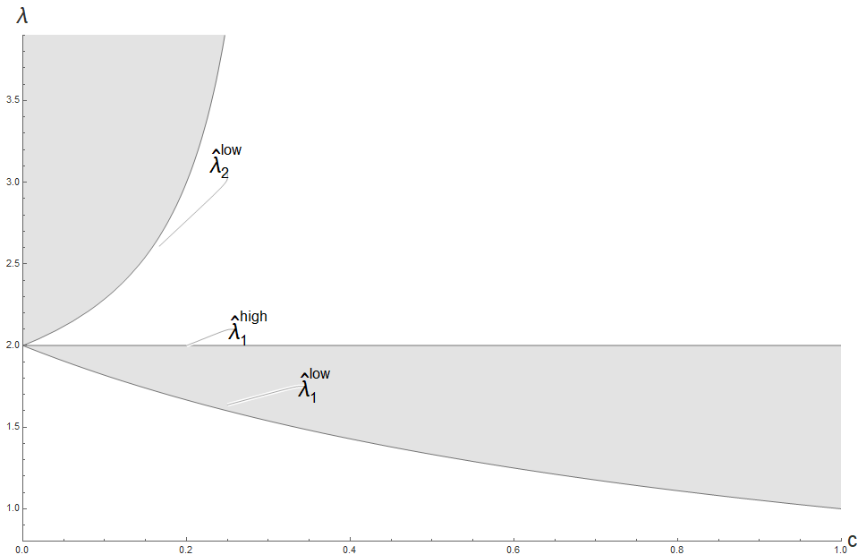


Figure 3

This plot shows the range of all values of λ under which lotteries are profit increasing. On the x-axis are the realizations of c and on the y-axis are the values of the loss aversion coefficient λ. The gain-loss utility parameter η is assumed to be 1. The range of λ, for which lotteries are profitable, are filled in gray.

For any c, there is a non-empty set of values for λ, where offering a lottery maximizes profits and thus proves the second part of my proposition. The proof for the third part of Proposition 2 follows from looking at the expression for λ_1^{low}. For any marginal cost $c > 0$, the partial derivative of λ_1^{low} in Equation (7) with respect to c can be written as:

$$\frac{\partial \hat{\lambda}_1^{low}}{\partial c} = \frac{-2}{\eta \cdot (1+c)^2} < \ 0. \tag{9}$$

The expression in (9) is negative for all feasible values of c and η, thus proving that the lower critical threshold of loss aversion decreases in c. The intuition behind this result is that when marginal production costs are high, the monopolist can generate cost savings by offering a lottery instead of a high cost premium product. Put differently, if firms can increase quality at no additional cost, a firm is less likely to offer a low quality product which leads to a more restrictive lower threshold. Conversely, higher values of c relax the lower bound on the required level of consumer loss aversion. In addition, Figure 3 clearly indicates that the set of degree of consumer loss aversion, for which lotteries are profitable, is staggered over the possible parametrizations of the cost function $C(\cdot)$. The reason for the existence of an upper limit of loss aversion λ_1^{high} for all $c > 0$ is that for high levels of loss aversion, the willingness to pay for the lottery is lower than its production costs. This implies that the lottery cannot be attractive to consumers and be profitable at the same time.

In general, Proposition 2 illustrates the power of product lotteries as a screening tool when consumers are sufficiently loss averse. Intuitively, expected gains and losses from the lottery are higher for consumers with a high appetite for quality. Compared to the deterministic case, the monopolist is able to screen consumers by offering a product lottery with a relatively high expected quality compared to its price. Consumers with low appetite for quality are less sensitive to randomness as they incur smaller expected gains and loss and therefore prefer the lottery over the premium product. Consumers with high appetite for quality, however, incur substantially larger gains and losses, making them less prone to self-select into the lower segment.

3.2.3 Characterization of optimal menu with lotteries

Proposition 2 shows that depending on the consumers' degree of loss aversion the profit maximizing quality-price schedule involves a lottery. It does not, however, fully describe the optimal strategy of the monopolist. In this section I will provide a closed-form solution of the monopolistic strategy for the case where the firm can increase quality at no additional cost, i.e. $c = 0$.

This case is interesting for three reasons. First, as pointed out in the third part of Proposition 2, the lower threshold for loss aversion λ_1^{low} decreases in c. Assuming $c = 0$ therefore sheds light on the most restrictive case within my model and hence provides a lower bound for the relevance of product lotteries. Second, the assumption of c=0 reduces the analytical complexity of the problem and thus results in a solution that is easier to interpret. Third, the assumption of low (incremental) production costs corresponds most closely to the case in reality where firms try to sell previously build-up inventory.

Note that a value of $\underline{\beta} = 0$ is equal to the value of the outside option and therefore, for the sake of the underlying argument, serves as the lottery's lowest possible outcome. In practice, however, the value of a lottery's "worst case" may often be higher than that of the outside option, i.e. $\underline{\beta} > 0$. The fact that the lottery outcomes and the outside option are perceived to be distinct would, in practical applications, support the formation of a probabilistic reference point.

