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Bank as a Venture Capitalist

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Abstract

Banks all over the world show interest in acting as venture capitalists. In this paper, I argue that banks offer venture capital (VC) financing along with traditional (collateralized) loans in response to the natural constraints of the hidden information that they face. Innovative entrepreneurs pursue new technology that promises high return but runs a high risk of failure. The more innovative entrepreneurs also have higher reservation utility. This interaction between type-dependent returns and reservation utility creates a situation where collateral alone is not sufficient to screen entrepreneurs, and the uninformed bank needs an additional screening device. VC fulfils that role.

JEL Classification: G21, G24, D86

Keywords: Bank, Venture Capital, Collateral, Debt, Screening

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1 Introduction

An economy supports innovation by providing capital to start-ups. A well-functioning market for entrepreneurial capital requires that financiers overcome several informational frictions. That is why the market requires specialist investors, such as venture capitalists. Venture capitalists specialize in screening projects and in providing value-adding services to start-ups. Banks have always shown interest in pursuing venture capital (VC) financing alongside their core lending business. However, banks face several regulatory hurdles that limit their participation in VC financing. In this paper, I argue that in a start-up capital market characterized by hidden information problem, a bank can efficiently screen borrowers by including both collateralized credit and VC financing in its offerings.

I present a model of intermediation in which several entrepreneurs face one bank that enjoys all the bargaining power. Entrepreneurs differ in how *innovative* they are. The more innovative entrepreneurs own start-ups (projects) that generate high returns when they are successful. However, the more innovative projects involve the application of new, untested technology, and therefore they run a higher risk of failure. More innovative entrepreneurs also have a higher reservation utility. An entrepreneur's *type* is their *degree* of innovativeness, which is their private information. There are three different types of entrepreneurs, but they all appear the same to the bank. I show that when the reservation utility of the entrepreneurs is sufficiently responsive to innovativeness, there is an incentive for entrepreneurs of the medium type to mimic *both* the low and the high types.

In such a scenario, offering only collateralized credit in its menu of contracts does not help the bank in *fully* separating the types. Appending this menu with a new contract that offers value-adding benefits to the entrepreneurs against a *high* repayment solves the problem of separation. This new contract is essentially a VC contract. With the extended menu on offer, the high-innovative types *self-select* into a VC contract, whereas the low-innovative types self-select into collateralized credit and the medium-innovation types into uncollateralized credit. As a corollary, not allowing the bank to offer a VC contract means the bank will have to *pool* some types. If the bank is unwilling to pool, it may price out the more innovative types, resulting in an inefficiently low level of start-up financing.

The novel mechanism in my model that necessitates an additional screening device is that the preference to mimic is not monotonic. This is a consequence of the interaction between the type-dependent project returns and the type-dependent reservation utility. To understand this, consider the benchmark when the bank is informed in the sense that it can observe the type of the entrepreneur. In this case, to maximize its surplus, the bank would charge a type-specific interest rate, such that each entrepreneurial type is just indifferent between borrowing and not borrowing. If an entrepreneur decides not to borrow, they derive the reservation utility associated with their type.

Now, note that the return on the project increases with the degree of innovativeness. Therefore, an informed bank would want to charge a higher interest to more innovative types. However, more innovative types have higher reservation utility and therefore, to attract them, the bank needs to *sweeten* their offer to them. This puts downward pressure on the interest the informed bank can charge to higher types. These two opposing forces may lead to a situation where the informed bank charges lower interest not only to the low type but also to the high types owing to the dominance of the latter force, while the medium type pays the highest interest rate. If the bank offered the same contracts when the type is unknown, the medium type would have incentive to imitate both low and the high types as they both pay lower rate than the medium type.

In this situation, employing collateral alone would not be effective in dissuading the medium type from imitating the other two types. By introducing a high-collateral-low-interest contract for the low type of entrepreneur, the bank may be able to deter the medium type from mimicking the low type. This is because the low types engage in safer projects than the medium types and, therefore, the low types are *relatively* more willing to pledge collateral than the medium type of entrepreneurs, whose projects are riskier. Thus, the bank could devise a contract for the low types requiring collateral *high enough* that the medium types would find it unattractive. However, requiring collateral from high types does not work when the bank attempts to dissuade the medium types from mimicking the high types. This is because the most innovative types also have the riskiest projects, and therefore they are *relatively* less willing to pledge collateral than the medium types. Thus, any high-collateral-low-interest contract that the bank devises for the high types, respecting their aversion for collateral, will also be attractive to the medium types.

What would effectively separate the medium types from the high types? The medium types run safer projects than the highly innovative types, therefore, they are more likely to repay than the high types. Therefore, medium types are *relatively less* willing than the high types to pledge a large repayment in the event of success. How could the bank ask for a large repayment from the high types without making the contract unattractive for them? They could do this by including in the contract some "unconditional benefits" for the entrepreneurs. Unconditional benefits refer to those benefits that the entrepreneur receives regardless of (and possibly before) the realization of the project outcome. If the bank designs a contract for the high types under which it transfers to them (in addition to the capital) some value-adding benefits and in return charges them a high repayment amount in the case of success, then the medium type would be dissuaded from choosing that contract. A contract that combines value-adding benefits for the entrepreneurs with a high repayment requirement is essentially a VC contract.

As evident from the above discussion, in my model, the value-adding investment that yields unconditional benefits for the entrepreneur is the defining feature of the VC contract. In fact, it is these benefits that the bank uses as a device to screen the borrowers. A real-life example of unconditional benefits provided by venture capitalists is the cash compensation for the founder (Ewens, Nanda and Stanton, 2020; Bengtsson and Hand, 2011). Other examples of benefits include several mentoring and professionalization services that the VC provides, as well as benefits that stem from certification or reputation (Hsu, 2004; Hellmann and Puri, 2002; Gorman and Sahlman, 1989; Sahlman, 1990).

Contrary to the widely held belief that banks finance only relatively mature firms, the recent literature has established that banks are also quite active in financing start-ups. However, due to various restrictions on their private equity activities, banks generally finance start-ups through the debt market. In an influential study on start-ups in the United States, Robb and Robinson (2014) find that bank credit is the most prominent form of financing for the newly founded firms. Given that most start-ups do not have a ready cash flow at the time of borrowing, most of this borrowing is through the personal collateral of the entrepreneur (Robb and Robinson, 2014; Avery, Bostic and Samolyk, 1998). The personal collateral is most often the real-estate assets of the entrepreneur (Schmalz, Sraer and Thesmar, 2017). In fact, many firms in the hi-tech sector have also begun to pledge their patents as collateral (Hochberg, Serrano and Ziedonis, 2018; Mann, 2018; Kerr and Nanda, 2015; Ibrahim, 2010).

As against the widely held belief that banks finance only relatively mature firms, recent literature has established that banks are also quite active in financing start-ups. Although, due to various restrictions on their private-equity activities, banks finance start-ups majorly through the debt market. In an influential study on start-ups in the USA, Robb and Robinson (2014) find that bank credit is the most prominent form of financing for the newly founded firms. Given that most start-ups do not have a ready cash flow at the time of borrowing, most of these borrowing are through personal collateral of the entrepreneur (Robb and Robinson, 2014; Avery, Bostic and Samolyk, 1998). The personal collateral mostly takes the form of real-estate assets of the entrepreneur (Schmalz, Sraer and Thesmar, 2017). In fact, many firms in the hi-tech sectors have also started to pledge their patents as collateral (Hochberg, Serrano and Ziedonis, 2018; Mann, 2018; Kerr and Nanda, 2015; Ibrahim, 2010).

My results point out that in the start-up capital market, banks should actively participate, not only through collateralized debt, but also through VC financing. However, banks' private equity activities are highly regulated, and they face restrictions that relate to their involvement as an equity holder in a venture (Hellmann, Lindsey and Puri, 2007). Consistent with my model predictions, however, we observe that banks are interested in VC financing as well. This is evident from the fact that despite several regulatory hurdles, banks have always found ways to carry out some form of VC financing. For instance, in the United States, before the Gramm-Leach-Bliley Act of 1999 allowed them to engage in various *private equity* related activities, banks undertook VC financing by setting up small business investment corporations.[1] Again, after the Volcker rule under the Dodd-Frank Act of 2010 in the United States, which restricted banks' participation in private equity, banks resorted to finding ways around these sanctions (Rothacker, 2013). Perhaps due to these concerns, US bank regulators eventually eased some provisions of the Volcker rule that related to banks' VC investments.[2]

Most major banks in advanced economies conduct their (restricted) VC activities through their VC arms.[3] Because banks frequently resort to making use of regulatory loopholes to make indirect VC investments, it is difficult to gauge the precise size of their VC investments. Therefore, the best we can do is to look at the officially announced investments made directly by banks, for example, through their VC arms. In studying this data from the Thomson ONE database, we observe a large cross-regional variation: bank investment accounted for around 6% of the total VC investment in Western Europe and East Asia, but around 2% percent in North America during the period 2013-17 (Figure A.1 in Appendix).[4] Indeed, recent evidence points out that banks have become even more active with their VC investments following the boom in fintech start-ups (Barba and Macheel, 2016; Lorenzetti, 2014).

Literature and contributions: Hellmann, Lindsey and Puri (2007) put forward an explanation of banks' foray into the VC market. They propose that making VC investments allows

[1]See the discussion in Hellmann, Lindsey and Puri (2007) and Hellmann (1997).

[2]See the press release on the easing of the Volcker rule (Federal Reserve Board, 2020).

[3]Citibank with its VC arm, Citi Ventures, and Mitsubishi UFJ Financial Group with its VC arm Mitsubishi UFJ Capital are some examples. In emerging economies as well, banks are becoming increasingly active in VC financing. For instance, in India and Indonesia, large banks have entered the VC market with the intent to fund start-ups (Shukla and Vyas, 2016; The Jakarta Post, 2017). In China, the largest banks were allowed by the China Banking Regulatory Commission to enter VC markets on a pilot basis in 2016 (Mak, 2016). However, Chinese banks had been making VC investments through their offshore investment affiliates before that as well (Reuters, 2016).

[4]Other than missing the indirect investments that the banks do in VC due to regulatory reasons, Thomson ONE also has another shortcoming in that its categorization of investments into different type of institutions is not perfect (Hellmann, Lindsey and Puri, 2007). An alternative source, for the European countries, is the annual report of *Invest Europe*. They report that, of the Euro 7.7 billion raised by VC funds in 2017, banks contributed around 6% percent (Invest Europe, 2018). For the United States, the National Venture Capital Association (NVCA) states that this number was 7% in 2011, and in the areas outside of the traditional VC centers, this share was even larger, for example, it might have been 13% in the upper Midwest region (NVCA, 2017).

banks to build relationships with firms that they see as potential future customers of their banking services. Using data from the United States, Hellmann, Lindsey and Puri (2007) find that, compared to the start-ups financed by non-banks, start-ups that have banks as a VC investor are more likely to take a loan from the investor bank at a future date. They also find that although banks invest more in the early stages of a start-up than in later stages, relative to individual venture capitalists, banks' investment is more concentrated in the later stages. The authors then relate this observation to their hypothesis that banks' main motive in getting involved in a start-up is to have a captive borrower base for the future.

In contrast to Hellmann, Lindsey and Puri (2007), my model predicts that banks will also be active in the early (seed) stages of a venture, at which time the bank screens different start-ups. While we do see banks making a major chunk of their VC investment at the seed stage, their lower activity at the early stage relative to venture capitalists could be driven by regulatory reasons. For instance, due to fixed costs of regulations, banks may find it remunerative to invest only large sums of money, which they may prefer to invest only after the start-up has matured and uncertainties about its value have resolved.

The results of my model imply that allowing banks to make VC investments solves the adverse selection problem. Therefore, expanding the types of contracts that banks can offer may increase access to financing for start-ups. With this, I also contribute to the literature that has studied the normative question of whether a bank should be allowed to make VC investments. To be clear, this normative question requires studying costs and benefits of deregulating banks' VC business along several dimensions. My contribution is to put forward a novel channel through which the deregulation of banks' VC investment may achieve benefit. There are other considerations, such as risk-taking issues and relationship benefits, which are not modeled in this paper but are relevant for this question. Theoretical and empirical papers that discuss the pros and cons of bank's equity holdings in start-ups include Von Beschwitz and Foos (2018); Ferreira and Matos (2012); Berlin, John and Saunders (1996); Santos (1999); John, John and Saunders (1994); Mahrt-Smith (2006); Boyd, Chang and Smith (1998).

The early literature on VC focused on how entrepreneurs choose *between* a bank and an individual venture capitalist. This literature, therefore, highlighted the comparative advantages of the two kinds of financial institutions. Ueda (2004), for instance, assumes that venture capitalist and bank differ in their abilities to screen entrepreneurs: a VC can perfectly observe the borrower type, whereas a bank can do so only imperfectly. However, a venture capitalist poses the threat that it can expropriate the project from the entrepreneur. In Winton and Yerramilli (2008), venture capitalists are assumed to have better monitoring capability, but a higher cost of capital as compared with banks. In De Bettignies and Brander (2007), bank and VC differ in the extra-financial support they provide: the venture capitalist actively participates in the entrepreneur's business by providing value-adding services, such as managerial inputs, while the bank is only a passive investor. The venture capitalist's input directly affects the output of the firm it invests in. This study derives conditions under which the start-up chooses one kind of financing over the other. Another paper in this strand of literature is Andrieu and Groh (2012). In this paper, the authors formulate a model that compares bank-affiliated VC and independent VC. They highlight the trade-off that these two funding channels offer in a set-up where funding is required over different periods. An independent VC firm provides better support quality but has limited resources, which does not guarantee future support for the start-up. A bank-affiliated VC has deep pockets but has less expertise in providing any other kind of support. The model predicts, among other things, that firms with less sophisticated ventures would choose bank-affiliated VC.[5]

My paper differs with the above literature in the following way. In the literature cited above the focus is on the question of bank *versus* VC. Therefore, this literature models banks and venture capitalists as two different sources of institutional finance. My paper, explains why do we observe one single institution (the bank) offering financing choices, some of which resemble passive collateralized debt while other resemble active VC investments. In other words, the fundamental question I ask is the following: why would banks choose to offer VC financing to entrepreneurs when they also have the option to finance them with debt contracts? Therefore, I do not impose the types of contracts that the bank can offer exogenously. I endogenize the funding choices that the bank can use. The bank sees VC as an alternative form of finance to collateralized credit and includes both the types of financing in its menu of offerings to separate the borrower types.

Another strand of VC literature where banks feature, albeit indirectly, studies the differences between an independent VC and a corporate owned or a captive VC. A bank-affiliated VC falls into the latter category. Bottazzi, Da Rin and Hellmann (2008) study the differences in the level of activism of two types of VCs in terms of their involvement in monitoring and supporting the portfolio companies. They conclude that captive VCs–including the bank affiliated VCs–showed less activism in their portfolio companies than the privately held independent VCs. However, the objective of Bottazzi, Da Rin and Hellmann (2008) was not to explain why banks would choose to own a VC arm in the first place.

Another contribution of my study is in terms of the modeling risk-returns relationship. The existing literature broadly follows two paths when modeling project risk and its relationship to project returns. Models following Stiglitz and Weiss (1981) (henceforth SW) in a classical set-up

[5] Da Rin, Hellmann and Puri (2013) provide an excellent survey of this literature.

with one-dimensional heterogeneity assume projects differ only in risk, producing homogeneous *expected returns*. Other models allowing projects to differ in their expected returns as well as in risk, for simplicity, assume these two factors to be perfectly correlated (in contrast, this correlation is zero in SW by design). Further, the latter kind of model usually assumes that success-state returns are homogeneous across types. The most prominent studies based on this set-up are Besanko and Thakor (1987) (henceforth BT) and De Meza and Webb (1987) (henceforth DW). In Hellmann and Stiglitz (2000), entrepreneurs differ in two privately observed dimensions–both in risk and in expected returns, with no perfect correlation between the two dimensions.

In my paper, I continue with heterogeneity in both dimensions (risk and expected returns) with perfect correlation, however, I do not restrict entrepreneurs to producing the same returns conditional on success. I allow for a more general relationship between risk and expected return with SW-type no correlation between risk and expected return being one special case. Among other relations between risk and expected returns, my returns function is also capable of producing negative correlation between risk and expected returns, i.e., the case of risk-expected returns trade-off.

Finally, at a more technical level, my study generalizes the result in Besanko and Thakor (1987) and Lengwiler and Rishabh (2018) that establishes that a monopolist bank does not find collateral effective for screening borrowers. In this study, I show that when the monopolist bank faces entrepreneurs with type-dependent returns and type-dependent reservation utilities, we may find that collateral is not effective, fully effective, or only partially effective as a screening device. In my set-up, Besanko and Thakor (1987) and Lengwiler and Rishabh (2018) are a special case in which returns function and reservation utility are constant functions of types. In addition, I show that when collateral is only partially effective or fully ineffective, the bank can use VC financing as a tool to solve the adverse selection problem.

This paper is organized as follows: in Section 2, I describe the basic elements of the model. In Section 3, I analyze the full information benchmark with which all other results will be compared. In Section 4, I discuss the interactions of the uninformed bank with various entrepreneurs. This section contains the main results of the study, along with some numerical examples. In Section 5, I discuss the alternative assumptions to those that I make in earlier sections. I conclude the discussion in Section 6.

2 Model

The economy consists of risk-neutral entrepreneurs and a risk-neutral bank. Each entrepreneur has a start-up (project) that requires one unit of capital. However, the start-ups do not own any capital and, therefore, require external financing to run the project. For the bank, the gross financing cost of capital (which can be thought of as the cost of deposits) is the risk-free rate that I normalize to be equal to one. The economic activities are organized over two periods. In the first period, projects are financed, and in the second period, project outcomes are materialized.