Proposition 3. *Consider the monopolist's problem to find an optimal quality-price schedule consisting of two elements with marginal cost of quality $c = 0$. If consumers are sufficiently loss averse, i.e., $\lambda \geq \hat{\lambda}^{low}$, there exists a closed-form solution for the monopolistic strategy where the optimal lotteries are characterized by:*

1. $l_1^* = \left(\bar{\beta}_1^*, \underline{\beta}_1^*, a_1\right)$ *with* $\bar{\beta}_1^* = \underline{\beta}_1^* = 1$*, and* $a_1 \in [0,1]$

2. $l_2^* = \left(\bar{\beta}_2^*, \underline{\beta}_2^*, a_2\right)$ *with* $\bar{\beta}_2^* = 1$, $\underline{\beta}_2^* = 0$, *and* $a_2 = \frac{1}{2}\left(1 + \frac{1}{\eta(1-\lambda)}\right)$

Proof. For a full description of the proof, please refer to the Appendix sections 6.2 and 6.3.3

First, note that the optimal choice of quality in the premium segment, $\beta_1^* = 1$, follows directly from the general proof in Proposition 2 for any $0 \leq c < 1$. Second, I showed in Proposition 2 that for c=0 there is no upper limit to the range of feasible levels of consumer loss aversion. The screening efficiency of the lottery grows with the degree of consumer loss aversion. The monopolist therefore maximizes the quality differential between outcomes, by setting $\underline{\beta}_2 = 0$, $\bar{\beta}_2 = 1$ which leads to $b_2 = 1$. The expected value of the lottery, as well as expected gains and losses can be fully calibrated by choosing an appropriate a_2. Equation (10) captures the monopolist's profit maximization problem with optimal prices p_2 and p_1, and a_2 as choice variables.

$$\max_{a_2, p_1, p_2} \Pi_{1,2}(p_1, p_2, a_2; \eta, \lambda) \tag{10}$$

Given that $\lambda \geq \hat{\lambda}^{low}$, the monopolist optimally chooses:

$$p_1^* = \frac{(1 + \eta(6 + \eta(-1+\lambda))(-1+\lambda))^2}{(8\eta \cdot (3 - \eta \cdot (1-\lambda))(1 - 3\eta \cdot (1-\lambda))(-1+\lambda))}, \tag{11}$$

$$p_2^* = \frac{((1 + \eta(6 - \eta(1-\lambda))(-1+\lambda))(1 + \eta(1-\lambda))^2)}{(8\eta \cdot (3 - \eta \cdot (1-\lambda))(1 - 3\eta \cdot (1-\lambda))(1-\lambda))}, \tag{12}$$

$$a_2 = \frac{1}{2}\left(1 + \frac{1}{\eta(1-\lambda)}\right). \tag{13}$$

The intuition behind that result is that if consumers are sufficiently loss averse, their perception of gains and losses acts as a screening mechanism. At the margin, the monopolist gains new, (previously unserved) consumers who generate more profit than the monopolist loses from consumers in the premium segment switching to the cheaper option. The optimal profit is given by:

$$\Pi_{1,2}(\cdot; \lambda, \eta) = \frac{(-3 + \eta(6 + \eta \cdot (1-\lambda))(1-\lambda))^2}{16\eta\left(-9 + \eta \cdot (10 + 3\eta \cdot (1-\lambda))(1-\lambda)\right) \cdot (-1+\lambda)} \tag{14}$$

If the consumer loss aversion coefficient is lower than the threshold above, the opposite happens:

The marginal profit increase from additional sales is less than the profit loss in the premium segment. In this case the lottery would not increase the profit of the monopolist and the optimal strategy is the same as in Proposition 1 with $\beta_1^* = 1, \beta_2^* = 0$ and $p_1^* = \frac{1}{2}$ for any $a_2 \in [0, 1]$.

4 Numerical Example

In this section, I examine the magnitude of the profit increase a monopolist can generate applying the optimal strategy derived in section 6.3.3. I choose representative values for the coefficient of loss aversion λ and the relative importance on gain-loss utility η. Figure 4 outlines the firm profit $\Pi(a_2; \lambda = 1, \eta = 1)$ as a function of the probability distribution between lottery outcomes captured by a_2, given optimal values for p_1, p_2, $\underline{\beta} = 0$ and $b = 1$. Consumer loss aversion is set equal to $\lambda = 1$ and thus, does not meet the threshold found in section 3.2.3. Offering a lottery would lead to a loss for the monopolist.

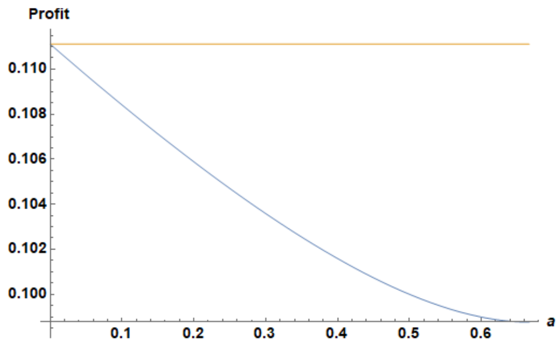
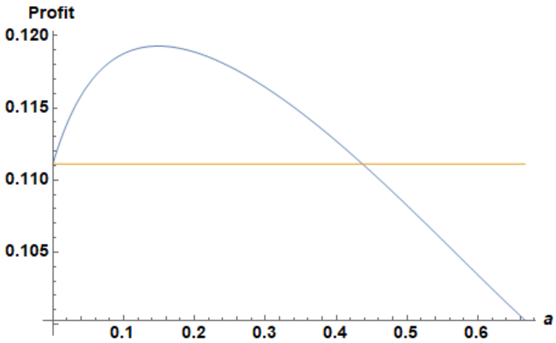


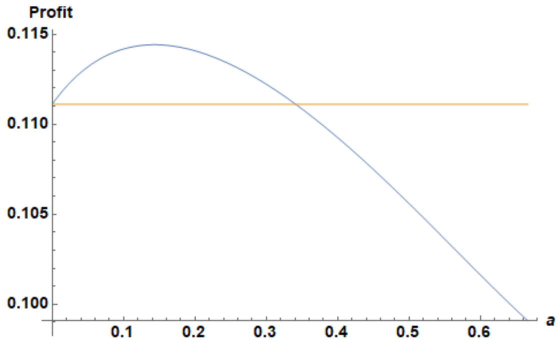
Figure 4
This graph shows the monopolistic profit for the parameter values of $c = 1/2$, $\eta = 1$ and $\lambda = 1$. The orange line corresponds to the monopolist's profit in the deterministic case, whereas the blue line shows the monopolist's profit from offering an optimal lottery as a function of the lottery parameter a.

Figure 5a with profit $\Pi(a; \lambda = 1.875, \eta = 1)$ and Figure 5b with profit $\Pi(a; \lambda = 1.75, \eta = 1)$, show the firm profits for values of $\lambda < 2$ as a function of the outcome distribution a in the lottery. In Figure 5a, the profit increase for a lottery with interior optimal prices and quality levels is clearly visible and around 10% than in the benchmark case. Moreover, the parametric assumption of $\eta = 1$ and $\lambda = 1.875$ is is in line with empirical estimations of WTP to WTA ratios.[9] It also meets the no-dominance criterion of gain loss utility, proposed in the behavioral literature (Herweg et al., 2010).

[9]Experimental estimates of the loss aversion parameter as the WTP to WTA ratio, range from 1 to 3 (Kahneman et al., 1991; Benartzi and Thaler, 1995; Thaler et al., 1997; Kahneman et al., 1999; Booij et al., 2010)



(a) $\lambda = 1.875$



(b) $\lambda = 1.75$

Figure 5

This graph shows the monopolistic profit for the parameter values of $c = 1/2$, $\eta = 1$ and $1 < \lambda < 2$. The orange line corresponds to the monopolist's profit in the deterministic case, whereas the blue line shows the monopolist's profit from offering an optimal lottery as a function of the lottery parameter a.

5 Discussion

Product lotteries or probabilistic selling strategies are a mode of sales with relevant real-world applications. Although it has already been shown that horizontal product lotteries can increase profits, there is a lack of understanding of why and how vertical product lotteries work. This article therefore examines the role of consumer loss aversion in vertical product lotteries.

My analysis of a profit-maximizing monopolist serving loss averse consumers has two major findings. First, I find that if consumers are moderately loss averse, vertical product lotteries can be profit increasing for a multitude of different cost functions. I identify two distinct dynamics regarding the loss aversion threshold. On the one hand, the lower bound for the level of consumer loss aversion decreases in the marginal cost of production because the monopolist can save costs from reducing quality. On the other hand, lottery profitability is limited by an upper bound on the feasible level of consumer loss aversion. This upper bound exists due to the expected value of the lottery decreasing in the level of consumer loss aversion and thus leading to negative profit contributions. However, it corresponds to the no-dominance criterion that can be found in the literature, e.g., in Herweg et al. (2010), and therefore does not have a strong, restrictive character. Second, I provide a closed-form solution to the monopolist's problem when the marginal cost of production is $c = 0$ and consumers are sufficiently loss averse. The optimal strategy consists of offering a premium product with high, deterministic quality and a product lottery with low and stochastic quality. In addition, I show that for levels of loss aversion in line with empirical estimates, the lottery can generate over 10% profit increases compared to the case without lottery.

My results hinge on specific assumptions. First, I make a strong and simplifying assumption about the distribution of consumers in the market in order to minimize technical complexities. Although this may seem like a severe limitation at first glance, note that for most of the presented results these distribution assumptions are not necessary. Whether or not a lottery can be profit increasing is determined by the demand gradients at the cutoff points between segments, thus technically allowing for a broader range of distributional assumptions. Second, in the closed-form solution of the monopolist's optimal menu, I assume that the value of the lottery's low outcome is equal to the consumers' outside option. One may question whether consumers do, in fact, build probabilistic reference points and do not simply take the value of the "base" option as reference point. It is important to note, however, that by choosing a worst case outcome higher than the outside option, may ensure that consumers do, in fact, build a probabilistic reference points and do not use the outside option's value as heuristic. In other words, even the most unattractive lottery outcomes on Priceline are offered at a discount compared to the normal retail price and would therefore likely be considered distinct from the outside option. Third, consumers are modeled as perfectly informed with frictionless access to all relevant information. In reality, however, numerous frictions such as imperfect information processing or status-quo biases, e.g., in terms of brand loyalty, may be guiding consumers' choices. Implications may be that in reality consumers at the margin between two segments are not as quick to switch their choices as assumed in this article. Such a dynamic may even lead to lotteries being more profitable than assumed in this context.

In reality, product lotteries seem to be predominantly used in the hospitality or tourism industry, e.g. for rental cars, hotel bookings, or even restaurants selling surprise meals at reduced prices.[10] There are different aspects that may help to understand why this phenomenon seems to be relevant in the aforementioned industries. First, firms require price setting market power in order to implement price setting schemes such as lotteries. My model uses the assumption of a monopolist, which can be easily extended to the case with monopolistic competition. Monopolistic competition in turn has been argued to give a fairly accurate description of industry structure especially in the hospitality sector (Cosman and Schiff, 2019). Second, the framework of reference-dependent utility requires that consumers both have expectations about their consumption utility as well as fairly sophisticated expectations about lottery outcomes. One could argue that people feel stronger about a free upgrade to their rental car than they do about the quality of the paper they put in their printer. Third, in situations where consumer identify strongly with the outcome of a lottery, i.e., show high levels of loss aversion, profitability is most easily ensured when the cost of additional quality is low. In the tourism and hospitality industries, production or investment decisions are often made in advance and thus may lead to unused resources that could be allocated at little additional cost. Lottery structures may then be adequate tools for firms to sell unused inventory without the risk to hurt their normal product's market share.