Entrepreneurs differ in how innovative they (or their projects) are. We index the entrepreneurial types with $e \in (0,1)$. The higher the e, the more entrepreneurial (innovative) the agent is. Entrepreneurial type, e, is information private to the entrepreneur, but the distribution of e is common knowledge. While bank may easily be able to differentiate an entrepreneur who plans to run a grocery store from one who requires capital to run a biotechnology firm, it may find it difficult to distinguish two firms with business plans that involve some futuristic biotechnology concept. We assume that the bank faces a market of the latter kind, where firms cannot be easily distinguished from each other based on observable features. Equivalently, the bank does not have the ability to fully understand the differences in the potential of these projects, making it difficult for it to distinguish between the entrepreneurial types. An entrepreneur's type, e, completely determines the risk-return characteristics of the project they runs as well as their reservation utility.

2.1 Project returns and risk

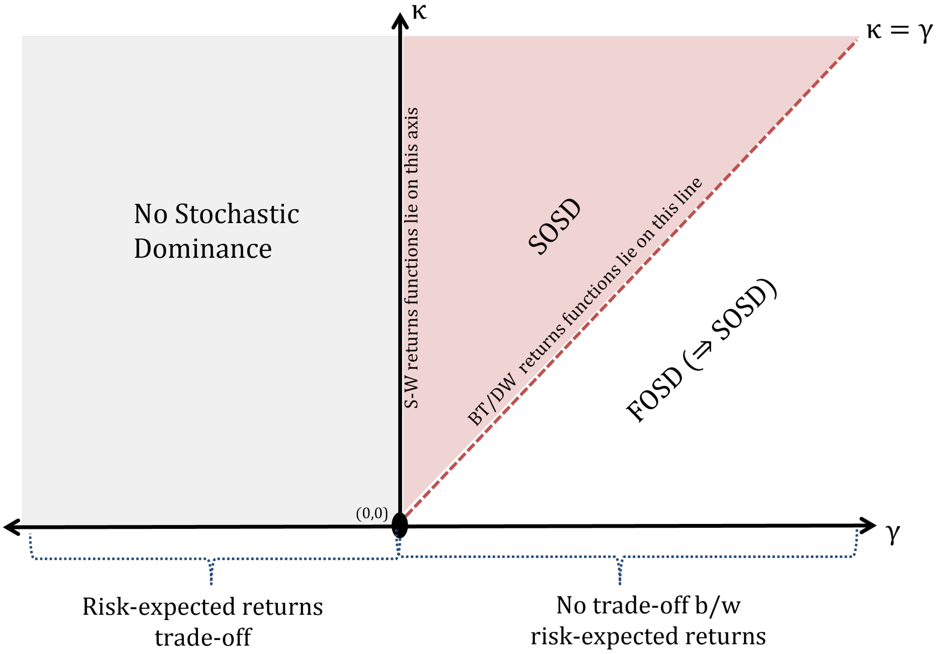
We assume a binomial outcome scenario where more innovative entrepreneurs run projects that promise high returns conditional on being successful but face a higher risk of failure. In the case of failure, a project produces zero returns. This situation can arise, for example, when the more innovative projects involve the application of an untested technology. Returns functions with this property are quite common in the banking literature. For instance, the binomial outcome version of the returns function in Stiglitz and Weiss (1981) has the same property. Other examples include Karlan and Zinman (2009); Minelli and Modica (2009); De Aghion and Gollier (2000); Ghatak (2000, 1999); House (2006).

An entrepreneur of type e runs a project that produces zero returns with probability e and $Y(e) > 0$ with probability $(1-e)$. That is, the degree of innovativeness (e) is perfectly correlated with the risk of the project. Therefore, we can also call the entrepreneurial types in our model the risk types, with higher e representing a higher risk of failure. For the type e entrepreneur, the project returns conditional on being successful are given by:

$$Y(e) = \frac{\kappa - \gamma e}{1 - e}; \quad \text{with} \quad \kappa \geq 0, \quad \kappa \geq \gamma. \tag{1}$$

The project returns function in 1 can generate several interesting risk-expected returns relations, including those commonly employed in the existing literature. Figure 1 presents all such relations that can be generated in the κ–γ plane. In the banking literature, there are broadly two distinct approaches. In the first—the BT/DW approach—returns conditional on success are assumed to be constant across all risk types (failure state returns, which are usually assumed to be zero, are by design the same across all risk types). Therefore, in this approach, the project returns are ordered in the first-order stochastic dominance (FOSD) sense (Besanko and Thakor, 1987; De Meza and Webb, 1987). This specification implies that expected returns decrease with risk. In Figure 1, the BT/DW type returns functions correspond to $\kappa = \gamma$.

Figure 1: Project returns functions



In the SW approach, *expected returns* are assumed to be constant across all risk types. All the projects produce the same expected returns, but the returns of the riskier project are more widely spread around the mean than the returns of the less risky project. This kind of mean-preserving spread implies that the returns are ordered in the second-order stochastic dominance (SOSD) sense (Stiglitz and Weiss, 1981). Minelli and Modica (2009) (henceforth MM) also allows returns to be ordered in the SOSD sense, but unlike SW, they do not require all the projects to produce the same expected returns. In Figure 1, MM return function lies in the region marked SOSD, which corresponds to the region $\kappa > \gamma > 0$. SW returns function corresponds to $\gamma = 0$ and is special in the sense that it lies at the boundary of returns functions that exhibit SOSD and those that exhibit no stochastic dominance. It is conceivable that other risk-return relations are possible. The most notable of these is the risk and expected-returns trade-off. With such a trade-off, there is no stochastic dominance relationship. In Figure 1, this trade-off is obtained in the region with $\gamma < 0$. Finally, note that $\kappa \geq \gamma$ ensures that the conditional return, $Y(e)$, is non-decreasing with the entrepreneurial type (risk). That means we consider returns functions in the two-colored regions in Figure 1.

2.2 Type-dependent reservation utility

More innovative entrepreneurs also find better opportunities for themselves if they do not run their project. This translates into a higher reservation utility for the more innovative entrepreneurs.

More precisely, if an entrepreneur does not implement their risky project, she earns non-stochastic returns $V(e)$. We consider a type-dependent reservation utility function that is increasing in e.

Assumption 1 *Reservation utility function is non-decreasing in e and is quadratic*

$$V(e) = v_0 + v_1 e^2 \qquad \textit{where} \quad v_0, v_1 \geq 0 \tag{2}$$

As in the previous case, we can see that reservation utility and entrepreneurial types are perfectly related. Therefore, we could interpret the entrepreneurial types (and risk types) as the reservation utility types. It is also interesting to think about the interpretation of the model if, instead of e, we had expressed entrepreneurial heterogeneity in terms of the reservation utility, V. In this case, our starting point would be entrepreneurs who differ in terms of their outside opportunities. For example, entrepreneurs differ in their earning potential from paid employment in the outside economy (Parker, 2003; Scheuer, 2013). Heterogeneous reservation utility could also arise because entrepreneurs belong to different job networks (such as university alumni networks) and enjoy differing levels of support from their networks, such that even for the same level of ability, people from more connected and supportive job networks get higher-paying jobs. In any case, having better paid employment opportunities increases the reservation utility of a prospective entrepreneur and impacts the risk and target return the entrepreneur can choose when running the project. An entrepreneur with high V would target a high return, Y, offered by more innovative technology, even if they present a higher risk of failure e. This creates the same correlation between e, Y, and V, as we described in the earlier case.

In contrast to our assumption, SW, BT, and DW assume a homogenous reservation utility ($v_1 = 0$). A type-dependent reservation utility function is used by Freixas and Rochet (2008) and Sengupta (2014) in their models of the credit market. However, both these studies assume that the reservation utility function is decreasing in e. The structure of their model is similar to that of Parker (2003), whose idea is that the higher ability of a person manifests as a higher wage for them in paid employment and as a *lower* failure probability if they choose to be an entrepreneur. This gives a decreasing $V(e)$ function. In all these models, the return function takes the form of the BT/DW returns function where success state returns (Y) do not increase in e.

An increasing $V(e)$ function appears to be a more natural assumption with the increasing $Y(e)$ function. The idea is that the most enterprising type of entrepreneurs are the ones pursuing innovative but unestablished technologies. These projects offer high returns if successful, but also have a higher probability of failure. This is a characterization of the technology in industries that are high-tech driven (for example, information technology, software, biotechnology, and now also financial services). The quality of the agents that we call entrepreneurial or innovative could be due to their ability or (for a given ability) the tendency to undertake more rewarding projects albeit at a higher risk. In other words, more capable entrepreneurs target the high returns that come with risky technologies. Such entrepreneurs also have a higher reservation utility.[6]

Our quadratic specification of the $V(\cdot)$ function is a simplification. All the main results in this paper will hold with many other specifications. We will discuss this in detail in Section 5.