In conclusion, this article provides a novel perspective on the role of loss aversion as a screening mechanism and suggests that it may be an important factor explaining the popular use of product lotteries as a sales channel. Even though my insights are focused on vertically differentiated product lotteries, loss aversion may be just as relevant for horizontally differentiated product lotteries. Future research may lead to important insights in two ways. Theoretical research may

[10]The app "Too Good To Go" lets users buy left-over meals at drastically reduced prices after normal business hours. Source: https://toogoodtogo.ch/

be able to find out whether more general forms of gain-loss utility lead to similar results as in this article. From an applied perspective, the insights from this article may help to generate hypotheses to empirically test the role of loss aversion in product lotteries.

References

Anderson, E. T. and J. D. Dana Jr (2009). When is price discrimination profitable? *Management Science 55*(6), 980–989.

Anderson, S. P. and L. Celik (2015). Product line design. *Journal of Economic Theory 157*, 517–526.

Anderson, S. P. and L. Celik (2020). Opaque selling. *Information Economics and Policy 52*, 100869.

Balestrieri, F., S. Izmalkov, and J. Leao (2021). The market for surprises: Selling substitute goods through lotteries. *Journal of the European Economic Association 19*(1), 509–535.

Banerji, A. and N. Gupta (2014). Detection, identification, and estimation of loss aversion: Evidence from an auction experiment. *American Economic Journal: Microeconomics 6*(1), 91–133.

Beccuti, J. and M. Möller (2018). Dynamic adverse selection with a patient seller. *Journal of Economic Theory 173*, 95–117.

Benartzi, S. and R. H. Thaler (1995). Myopic loss aversion and the equity premium puzzle. *The Quarterly Journal of Economics 110*(1), 73–92.

Bhargava, H. K. and V. Choudhary (2001). Information goods and vertical differentiation. *Journal of Management Information Systems 18*(2), 89–106.

Booij, A. S., B. Van Praag, and G. Van De Kuilen (2010). A parametric analysis of prospect theory's functionals for the general population. *Theory and Decision 68*(1), 115–148.

Camerer, C. (2005). Three cheers—psychological, theoretical, empirical—for loss aversion. *Journal of marketing research 42*(2), 129–133.

Carbajal, J. C. and J. C. Ely (2016). A model of price discrimination under loss aversion and state–contingent reference points. *Theoretical Economics 11*(2), 455–485.

Cosman, J. and N. Schiff (2019). Delivery in the city: evidence on monopolistic competition from new york restaurants.

Crawford, V. P. and J. Meng (2011). New york city cab drivers' labor supply revisited: Reference-dependent preferences with rational-expectations targets for hours and income. *American Economic Review 101*(5), 1912–32.

Deneckere, R. J. and R. P. McAfee (1996). Damaged goods. *Journal of Economics & Management Strategy 5*(2), 149–174.

Fay, S. and J. Xie (2008). Probabilistic goods: A creative way of selling products and services. *Marketing Science 27*(4), 674–690.

Fay, S. and J. Xie (2010). The economics of buyer uncertainty: Advance selling vs. probabilistic selling. *Marketing Science 29*(6), 1040–1057.

Gill, D. and V. Prowse (2012). A structural analysis of disappointment aversion in a real effort competition. *American Economic Review 102*(1), 469–503.

Gneezy, U., L. Goette, C. Sprenger, and F. Zimmermann (2017). The limits of expectations-based reference dependence. *Journal of the European Economic Association 15*(4), 861–876.

Hart, S. and N. Nisan (2019). Selling multiple correlated goods: Revenue maximization and menu-size complexity. *Journal of Economic Theory 183*, 991–1029.

Hart, S. and P. J. Reny (2015). Maximal revenue with multiple goods: Nonmonotonicity and other observations. *Theoretical Economics 10*(3), 893–922.

Heidhues, P. and B. Kőszegi (2008). Competition and price variation when consumers are loss averse. *American Economic Review 98*(4), 1245–1268.

Heidhues, P. and B. Kőszegi (2014). Regular prices and sales. *Theoretical Economics 9*(1), 217–251.

Herweg, F., D. Müller, and P. Weinschenk (2010). Binary payment schemes: Moral hazard and loss aversion. *American Economic Review 100*(5), 2451–77.

Huang, T. and Y. Yu (2014). Sell probabilistic goods? A behavioral explanation for opaque selling. *Marketing Science 33*(5), 743–759.

Jerath, K., S. Netessine, and S. K. Veeraraghavan (2010). Revenue management with strategic customers: Last-minute selling and opaque selling. *Management Science 56*(3), 430–448.

Johnson, J. P. and D. P. Myatt (2003). Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review 93*(3), 748–774.

Johnson, J. P. and D. P. Myatt (2018). The determinants of product lines. *The RAND Journal of Economics 49*(3), 541–573.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1991). Anomalies: The endowment effect, loss aversion, and status quo bias. *Journal of Economic perspectives 5*(1), 193–206.

Kahneman, D., I. Ritov, D. Schkade, S. J. Sherman, and H. R. Varian (1999). Economic preferences or attitude expressions? An analysis of dollar responses to public issues. In *Elicitation of preferences*, pp. 203–242. Springer.

Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica 47*(2), 363–391.

Kahneman, D. and A. Tversky (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty 5*(4), 297–323.

Karle, H., G. Kirchsteiger, and M. Peitz (2015). Loss aversion and consumption choice: Theory and experimental evidence. *American Economic Journal: Microeconomics 7*(2), 101–20.

Kőszegi, B. and M. Rabin (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics 121*(4), 1133–1165.

Manelli, A. M. and D. R. Vincent (2007). Multidimensional mechanism design: Revenue maximization and the multiple-good monopoly. *Journal of Economic Theory 137*(1), 153–185.

Marzilli Ericson, K. M. and A. Fuster (2011). Expectations as endowments: Evidence on reference-dependent preferences from exchange and valuation experiments. *The Quarterly Journal of Economics 126*(4), 1879–1907.

McAfee, R. P. and J. McMillan (1988). Multidimensional incentive compatibility and mechanism design.

Journal of Economic Theory 46(2), 335–354.

McAfee, R. P., J. McMillan, and M. D. Whinston (1989). Multiproduct monopoly, commodity bundling, and correlation of values. *The Quarterly Journal of Economics 104*(2), 371–383.

Mussa, M. and S. Rosen (1978). Monopoly and product quality. *Journal of Economic Theory 18*(2), 301–317.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research 6*(1), 58–73.

Pavlov, G. (2011). A property of solutions to linear monopoly problems. *The BE Journal of Theoretical Economics 11*(1).

Rabin, M. (2013). Risk aversion and expected-utility theory: A calibration theorem. In *Handbook of the fundamentals of financial decision making: Part I*, pp. 241–252. World Scientific.

Riley, J. and R. Zeckhauser (1983). Optimal selling strategies: When to haggle, when to hold firm. *The Quarterly Journal of Economics 98*(2), 267–289.

Rochet, J. and P. Choné (1998). Ironing, sweeping, and multidimensional screening. *Econometrica 66*(4), 783–826.

Rochet, J.-C. (1987). A necessary and sufficient condition for rationalizability in a quasi-linear context. *Journal of Mathematical Economics 16*(2), 191–200.

Rochet, J.-C. and J. Thanassoulis (2019). Intertemporal price discrimination with two products. *The RAND Journal of Economics 50*(4), 951–973.

Rosato, A. (2016). Selling substitute goods to loss–averse consumers: limited availability, bargains, and rip–offs. *The RAND Journal of Economics 47*(3), 709–733.

Thaler, R. H., A. Tversky, D. Kahneman, and A. Schwartz (1997). The effect of myopia and loss aversion on risk taking: An experimental test. *The Quarterly Journal of Economics 112*(2), 647–661.

Thanassoulis, J. (2004). Haggling over substitutes. *Journal of Economic Theory 117*(2), 217–245.

Zhang, Z., K. Joseph, and R. Subramaniam (2015). Probabilistic selling in quality-differentiated markets. *Management science 61*(8), 1959–1977.

Zheng, Q., X. A. Pan, and J. E. Carrillo (2019). Probabilistic selling for vertically differentiated products with salient thinkers. *Marketing Science 38*(3), 442–460.

6 Appendix

6.1 Consumer problem

In this article, consumers are assumed to be loss averse expected utility maximizers. I consider the monopolist's problem to define an optimal menu with two menu positions $m_k = (l_k, p_k)$ with $k = 1, 2$, consisting of a quality lottery l_k and a price p_k. Furthermore, I restrict my analysis to a simple lottery setup where $l_k(\bar{\beta}_k, \underline{\beta}_k, a_k)$ has two outcomes $\bar{\beta}_k$ and $\underline{\beta}_k$. The reason for that is that in this model gain-loss utility is linear around the reference point with different slopes for gains and losses. A full characterization of expected gains and losses thus consists the reference point, as well as the first moment above and below the reference point. The monopolist is able optimally calibrate R_k, g_k, and h_k by choosing low and high lottery outcomes $\underline{\beta}_k, \bar{\beta}_k \in [0, 1]$, as well as their entry probabilities $a_k = P(\bar{\beta}_k)$ and $(1 - a_k) = P(\underline{\beta}_k)$. Furthermore, let b_k be the quality differential between the high and the low outcome and thus,

$$b_k = \bar{\beta}_k - \underline{\beta}_k. \tag{15}$$

A lottery l_k is fully characterized by:

1. Expected outcome of the lottery as the unconditional expected value of the quality distribution:

$$R_k = \mathbb{E}[l_k] = \left(\underline{\beta}_k + a_k \cdot b_k\right) \tag{16}$$

2. Expected gain as the conditional expectation for the quality variable to exceed the expected value of the lottery:

$$g_k = \mathbb{E}\left[l_k | l_k \geq R_k\right] = a_k \cdot (1 - a_k) \cdot b_k \left(\underline{\beta}_k + a_k \cdot b_k\right) \tag{17}$$

3. Expected loss as the conditional expectation for the quality variable to be below the expected value of the lottery:

$$h_k = \mathbb{E}\left[l_k | l_k < R_k\right] = a_k \cdot (a_k - 1) \cdot b_k \tag{18}$$

Given any consumer's individual appetite for quality v_i, purchasing a lottery l_k therefore leads to expected gross utility of:

$$\mathbb{E}\left[u_i(\cdot)\right] = v_i \cdot (R_k + \eta \cdot (g_k + \lambda \cdot h_k)) - p_k \tag{19}$$

6.2 Generic firm problem solution

In line with Expression 2, the monopolist's profit can be written as the sum of the profits made with each menu position m_k, subject to the parameters for the cost of production and the consumers' degree of loss aversion:

$$\sum_{k=1}^{2} \Pi_k(m_k; c, \lambda, \eta) \tag{20}$$

I solve this problem sequentially by first writing the profit as a function of all relevant variables and parameters. Then I optimize the profit over both menu items to find the interior optima for p_2 and p_1. I then examine the resulting expression for the monopolist's profit in terms of the optimal quality choices in the two menu positions m_k with $k = 1, 2$.