2.3 Collateralized credit and venture capital finance

2.3.1 Collateralized credit

A typical bank credit relation involves a loan of capital by the bank to the entrepreneur against a contract outlining an interest repayment (R) in the success state, and the amount of collateral (C) that the bank gets in case the borrower fails and defaults on repayment. There is a dichotomy in collateral valuation. Bank valuation of the pledged collateral, C, is always βC, with $\beta < 1$ to

[6]Type-dependent utility functions are also common in models that study occupational choice and taxation in the presence of credit market imperfections. For instance, Ghatak, Morelli and Sjöström (2007) present a setting with one-dimensional heterogeneity, where high-risk types have lower outside opportunity. However, consistent with our explanation above, the success output is either constant or decreasing (in different cases) in risk in their model as well. Scheuer (2013) also allows type-dependent reservation utility in an occupational choice model with two-dimensional heterogeneity. In his model too, entrepreneurs differ in both risk and reservation utility, but unlike our model, these two heterogeneities are not necessarily perfectly correlated in his model. However, in Scheuer (2013), returns are ordered in the FOSD sense, similar to BT/DW.

account for the liquidation costs of collateral that the bank incurs when there is a default (Besanko and Thakor, 1987; Sengupta, 2014; Barro, 1976).

2.3.2 Venture capital finance

An investor-entrepreneur relationship under a VC contract is more involved than the same relationship through a credit contract. First, under a VC contract, in addition to the capital, the investor (bank) also provides several other value-adding services to the start-up. Some of these value-adding investments result in private benefits for the entrepreneur, including sometimes from outright cash compensation paid by the investor. Second, although securities governing venture capitalist's cash flow and control rights may take varied forms in different jurisdictions, under VC investments generally, the investor exhibits active participation in the venture, for example, by taking board seats. We discuss each of these in detail below.

A crucial feature of venture capitalist-entrepreneur relationship is the value-adding services that the venture capitalist provides to the venture–the benefits of which are likely to accrue to the entrepreneur as well (Da Rin, Hellmann and Puri, 2013). These value-adding services may take several forms: professionalization of the company (Hellmann and Puri, 2002), mentoring of the entrepreneur, taking board seats, and performing monitoring (Gorman and Sahlman, 1989; Sahlman, 1990). The entrepreneur and their project may also benefit from the reputation of the investing venture capitalist through a certification effect and from the existing professional network of the venture capitalist (Hsu, 2004). While some of these benefits cannot be enforced and therefore contracted upon, they still play an important role in the contractual relationship between the two parties. Hsu (2004) finds that venture capitalists of repute acquire start-up equity at a 10–14% discount.

Another form of private benefit accruing to the entrepreneur from a VC investment is derived from the cash compensation paid by the venture capitalist (Ewens, Nanda and Stanton, 2020; Bengtsson and Hand, 2011; Wasserman, 2006). According to a study of US ventures by Ewens, Nanda and Stanton (2020), founder CEOs receive, on average, a cash compensation of USD 100,000/year in the seed stage, with cash compensation rising as the start-up matures. Bengtsson and Hand (2011) also find that although cash compensation is small in value terms compared with the implied value of the equity that the entrepreneur holds in the venture, the venture capitalist provides cash compensation because equity is a noisy and risky kind of compensation.

In what follows, we will denote by B the additional value-adding VC investment. Here the term *value-adding investment* is used in a broad sense. For example, it includes the cash compensation that the VC (bank) commits, and the outlays that the bank makes to offer mentoring or professionalization services, and its board activities. The VC investment of amount B has two effects. First, similar to De Bettignies and Brander (2007), it increases the output conditional on success by an amount δB, where $\delta \geq 0$. Thus, in the case of success, the total output is $Y + \delta B$. Second, a VC investment of amount B gives the entrepreneur a private benefit of λB, where $\lambda > 0$. This benefit includes the cash compensation and other benefits that may accrue to the entrepreneur due to mentoring and managerial inputs from the VC.[7]

VC investment is costly for the bank. The bank incurs a cost ϕ, with $\phi > 1$, to provide one unit of VC investment. Note that this cost is higher than the capital cost (fixed at one), as it includes the additional cost that the bank must incur for its active participation in the venture. An $\phi > 1$ may also reflect the higher financing cost of VC due to stricter capital requirements associated with the private equity business of the banks. Indeed, we would assume that a bank's engagement in VC business is socially costly in the sense that the cost of VC offered by the bank is higher than the benefits it brings to the venture and the entrepreneur. That is, $\phi > \lambda + \delta(1-e)$ for all e. What is interesting is that we show that, despite the bank's VC activity being socially costly, it may still lead to an improvement in the social welfare (compared to the case when VC investment by the bank is not permitted) by allowing the bank to serve high-type entrepreneurs. This assumption is also consistent with the evidence found in the literature that banks are less effective as a VC investor (Bottazzi, Da Rin and Hellmann, 2008; Andrieu and Groh, 2012; Cumming and Murtinu, 2016). The results in the literature suggest that banks may face a higher cost of VC business, and thus undertake that activity only to the extent that it helps them distinguish between entrepreneurs, as explained by our model.

2.3.3 Securities in VC finance

Another aspect of VC financing is the securities that govern cash flow and control the rights of the venture capitalist in the venture. Interestingly, there is variation across countries with respect to which securities are used under VC (Kaplan, Martel and Strömberg, 2007; Cumming, 2005*a*,*b*; Bottazzi, Da Rin and Hellmann, 2009).[8] Although equity is a commonly used security, VCs also

[7]Ours is a static model, while some of the value-adding services that we have talked about are dynamic in nature. We assume that the entrepreneurs still find them valuable and that these enter their utility function. One reason for this is that entrepreneurs can use the skills acquired through mentoring and professionalization in their next venture or upon their return to paid employment following an unsuccessful venture. A study by Gompers et al. (2010) of VC-backed start-ups in the United States suggests that in the 1990s, almost 10% of start-up founders were serial entrepreneurs, and their ventures performed better than the ventures of first-time entrepreneurs. Ewens, Nanda and Stanton (2020) and Manso (2016) also find that, upon returning to paid employment following a failed venture, entrepreneurs do not face any penalty.

[8]These differences are explained by the tax treatments of capital gains (Gilson and Schizer, 2003) and legal and institutional set-ups (Kaplan, Martel and Strömberg, 2007). For a review of the literature on optimal contract *within* VC financing, see Da Rin, Hellmann and Puri (2013) and Burchardt et al. (2016).

use other forms of securities, such as convertible preferred equity, that combine debt-like features with equity-like features.

Instead of mulling over the nuances of the differences in various forms of securities in VC financing, we prefer to follow a simple approach. We interpret a VC contract as one that involves the bank providing, in addition to the capital, some value-adding services that also accrue private benefits to the entrepreneur. In return, the bank may get 'repaid' through an equity or debt, and we leave that interpretation open, as our set-up allows us to capture the complex debt and equity contracts in a simple way. In the binomial outcome possibility that we have, we need not write equity contracts explicitly as a share of output and rather express them like any other debt contracts ex-ante. We can interpret them ex-post based on the repayment conditions that emerge in the equilibrium (Tirole, 2010, p. 119). An equilibrium contract that requires interest repayment in the success state and asks for collateral to be put up for the failure state is a standard debt contract. An equilibrium contract that does not require any collateral payment in the failure state, but requires only a repayment in the success state, then, can be interpreted as an equity. Therefore, in this sense, an uncollateralized loan contract can also be interpreted as an equity contract in our model. In the same vein, an equilibrium contract that requires no collateral and entails a provision of an additional VC investment—the contract that we will call a VC contract—can also be interpreted as an equity contract.

However, we will reserve the option of labeling a contract an equity contract only when it involves a VC investment from the bank. The reason for that is as follows. An equity contract makes sense only if the exact return is observable to the investor, whereas a debt contract requires that only the event of success or failure is verifiable by the lender, not necessarily the return. It would indeed be an unrealistic assumption that, for a small start-up, the exact return be observable to the *outside investor*. I will, therefore, assume that while the bank can observe, without cost, whether a firm has been successful or not, it can observe the success state *return* only if it has signed a VC contract with the entrepreneur. This is because the bank as a venture capitalist is actively engaged in the start-up's business, and it may actively monitor the start-up by having a seat on the start-up's board of directors (Gorman and Sahlman 1989, Kaplan and Strömberg 2004, Tirole 2010, pp. 90-91). Close monitoring makes the start-up's cash flow observable to the bank and any equity contract enforceable.

2.4 Model timing and agent pay-offs

We do not distinguish between a credit contract and a VC contract at the outset. We start from a possibility that the bank can offer a contract that has elements of both kinds of financing and it is determined endogenously whether the chosen contract is a credit contract or a VC contract. Therefore, we allow the bank to offer contracts that specify an Interest rate (R), Collateral requirement (C) and VC investment (B). R is the gross repayment that the firm has to make to the bank in case of success. The bank captures the pledged collateral (C), if the project is unsuccessful and there is a default. The bank makes the provision of VC investment (B) at the same time as transferring capital to the borrowing entrepreneur. Indeed, we will see later that it is an equilibrium outcome that the bank never offers a contract where it requires a collateral (a characteristic of a bank debt finance) and also makes a VC investment. An equilibrium contract with $B > 0$ and $C = 0$ is a VC contract. As discussed in the previous section, when an entrepreneur gets a contract with $B > 0$ the repayment, R, can also be interpreted as gross returns on equity. A contract with $B = 0$ and $C > 0$ is a standard bank debt contract and an equilibrium contract with $B = 0$ and $C = 0$ is an uncollateralized debt contract.

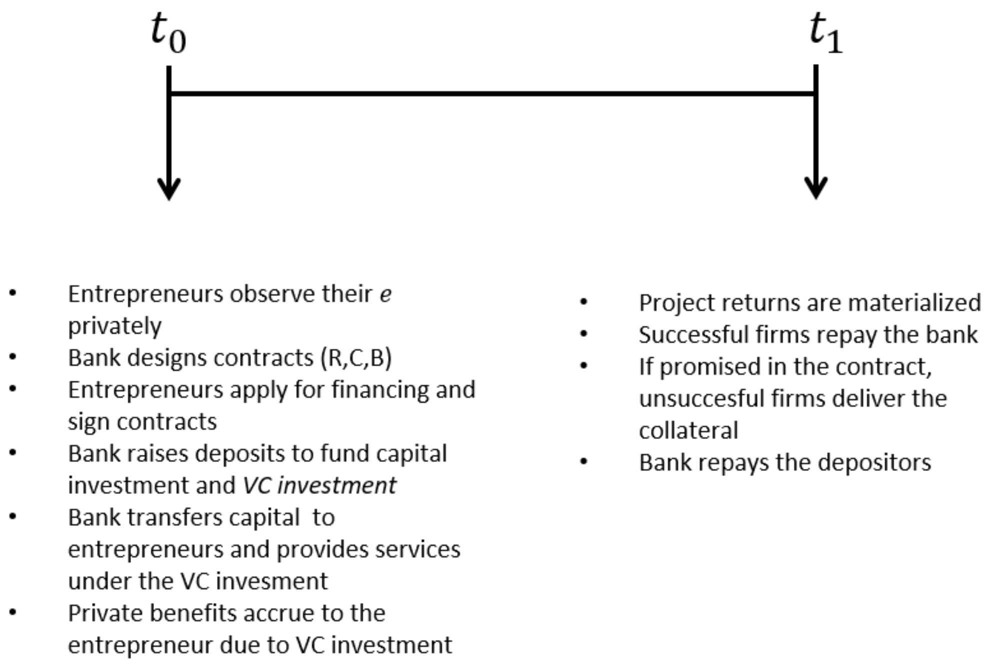
There are two periods, t_0 and t_1. At t_0, the bank offers (possibly a menu of) take-it-or-leave-it contracts to the entrepreneurs. Contracts are signed (if accepted by the entrepreneur), capital and additional VC investment are provided in period t_0. Consequent on the VC investment, the entrepreneur draws the private benefit in the period t_0 regardless of the outcome of the project.

States and project outcomes realize in period t_1. Conditional on the outcome and depending on the contract signed in t_0 firms take action. A successful entrepreneur repays, a failed entrepreneur may lose collateral if they pledged it under the contract. If an entrepreneur of type e does not accept any contract then she gets her outside opportunity $V(e)$ in t_1. Timing of the model is summarized in Figure 2.

In order to make capital investment, the bank, collects deposits at t_0 and promises to repay at a gross market risk free interest rate of one to the depositors at t_1. To finance VC investment, the bank raises deposits but also incur additional cost, giving us a cost of VC $\phi > 1$. We also assume the discount rate is the same as the risk free rate i.e. equal to one.

We assume for simplicity that the entrepreneurs do not have any constraints in putting up collateral and their ability to put up collateral is the same. For instance, all the entrepreneurs have the ability to pledge collateral from their wealth in the form of real-estate or even from the patents they hold. The idea is that bank needs to differentiate between different entrepreneurs *not* based on their *ability* to pledge collateral but based on their *willingness*. Since, the willingness to pledge collateral would be smaller for more innovative entrepreneurs as their projects are more risky, bank can *potentially* exploit this difference in willingness to elicit information about the types. To what extent can the bank do that is the issue we deal with in section 4. .

Figure 2: Model Timing



The social surplus associated with the project of a type e firm is:

$$S(e) = (1-e)Y(e) - V(e) - 1 \tag{3}$$

The social surplus of type e entrepreneur is the expected return for type e net of that type's reservation utility ($V(e)$) and the opportunity cost of capital, which is one.

For a given contract (R, C, B), the expected profit of a type e firm in excess of the outside opportunity $V(e)$ is

$$u(R,C,B;e) = \lambda B + (1-e)\{Y(e) + \delta B - R\} - eC - V(e) \tag{4}$$

$$= (1-e)\{Y(e) - R\} - eC + \{\lambda + \delta(1-e)\}B - V(e), \tag{5}$$

Bank's expected profit from the same contract can be compactly written as

$$\pi(R,C,B;e) = (1-e)R + e\beta C - \phi B - 1. \tag{6}$$

Therefore the surplus generated in the trade with the type e entrepreneur signing the contract

(R,C,B) is:

$$u(R,C,B;e)+\pi(R,C,B;e) = (1-e)Y(e)-(1-\beta)eC-\{\phi-\lambda-\delta(1-e)\}B-V(e)-1$$
$$= S(e)-(1-\beta)eC-\{\phi-\lambda-\delta(1-e)\}B \tag{7}$$

From our assumptions that $\beta < 1$ and $\phi > \lambda+\delta(1-e)$, equation (7) demonstrates that using collateral and VC investment entails social loss.

3 The symmetric information benchmark

Since offering collateralized credit and making VC investment leads to social loss, a fully informed social planner would always choose $C(e) = 0 = B(e)$ for all e. Further, to maximize the aggregate social surplus, a social planner would finance all the projects that have $S(e) \geq 0$. This is the first best allocation.

In the case when the bank is fully informed, it will offer type specific contracts such that it captures the entire surplus from a borrower. Again, since there is a deadweight loss if the bank offers collateral or VC investment, the bank would choose $C(e) = 0$ and $B(e) = 0$ for all e. Therefore the bank would offer contracts with type specific interest repayment such that $u(R(e),0,0;e) = 0$. The full information contracts are therefore given as:

$$(I(e),0,0) := \left(Y(e)-\frac{V(e)}{1-e},0,0\right) = \left(\frac{\kappa-\gamma e-v_0-v_1e^2}{1-e},0,0\right) \tag{8}$$

Where $I(e)$ is the interest repayment that the type e firm pays in the success state to the fully informed bank. This says that the symmetric information contracts are uncollateralized debt contracts.

Since the bank's profit from a given borrower type now is the same as social surplus from that type, it chooses to finance all the projects with $S(e) \geq 0$. Therefore, the fully informed bank implements the first best allocation.

Assumption 2 *We make the following assumption to ensure that $I(e)$ has a unique global maximum*

$$v_1 > \kappa-\gamma-v_0 > 0$$

Let the global maximum is achieved at e^*, where

$$0 < e^* := 1 - \frac{\sqrt{v_1(v_1 + v_0 + \gamma - \kappa)}}{v_1} < 1$$

Thus, contract in (8) shows that with full information, initially with rising innovativenes, e, interest rate rises but after a certain level of innovativeness (e^*), interest rates fall with higher e. This happens because the success state return, Y, increases in e giving bank an opportunity to charge high interest. However, borrowers with high e can also earn higher amount if they do not run the project ($V(e)$ is increasing in e). So to attract the higher e types to borrow, the bank also needs to charge smaller interest rate. The interaction of these two forces results in the inverted U shape of the $I(e)$ function. This is an interesting contrast with more conventional models like Besanko and Thakor (1987) and Lengwiler and Rishabh (2018) where the less risky (less innovative) types always pay higher interest rate under full information (i.e. $I(e)$ function is monotonically decreasing in e). This is a special case in our model. If we assume $\kappa = \gamma > 0$ and $v_1 = 0$ we get the same $I(\cdot)$ function as in these papers.

4 Asymmetric information

In a more realistic scenario, the bank cannot observe the types and therefore it cannot offer personalized contracts as above. This information asymmetry gives incentives to the entrepreneurs to lie about their true types. In what follows we assume that there are three types of agents - low (e_l), medium (e_m) and high (e_h) entrepreneurial types, where $0 < e_l < e_m < e_h < 1$. The share of the types e_l, e_m and e_h are f_l, f_m and f_h respectively where $f_i > 0$ for $i \in \{l, m, h\}$ and $\sum_i f_i = 1$.

For notational convenience we will change the index of types from e to i. That is we denote $S(e_i)$ as S_i, $V(e_i)$ as V_i, $Y(e_i)$ as Y_i and so on. We also assume for that all the types are socially productive, producing positive social surplus.

Assumption 3 $S_i \equiv S(e_i) > 0 \, \forall i \in \{l, m, h\}$

Given that the $I(\cdot)$ is concave, with three types we can have four possible combinations of full information interest rate. These are (i) $I_l \leq I_h \leq I_m$ (ii) $I_h < I_l \leq I_m$ (iii) $I_l < I_m < I_h$ and (iv) $I_h < I_m < I_l$. Given es are distinct there can be at most one equality in (i). To keep the analysis concise, we will focus on only cases (i) and (ii). Once we have dealt with the first two cases, results for cases (iii) and (iv) will follow easily. We discuss cases (iii) and (iv) in section 5 on alternative assumptions.