I solve the firm problem sequentially where I first maximize profit and derive interior optima for the prices p_1 and p_2. Using these optimal prices, the monopolist's profit is then written as a function of the different quality choices of the monopolist. As shown in Section 3.2.2, the monopolist in the premium segment always offers a deterministic level of product quality $\beta^* = 1$. *Note that for further reference and better distinction, the premium segment is denoted by P and the lottery as L.* In the premium segment, expected gains and losses are zero and hence, $g_P = h_P = 0$ with $R_P = 1$. Gross utility in the premium segment thus equals the individual valuation for quality minus the price, i.e., $v_i - p_P$.

As highlighted in Figure (1), the boundaries between the lottery segment and the premium segment are given by $\hat{v}_L(\cdot)$ and $\hat{v}_P(\cdot)$. The share of consumers opting for the lottery is then given by $\hat{v}_P - \hat{v}_L$, where $\hat{v}_P$ is the consumer's valuation that is indifferent between the lottery and the premium product, whereas $\hat{v}_L$ is the consumer's valuation that is indifferent between the lottery and the outside option. The lower boundary $\hat{v}_L(\cdot)$ can be understood as the marginal consumers for which the participation constraint is met in order to be served by the monopolist. It requires that the marginal consumer choosing the lottery experiences utility at least high as the outside option of 0:

$$v_i \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) - p_L \geq 0$$

The consumer located at the cutoff $\hat{v}_L$ is then given by:

$$\hat{v}_L \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) - p_L \stackrel{!}{=} 0. \tag{21}$$

Solving Expression (21) for $\hat{v}_L$ gives the consumer that is just indifferent between the lottery and the outside option:

$$\hat{v}_L = \frac{p_L}{(R_L + \eta \cdot (g_L + \lambda \cdot h_L))} \tag{22}$$

The cost of quality in the lottery segment is $c \cdot R_L$ and thus, the profit in the lower segment can be written as:

$$\Pi_L(p_L, \cdot) = (p_L - c \cdot R_L)\,(\hat{v}_P - \hat{v}_L) \tag{23}$$

Replacing $\hat{v}_L$ from Expression (22) in Expression (23) and optimizing for p_L gives the following FOC:

$$\frac{\partial \Pi_L(p_L, \cdot)}{\partial p_L} \stackrel{!}{=} 0$$

$$\iff p_L^*(\hat{v}_p) = \frac{1}{2} \cdot (c \cdot R_L + \hat{v}_p \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L))) \tag{24}$$

The marginal consumer $\hat{v}_P$, indifferent between purchasing the premium product at price p_P and the lottery at price p_L^* is characterized by the following equality:

$$\hat{v}_P - p_p \stackrel{!}{=} \hat{v}_P \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) - p_L^* \tag{25}$$

Replacing p_L^* with Expression (24) and solving for $\hat{v}_P$ gives us:

$$\hat{v}_P - p_p \overset{!}{=} \hat{v}_P \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) - \frac{1}{2} \cdot (c \cdot E(\beta) + \hat{v}_p \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)))$$

$$\iff \hat{v}_P - p_p \overset{!}{=} \hat{v}_P \cdot \left(1 - \frac{1}{2}\right) \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) - \frac{1}{2} c \cdot R_L$$

$$\iff \hat{v}_P \overset{!}{=} \frac{1}{2} \cdot \hat{v}_P \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L)) + p_p - \frac{1}{2} c \cdot R_L$$

$$\iff \hat{v}_P \cdot \left(1 - \frac{1}{2}(R_L + \eta \cdot (g_L + \lambda \cdot h_L))\right) \overset{!}{=} p_p$$

$$\iff \hat{v}_P \overset{!}{=} \frac{(2 \cdot p_p - c \cdot R_L)}{(2 - (R_L + \eta \cdot (g_L + \lambda \cdot h_L)))} \tag{26}$$

Plugging in Expression (26) for $\hat{v}_P$ into the solution for $p_L^*(\hat{v}_P)$ in Expression (24) and $\hat{v}_L(p_L)$, we can rewrite:

$$p_L^*(\hat{v}_p, \cdot) = c \cdot R_L - p_p + \frac{(2 \cdot p_p - c \cdot R_L)}{(2 - (R_L + \eta \cdot (g_L + \lambda \cdot h_L)))} \tag{27}$$

$$\hat{v}_L = \frac{1}{2} \cdot \left(\frac{2 \cdot p_p - c \cdot R_L}{2 - (R_L + \eta \cdot (g_L + \lambda \cdot h_L))} + c \cdot \frac{c \cdot R_L}{(R_L + \eta \cdot (g_L + \lambda \cdot h_L))} \right) \tag{28}$$

I now look at total profit to find the interior optimum for p_P^*:

$$\Pi_{P,L}(\cdot) = (p_P - c)(1 - \hat{v}_p) + (p_L - c \cdot R_L) \cdot (\hat{v}_P - \hat{v}_L)$$

Plugging the solutions from Expressions (26), (27), and (28) gives:

$$\Pi_{P,L}(\cdot) = (p_P - c) \cdot \left(1 + \frac{(2 \cdot p_P - c \cdot R_L)}{(2 - (R_L + \eta \cdot (g_L + \lambda \cdot h_L)))}\right) +$$
$$\frac{\Big(c \cdot R_L - p_P(R_L + \eta \cdot (g_L + \lambda \cdot h_L))\Big)}{\big(2 - (R_L + \eta \cdot (g_L + \lambda \cdot h_L))\big)^2 \cdot (R_L + \eta \cdot (g_L + \lambda \cdot h_L))} \tag{29}$$