Assumption 4 *Entrepreneurial types, e, are such that either $I_h < I_l \leq I_m$ or $I_l \leq I_h \leq I_m$ with at most one equality.*

That is, we assume e_m is the *top type*, i.e. agent with highest $I(e)$) throughout our analysis. We do not make any assumption about the bottom type (the borrower with lowest $I(e)$). In what follows we ignore the possibility of equal Is as it requires us to qualify every exposition every time with two distinct possible equalities. Since the results with equalities are going to be obvious as we will see in section 4.1.1, we skip any qualification on that aspect for the rest of the paper.

Now consider what will happen, if the bank offers the full information contracts, as given in (8), in the presence of asymmetric information. If we have $I_l < I_h < I_m$ then e_m and e_h types have incentive to mimic the e_l type as it is the e_l that is asked to pay the lowest interest rate. Therefore, all the types will pool at the contract for e_l. This will give e_m and e_h informational rent, $u(I_l, 0, 0; e_i) = (1 - e_i)[I_i - I_l] > 0$, for $i = m, h$. Clearly bank looses out compared to the full information case as now it cannot capture the entire surplus from e_m and e_h types. Symmetrically, e_l and e_m types earn informational rents by pooling at the contract for e_h type, if $I_h < I_l < I_m$.

The bank will certainly want to do better by possibly separating the types by designing a self selecting menu of contracts. A menu of contracts would contain the triples, (R_i, C_i, B_i) i.e. where R, C and B are functions of the entrepreneurial types. To look at the optimal contracts menu, using the revelation principle, we only need to focus on the menus that give rise to truth telling (Myerson, 1979). A menu of contracts induces truth telling by ensuring that the contracts are incentive compatible and satisfy participation constraints (PC) of the agents. An incentive compatibility constraint, IC$_{ij}$, requires that the type e_i firm finds the contract designed for herself at least as attractive as the contract designed for any other type, e_j. That is:

$$u(R_i, C_i, B_i; e_i) \geq u(R_j, C_j, B_j; e_i) \qquad \forall i, j \in \{l, m, h\}$$

Which can be written as,

$$R_i + \frac{e_i}{1-e_i} C_i - \left[\delta + \frac{\lambda}{1-e_i}\right] B_i \leq R_j + \frac{e_i}{1-e_i} C_j - \left[\delta + \frac{\lambda}{1-e_i}\right] B_j \qquad \forall i, j \in \{l, m, h\} \tag{9}$$

The PC requires that each type prefers to run the project than exercise her outside option. The bank achieves this by ensuring

$$u(R_i, C_i, B_i; e_i) \geq 0 \qquad \forall i \in \{l, m, h\}. \tag{10}$$

Bank's profit maximization problem, therefore, can be written as:

$$\left.\begin{aligned}
&\max_{R_i,C_i,B_i} \sum_{i\in\{l,m,h\}} \left[(1-e_i)R_i + e_i\beta C_i - \phi B_i - 1\right] f_i && \text{subject to}\\
&R_i + \frac{e_i}{1-e_i}C_i - \left[\delta + \frac{\lambda}{1-e_i}\right]B_i \le R_j + \frac{e_i}{1-e_i}C_j - \left[\delta + \frac{\lambda}{1-e_i}\right]B_j && \forall i,j \in \{l,m,h\}\\
&u(R_i,C_i,B_i;e_i) \ge 0 && \forall i \in \{l,m,h\}\\
&R_i \le Y_i && \forall i \in \{l,m,h\}\\
&R_i,B_i,C_i \ge 0 && \forall i \in \{l,m,h\}
\end{aligned}\right\} \quad \texttt{(P-I)}$$

The first constraint is the set of IC constraints discussed above. There are six such constraints for the three types. Second is the PC that ensures that the agents find running the project worthwhile compared to the next best alternatives they have. There are three participation constraints, one for each type. Next is the constraint requires that, for each type, repayment in the success state cannot be higher than the income of the entrepreneur. The last set of constraints impose non-negativity for all the choice variables.

In the following discussion, we will find the conditions under which the bank can separate the types and solve for the optimal contracts in separation. Afterwards we discuss the equilibrium outcomes if the complete separation conditions are not met.

4.1 Complete separation

Instead of solving the problem (P-I) we solve a relaxed problem with fewer constraints and show that the solution satisfies all the constraints in (P-I). The relaxed problem is given as:

$$\left.\begin{aligned}
&\max_{R_i,C_i,B_i} \sum_{i\in\{l,m,h\}} \left[(1-e_i)R_i + e_i\beta C_i - \phi B_i - 1\right] f_i && \text{subject to}\\
&R_m + \frac{e_m}{1-e_m}C_m - \left[\delta + \frac{\lambda}{1-e_m}\right]B_m \le R_l + \frac{e_m}{1-e_m}C_l - \left[\delta + \frac{\lambda}{1-e_m}\right]B_l &&\\
&R_m + \frac{e_m}{1-e_m}C_m - \left[\delta + \frac{\lambda}{1-e_m}\right]B_m \le R_h + \frac{e_m}{1-e_m}C_h - \left[\delta + \frac{\lambda}{1-e_m}\right]B_h &&\\
&u(R_i,C_i,B_i;e_i) \ge 0 && \forall i \in \{l,m,h\}\\
&R_i,B_i,C_i \ge 0 && \forall i \in \{l,m,h\}
\end{aligned}\right\} \quad \texttt{(P-II)}$$

Problem (P-II) has only two of the six IC constraints viz. IC_{ml} and IC_{mh} and contains no viability constraint as in problem (P-I). Indeed we will ignore this condition for the rest the paper as it can be checked for e_m and e_l types in any equilibrium this condition is satisfied. In the case of

e_h and when the complete separation is achieved it reduces into a technical condition[9], that we will assume to be satisfied always. Indeed, in all the numerical examples presented later, this condition is easily satisfied. .

Lemma 1 *In any solution to the relaxed problem* `P-II`*, the participation constraints for e_h and e_l must bind.*

Proof. Suppose not. Then there is a solution to `P-II` in which for some $k \in \{l, h\}$ we have:

$$\begin{aligned} & u(R_k, C_k, B_k; e_k) > 0 \\ \Rightarrow \quad & \lambda B_k + (1-e_k)[Y_k + \delta B_k - R_k] - e_k C_k - V_k := \varepsilon_k > 0 \end{aligned}$$

Consider the new contract $(\overline{R_k}, \overline{C_k}, \overline{B_k})$ such that $\overline{R_k} = R_k + \varepsilon_k/(1-e_k)$, $\overline{C_k} = C_k$ and $\overline{B_k} = B_k$. The new contract preserves the participation constraint for the type e_k, continues to satisfy the two ICs and does not impact PC for any other types. However, this leads to an unambiguous increase in profit for the bank. QED

Next, substituting the binding participation constraints for e_h and e_l in the relaxed problem `P-II`, we find the following optimal contracts.

Lemma 2 *The solution to the relaxed problem* `P-II` *is given by:*

$$\begin{aligned} R_l &= I_l - \frac{e_l}{1-e_l} C_l, & C_l &= \frac{I_m - I_l}{\Delta_{ml}}, & B_l &= 0 \\ R_m &= I_m, & C_m &= 0, & B_m &= 0 \\ R_h &= I_h + \left[\delta + \frac{\lambda}{1-e_h}\right] B_h, & C_h &= 0, & B_h &= \frac{I_m - I_h}{\lambda \Delta_{hm}} \end{aligned}$$

Where $\Delta_{ij} := \dfrac{e_i - e_j}{(1-e_i)(1-e_j)} = \left[\dfrac{e_i}{1-e_i} - \dfrac{e_j}{1-e_j}\right] = \left[\dfrac{1}{1-e_i} - \dfrac{1}{1-e_j}\right]$ *and* $\Delta_{ml} > 0$, $\Delta_{hm} > 0$.

Proof. See appendix. QED

As a next step, we demonstrate in the appendix, the solution to `P-II` is also a solution to the original problem, `P-I`, by showing that this solution satisfies all the constraints in `P-I`.

Lemma 3 *Solution to* `P-II` *is also a solution to* `P-I`*.*

Proof. See appendix. QED

[9]The condition is given as $\delta(1-e_h) + \lambda \le \frac{V_h}{v_1(e_h + e_m - e_h e_m) - (\kappa - \gamma - v_0)}$. Essentially it is a condition on the slope of the participation constraint (indifference curve), relative to its intercept in the R–C–B space.

Finally, the following proposition summarizes all the results relating to complete separation including the conditions that ensure that.