Optimizing Expression (29) for the interior optimum of p_P then gives us the following FOC:

$$\frac{\partial \Pi_{P,L}(\cdot)}{\partial p_P} \stackrel{!}{=} 0$$

$$\iff p_P^* = \frac{(2-(R_L+\eta\cdot(g_L+\lambda\cdot h_L)))^2}{(-8+6\,(R_L+\eta\cdot(g_L+\lambda\cdot h_L)))}$$

$$+\frac{c\cdot(-4+2\eta\cdot g_L+R_L(2+R_L+\eta\cdot g_L))+(2+R_L)\cdot\eta\lambda\cdot h_L)}{(-8+6\,(R_L+\eta\cdot(g_L+\lambda\cdot h_L)))} \tag{30}$$

Plugging Expression (30) into the monopolist's profit function in Expression (29):

$$\Pi_{P,L}(\cdot) = \frac{1}{36}\cdot\Big(8-24c+6c\cdot R_L-3(R_L+\eta\cdot(g_L+\lambda\cdot h_L))+$$
$$\frac{9c^2\cdot R_L^2}{R_L+\eta\cdot(g_L+\lambda\cdot h_L)}+\frac{4(1-3c(-1+R_L))^2}{4-3(R_L+\eta(g_L+\lambda h_L))}\Big) \tag{31}$$

6.3 Calculations for model findings

6.3.1 Optimal strategy with deterministic product quality

From the previous analysis I know that the optimal strategy of a monopolist in the deterministic case consists of one premium segment with maximum quality, $\beta_P^*=1$ and $\beta_L^*=0$. In a first step, I look at the case where the monopolist offers a menu with two degenerate lotteries in order to model deterministic quality levels. Note that if quality is deterministic, consumers do not experience reference dependent gains or losses, as expectation about quality and realized quality perfectly coincide, i.e. $b=0, a\in[0,1]$ and $R_k=\beta_k$. Substituting the Expressions (15), (19), (17), and (18) into the profit function given in Expression (31), gives Expression (32). This expression is the monopolistic profit as a function of the deterministic quality level β_P, with optimal (interior) quality $\beta_P^*=1$ and prices p_P^* and p_L^*.

$$\Pi_{P,L}(\cdot) = \frac{1}{36}\cdot\left(8-24c+6c\cdot\underline{\beta}_L-3\underline{\beta}_L+\frac{9c^2\cdot\underline{\beta}_L^2}{\underline{\beta}_L}+\frac{4(1-3c\cdot(-1+\underline{\beta}_L))^2}{(4-3\underline{\beta}_L)}\right)$$

$$\iff \Pi_{P,L}(\cdot) = \frac{(1-c)^2\cdot(2-\underline{\beta}_L)^2}{4(4-3\underline{\beta}_L)} \tag{32}$$

The overall decrease in profit shows that there is no interior optimum for β_L and the firm's optimal choice is $\beta_L^* = 0$ or $\beta_L^* = 1$, which leads to:

$$\Pi_{P,L}(\beta_P, \underline{\beta}_L, \bar{\beta}_L) = \frac{(1-c)^2}{4} \tag{33}$$

6.3.2 Calculations for conditions for optimality of lotteries

Again, we know that the optimal strategy of a monopolist in the deterministic case consists of one premium segment with maximum quality, $\beta_P^* = 1$ and $\beta_L^* = 0$. The profit expression (33) serves as benchmark in order to determine under which condition lotteries are a beneficial addition to the firm strategy. In fact, profit increases due to randomness are possible if a marginal increase of randomness, leads to marginal net growth of profit compared to the optimal solution for deterministic quality choices. I therefore express the monopolist's profit function with optimal prices p_P^* and p_L^* as a function of the lottery parameters a_L and b_L:

$$\begin{gathered}\Pi_{P,L}(a_L, b_L; \eta, \lambda, c) = \\ \Big[-((a_L-1)a_L b_L \eta\lambda + a_L\eta(b_L - a_L b_L) + a_L b_L + \underline{\beta}_L - 2)^2 + \\ c\Big(2a_L\eta(b_L - a_L b_L) + (a_L - 1)a_L b_L \eta\lambda(a_L b_L + \underline{\beta}_L + 2) + (a_L b_L + \underline{\beta}_L)\Big) + \\ c\Big((a_L b_L + \underline{\beta}_L)(a_L\eta(b_L - a_L b_L) + a_L b_L + \underline{\beta}_L + 2) - 4)\Big)\Big] \cdot \\ \frac{1}{\Big(6\eta((a_L-1)a_L b_L\lambda + a_L(b_L - a_L b_L)) + 6(a_L b_L + \underline{\beta}_L) - 8\Big)}\end{gathered} \tag{34}$$

I now induce marginal uncertainty into the former optimum in Expression (34) by choosing $a_L \in (0,1)$ and by allowing for positive quality differential between the high and the low lottery outcome $b_L = \bar{\beta} - \underline{\beta}$. I then compute the partial derivative of $\Pi_{P,L}$ with respect to b_L and evaluate it at $b_L = 0$ whether a positive quality differential leads to a profit increment:

$$\begin{gathered}\frac{\partial \Pi_{P,L}(\cdot)}{\partial b_L}\Big|_{\beta_1^*=1, \underline{\beta}=0, b_L=0} = \\ \frac{a_L(1 + c(-1 + (1-a_L)\eta(1-\lambda))) + (1-a_L)\eta(1-\lambda)}{16(1 + (1-a_L)\eta(1-\lambda))} \\ \cdot \frac{(-1 + c + 3c\cdot(1-a_L)\eta(1-\lambda) - \eta(1 - a_L - \lambda + a_L\lambda))}{1}\end{gathered} \tag{35}$$

If the partial derivative in Expression (35) is positive, it implies that increasing the quality differential between the two outcomes and hence offering a non-trivial product lottery, leads to an overall increase of profit over both segments. Evaluating the lottery profitability condition in Expression (36)

$$\frac{\partial \Pi_{P,L}(\cdot)}{\partial b_L}|_{\beta_1^*=1,\underline{\beta}=0,a_L\in(0,1),b_L=0} > 0 \tag{36}$$

gives three distinct sets of conditions under which a product lottery is profit maximizing:

1. For $c = 0$, marginal profit increases in b_L for any level of loss aversion exceeding the lower threshold of

$$\underbrace{1+\frac{1}{\eta}}_{\hat{\lambda}^{low}} \quad < \quad \lambda \tag{37}$$

where

$$\lim_{a_L\to 0} \hat{\lambda}^{low} = 1+\frac{1}{\eta}. \tag{38}$$

2. For $c \in (0, 1)$, marginal profit increases in b if consumer loss aversion lies between the lower threshold λ_1^{low} and the upper threshold λ_1^{high}

$$\underbrace{1+\frac{(1-c)}{\eta(1-a_L)(1+c)}}_{\lambda_1^{low}} \quad < \quad \lambda \quad < \quad \underbrace{1+\frac{1}{\eta(1-a_L)}}_{\lambda_1^{high}} \tag{39}$$

where

$$\lim_{a_L\to 0} \lambda_1^{low} = 1+\frac{1-c}{\eta(1+c)}, \tag{40}$$

$$\lim_{a_L\to 0} \lambda_1^{high} = 1+\frac{1}{\eta}. \tag{41}$$

3. Figure (3) shows how for $c \in (0, \frac{1}{3})$, there is an additional lower bound for loss aversion λ_2^{low}, with $\lambda_2^{low} > \lambda_1^{high}$. For this range of c, introducing randomness increases profits if either the coefficient of loss aversion lies in the range given by inequality (39) or it exceeds

the limit of λ_2^{low}

$$\underbrace{1 + \frac{(1-c)}{(1-a_L-3c(1-a_L))\eta}}_{\lambda_2^{low}} < \lambda \tag{42}$$

where

$$\lim_{a_L \to 0} \lambda_2^{low} = 1 + \frac{(1-c)}{(1-3c)\eta} \tag{43}$$

6.3.3 Calculations for characterization of optimal menu with lotteries

I consider the monopolists profit given in Expression (34) where the marginal cost of production is $c = 0$. If Expression (37) does not hold, i.e., $\lambda < 1 + \frac{1}{\eta}$, the profit-maximizing strategy for the monopolist is identical to the deterministic benchmark case (33). The reason is that in this case the lottery would not be able to segment consumers sufficiently. This would mean that the monopolist loses more profit in the premium segment than it gains in profit in the cheap segment of the market. Therefore, it would be optimal to offer no quality gradation and would simplify the monopolist's decision problem to the problem with deterministic product quality. I therefore focus now on the case where consumers are sufficiently loss averse and Expression (37) holds. Expression (44) shows that the monopolist's profit monotonously increases with b.

$$\frac{\partial \Pi_{P,L}(\cdot)}{\partial b_L} > 0 \tag{44}$$

The monopolist therefore optimally maximizes the quality differential in the lottery by choosing

$$\begin{aligned} \underline{\beta}_L^* &= 0, \\ \bar{\beta}_L^* &= 1, \end{aligned} \tag{45}$$

Plugging values from (45) into Expression (34) for the firm's profit simplifies to:

$$\Pi_{P,L}(a_L; \lambda, \eta) = \frac{(2 + a_L^2\eta(-1+\lambda) + a_L(1+\eta(1-\lambda)))^2}{4(4 + a_L(3-3\eta(-1+\lambda)) + 3a_L^2\eta(-1+\lambda))} \tag{46}$$

Maximizing Expression (35) for the optimal choice a gives

$$a_L^* = \frac{1}{2} + \frac{1}{(2\eta - 2\eta\lambda)}. \tag{47}$$

Using the Expressions (45) and (47) to specify the optimal prices ((30) and (24)) and the points of indifference between the premium product, the lottery, and the outside option ((26) and (28)) gives:

$$p_P^* = \frac{(1 + \eta(6 + \eta(-1 + \lambda))(-1 + \lambda))^2}{(8\eta \cdot (3 - \eta \cdot (1 - \lambda))(1 - 3\eta \cdot (1 - \lambda))(-1 + \lambda))} \tag{48}$$

$$p_L^* = \frac{\left((1 + \eta(6 - \eta(1 - \lambda))(-1 + \lambda))(1 + \eta(1 - \lambda))^2\right)}{(8\eta \cdot (3 - \eta \cdot (1 - \lambda))(1 - 3\eta \cdot (1 - \lambda))(1 - \lambda))} \tag{49}$$

$$v_p = \frac{1}{3} + \frac{1}{(3 - \eta(1 - \lambda))} - \frac{1}{(3 - 9\eta \cdot (1 - \lambda))} \tag{50}$$

$$v_L = \frac{1}{6} \cdot \left(1 + \frac{3}{3 - \eta(1 - \lambda)} + \frac{1}{(\eta - 3\eta\lambda)}\right) \tag{51}$$

Based on the solutions (48) to (51), the firm's profit can be expressed through λ and η:

$$\Pi_{P,L}(\cdot; \lambda, \eta) = \frac{(-3 + \eta(6 + \eta \cdot (1 - \lambda))(1 - \lambda))^2}{(16\eta(-9 + \eta \cdot (10 + 3\eta \cdot (1 - \lambda))(1 - \lambda)) \cdot (-1 + \lambda))} \tag{52}$$