Proposition 1 *Under the conditions,*

$$\frac{1}{f_m}\left[(1-\beta)e_l f_l\left(\frac{1-e_l}{e_m-e_l}\right)+\{\phi-\lambda-\delta(1-e_h)\}\frac{f_h}{\lambda}\left(\frac{1-e_h}{e_h-e_m}\right)\right]\leq 1 \tag{C.1}$$
$$\frac{I_m-I_l}{\Delta_{ml}}\leq\frac{S_l}{e_l\,(1-\beta)} \tag{C.2}$$
$$\frac{I_m-I_h}{\lambda\Delta_{hm}}\leq\frac{S_h}{\phi-\lambda-\delta(1-e_h)} \tag{C.3}$$

the bank separates all the three types offering the contracts:

$$\begin{aligned}
&R_l^s=I_l-\frac{e_l}{1-e_l}C_l^s, && C_l^s=\frac{I_m-I_l}{\Delta_{ml}}, && B_l^s=0\\
&R_m^s=I_m, && C_m^s=0, && B_m^s=0\\
&R_h^s=I_h+\left[\delta+\frac{\lambda}{1-e_h}\right]B_h^s, && C_h^s=0, && B_h^s=\frac{I_m-I_h}{\lambda\Delta_{hm}}
\end{aligned} \tag{11}$$

Proof. Follows directly from solution to `P-II` and Lemma 3. QED

The superscript 's' stands for separation.

4.1.1 Discussion of the separating equilibrium

There are some interesting properties of the separating contracts in (11), that we can readily see. First is that the medium type gets the full information contract i.e. it pays no collateral, gets no additional VC investment and pays the interest I_m. This result is called *'no distortion at top'*, for e_m is the top type in our analysis. If the bank offered full information contracts, no type would try to imitate this type, as under full information e_m type pays the highest interest rate. It is for this reason there is no need for the bank to distort the full information contract for e_m type.

The optimal contracts for e_h and e_l types are determined by the differences in the sensitivity of different types towards C and B. The sensitivity is essentially captured by the slope of the indifference curve. The indifference curve (precisely, three dimensional *indifference plane*) for any given type e is given by (5). Using, (5), we have the slope of indifference curve in $C-R$ plane as

$$\frac{dR}{dC}=-\left[\frac{\frac{\partial u(e)}{\partial C}}{\frac{\partial u(e)}{\partial R}}\right]=-\left[\frac{e}{1-e}\right]<0$$

That is, given any B, the entrepreneur needs to be charged a lower R if a higher C is demanded while keeping the utility level unchanged (for example, at zero). Further, note this slope itself is decreasing in e, making the indifference curve in the $C-R$ plane steeper for higher e. In other words, marginal rate of substitution (MRS) between R and C, increases with e. Thus, more entrepreneurial types would be more willing to trade-off collateral for a higher interest rate. This is because the more entrepreneurial types have riskier projects and, thus, they prefer to avoid putting up collateral. Equivalently, we can say that types with lower e prefer to take-on more C for a slight reduction R. Since they are less likely to lose the collateral and more likely to pay up R, they value a reduction in R a lot even if it means putting up larger C.

Similarly from, (5), the slope of indifference curve in a $B-R$ plane is

$$\frac{dR}{dB} = -\left[\frac{\frac{\partial u(e)}{\partial B}}{\frac{\partial u(e)}{\partial R}}\right] = \frac{(1-e)\delta + \lambda}{1-e} > 0$$

That is, given any C, the entrepreneur needs to be charged a higher R to keep the utility level unchanged, if a higher B is provided. In addition, this slope itself is increasing in e, making the indifference curve along $B-R$ dimension steeper for higher e. Thus, more entrepreneurial types would be more willing to take-on higher interest charges for a unit increase in value-adding investment. This is because the private benefits from the value adding investments accrue regardless of the outcome of the project, while R needs to be paid only in successful states. Since the more entrepreneurial types are less likely to be paying R, due to higher risk of their projects, they prefer to rather receive these benefits upfront even at a higher R.

The bank exploits these differences in borrowers' MRS on different dimensions to design an optimal menu of contracts. For the optimal menu, the bank needs to distort the full information contract for the types other than the e_m type, such that no one has any incentive to mimic any one else (contracts are incentive compatible). As an illustration, suppose, e_h is the bottom type. In other words, e_h type has the *smallest* interest repayment under full information i.e. $I_h < I_l < I_m$. Therefore, e_l and e_m types jeopardize the full information contract of the e_h type, by imitating it under incomplete information, if the bank continues to offer the full information contracts.

How should the bank distort the full information contract for the e_h type to prevent e_m and e_l from mimicking it? Offering collateralized credit (that is necessarily combined with lower R along the PC) to e_h type will not help because the MRS between R and C is increasing in e. Both e_m and e_l types like contracts with collateral even more than the high type because these types are more sensitive to R and like any contract that has lower R even with higher collateral requirement. However, if the e_h type gets a contract with VC investment combined with a high R, the other types will be dissuaded to mimic it, as the other two types succeed (and hence are required to pay R) with larger probability.

Now consider the e_l type whose contract has to be distorted such that no other type mimics it. Making any VC investment in the e_l type (and charging high interest rate simultaneously) will not help as relatively more innovative type agent, e_m will find such a contract even more attractive since their projects are more likely to fail. While requiring high enough collateral (and reducing the interest required simultaneously) from e_l type might dissuade the other *riskier types* from mimicking it, as they are relatively more likely to lose their collateral.

This gives us a clean separation of types into different kinds of contracts. The high entrepreneurial types get a VC investment. The bank provides value adding services and in return gathers a 'high' R in the success state and receives nothing in the failure state but. The low innovative types on the other hand get a standard collateralized credit contract. The middle types get uncollateralized credit contracts. It is also important to note from the above discussion that the bank does not offer any hybrid contract involving both collateral and VC investment. In short, $C_iB_i = 0, \forall i \in \{l,m,h\}$. The reason, as discussed above, is that between VC investment (B) and collateral (C), only one is effective as a screening device for a given type. Finally, note that although all the types end up at their PCs (PCs of all three types bind), the bank still does not get the full social surplus because it has to incur deadweight cost of screening.

Figure 3 shows three-dimensional indifference curves associated with $u(R,C,B;e) = 0$, i.e. participation constraints – one for each type. By implication of our result that the bank will not offer any hybrid contract i.e., $C_iB_i = 0, \forall i \in \{l,m,h\}$, we can ignore all the contracts in the shaded part of the indifference curves. We need to focus only on the edges of the indifference curves lying in the C–R and B–R coordinate planes. These edges are depicted in bold lines in the figure. Indeed, this can be visualized in a two dimensional figure, making it easier to optically identify the optimal contracts.

Figure 4 plots C on the horizontal axis starting from origin and moving towards right, and B on the horizontal axis as well but starting from origin and moving towards left. Essentially in this figure, the horizontal axis plots two different variables, both taking non-negative values, with the values increasing farther you are on the axis in any direction.

In addition to the participation constraints drawn in bold lines, the figure 4 also includes the iso-profit lines (dashed lines) through the benchmark symmetric information contracts. Borrowers preference increases in the *south-west* direction, while the bank profits increase northwards. Clearly, if the bank offered the full information contracts which are depicted on the vertical (R) axis, medium and the low type would pick the same contract as high type, pooling at the contract $(I_h, 0, 0)$, as this contract is south of their own contracts. However, a separating menu is possible under the conditions C.1 through C.3 in Proposition 1. The three bold points depicted in figure

Figure 3: Participation Constraints

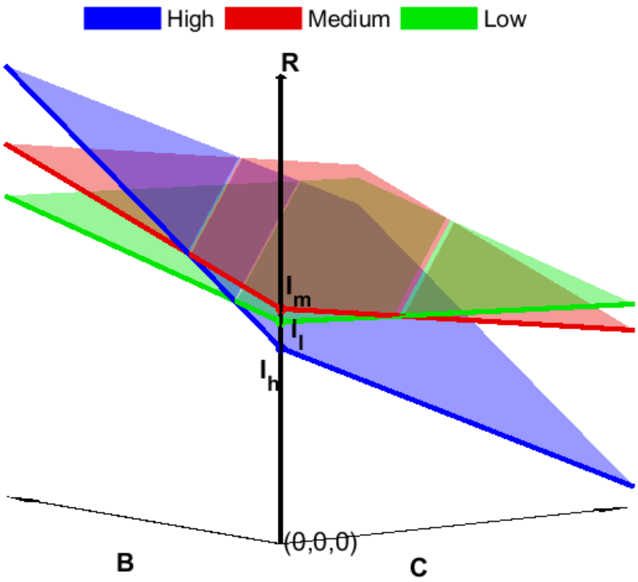
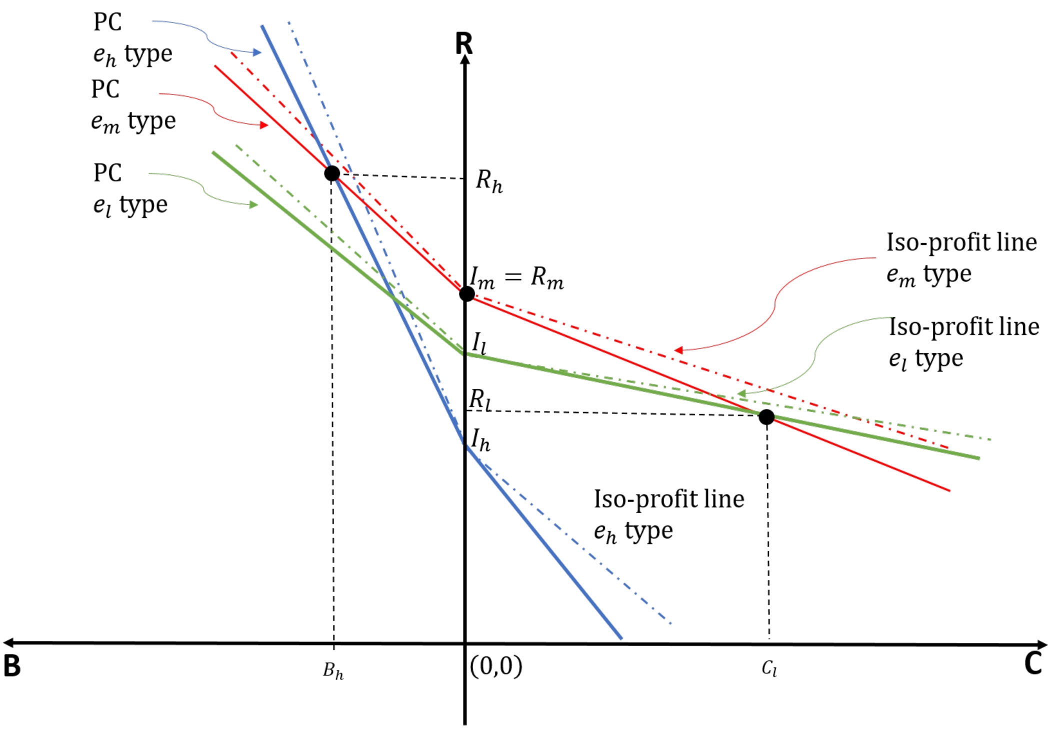


Figure 4: Participation Constraints and Separating Contracts



4 are the separating contracts. It can be quickly checked that although bank is able to push all the types to their respective participation constraints, it still earns less profit than the symmetric

information benchmark, except in the case of the e_m type, whose full information contract is not distorted. Note that figures 3 and 4 are drawn for the case $I_h < I_l < I_m$. As our solution shows above, for the alternative case $I_l < I_h < I_m$ how the conditions that determine the optimal contracts remain the same.

Let us look at the conditions for complete separation. (C.1) specifies conditions on parameters such as distribution of e, heterogeneity of types and relative cost of screening. The condition seems quite intuitive: *ceteris paribus* lower cost of collateralization $(1-\beta)$ and VC (ϕ) makes separation more likely. Also, a higher λ or δ reduce the net cost of VC investment and therefore make separation more likely. Similarly, other things remaining the same, the more heterogeneous the entrepreneurs are (i.e. bigger is the difference between e's) the more likely there will be a separation of types. Finally, higher the share of the top type (f_m) relative to f_h and f_l, higher are the chances that the bank will separate. The reason is that the bank does not have to distort the contract of the top type and hence there are no profit losses attributable to inefficiency of the screening process for the e_m type. Thus, larger the share of e_m types the more leeway the bank has on incurring cost of screening for the other two types. If this condition is satisfied with equality then the bank may give e_m types some informational rent (allows the PC of e_m types to be slack) by allowing some kind of pooling (partial or complete) involving e_m types with at least one other types. However, this gives the bank no additional profit. We will assume in such a case the bank will continue to separate completely such that the PC for e_m types also binds.

We also require conditions that ensure that by separating the types the bank does not make negative profits from the types whose full information contracts are distorted. Given the result from Lemma 1, that the types e_h and e_l do not earn any informational rent, this condition requires that the social surplus net of the deadweight loss of using collateral or VC investment should be non-negative. This is what conditions (C.2) and (C.3) encapsulate.

4.2 Partial and complete pooling

In the case any one of the conditions in proposition (1) is violated, bank will find it profitable to pool some or all the types. Let us begin by considering the violation of condition (C.1) assuming the other two conditions are satisfied. We know from the discussion above that bank will not distort the contract of the top-type (e_m type). Further, the bank will never find it useful to offer a collateralized debt contract to high type and a VC contract to low type. Additionally, it is also clear that any kind of partial pooling will involve the *top type* (e_m type), otherwise the bank cannot deter e_m type to mimic the other pooled agents. Therefore, the contract on which some or all types pool must correspond to the full information contract – involving no collateral and no VC investment.

Since, both collateral and VC contracts incur cost, and at most only one type needs to be separated in a scenario involving pooling, the bank needs to distort the full information contract of only that type, at most. Indeed, by the same reasoning, it does not need to distort the full information contract of any agent in the case of complete pooling.

Who should e_m type be pooled with? In case of partial pooling, the top two types should be pooled and the 'bottom' type should be separated, for any other combination of pooling of two agents, will be ineffective in separating the remaining type. In the example of figure 4, under partial pooling e_m and e_l types will be pooled at the full information contract of e_l, while the e_h will be separated with a VC contract, distinct from the one in (11). To elaborate on the previous points in the context of this example, from figure 4 note, if the e_h and e_l types are pooled at *any* contract, e_m will mimic them to pick the same contract, since there in no contract where the PC of both e_l and e_h types are satisfied and that contract in not to the south of the PC of the e_m type. Thus, the two bottom types will pooled only under complete pooling. Further, in this example, partial pooling of the high and medium type along with separation of low type could not be profit maximizing as the bank would be spending screening cost of VC financing on two types – e_m and e_h types rather than only for e_h type. Of course, there is also a condition, similar to (C.1) that needs to be satisfied for the bank to partially pool rather than completely pool the types. The following propositions sum up the discussion:

Proposition 2 *If* (C.1) *is violated while* (C.2) *and* (C.3) *are satisfied and* $I_h < I_l < I_m$ *holds, then under the condition*

$$\frac{1}{f_m}\left[\{\phi-\lambda-\delta(1-e_h)\}\frac{f_h}{\lambda}\left(\frac{1-e_h}{e_h-e_m}\right)-f_l\left(\frac{1-e_l}{1-e_m}\right)\right]\leq 1 \tag{C.4}$$

the bank pools e_m and e_l types, and separates e_h types offering the contracts:

$$\begin{aligned} &R_l^{pp}=R_m^{pp}=I_l, && C_l^{pp}=C_m^{pp}=0, && B_l^{pp}=B_m=0 \\ &R_h^{pp}=I_h+\left[\delta+\frac{\lambda}{1-e_h}\right]B_h^{pp}, && C_h^{pp}=0, && B_h^{pp}=\frac{I_l-I_h}{\lambda\Delta_{hm}} \end{aligned} \tag{12}$$

Proof. See appendix. QED

Where the superscript '*pp*' is for partial pooling.

Proposition 3 *If* (C.1) *is violated while* (C.2) *and* (C.3) *are satisfied and* $I_l < I_h < I_m$ *holds then* *under the condition*

$$\frac{1}{f_m}\left[(1-\beta)e_l f_l\left(\frac{1-e_l}{e_m-e_l}\right)-f_h\left(\frac{1-e_h}{1-e_m}\right)\right]\leq 1 \tag{C.5}$$

the bank pools e_m and e_h types, and separates e_l types offering the contracts:

$$\begin{aligned} &R_h^{pp}=R_m^{pp}=I_h, && C_h^{pp}=C_m^{pp}=0, && B_h^{pp}=B_m^{pp}=0\\ &R_l^{pp}=I_l-\frac{e_l}{1-e_l}C_l^{pp}, && C_l^{pp}=\frac{I_h-I_l}{\Delta_{ml}}, && B_l^{pp}=0 \end{aligned} \tag{13}$$

Proof. See appendix. QED

Propositions 2 and 3 show that only the bottom type's contract is distorted. In the first case, since highly entrepreneurial agents (e_h) are the bottom type, they are offered a VC contract while the other two types are offered an uncollateralized credit contract. In the latter case, smallest e types are the bottom types and they get the collaterlaized credit contract, while the other two types get the uncollateralized credit contract. In summary, in a partial pooling equilibrium, in combination with the uncollaterlized credit contract, either a VC contract is offered or a collateralized credit contract, but not both.

Another notable feature of optimal contracts under partial pooling is $R_m^{pp} < R_m^s$ in either case and $B_h^{pp} < B_h^s$ in the former case and $C_l^{pp} < C_l^s$ in the latter. This is because given that the bank is pooling the top two types, the amount of distortion required to keep them from mimicking the bottom type is small.

The conditions (C.4) and (C.5) for partial pooling require that medium types form a relatively high proportion of the population compared to the bottom type. Additionally, the cost of screening in either case should be sufficiently low and heterogeneity among entrepreneurs sufficiently high. The negative terms we see at the end are new in these conditions say that the higher is proportion of the type with whom the top type is pooled the more likely it is that the bank will partially pool rather than fully pool. It follows from the fact that larger is the number of these types, the more surplus the bank can extract potentially by separating them from the bottom type. No surprise that the violation of condition (C.4) in the first case and of condition (C.5) in the second case leads to complete pooling.

Proposition 4 *If* (C.1) *is violated while* (C.2) *and* (C.3) *are satisfied then*
(i) In case $I_h < I_l < I_m$ holds and (C.4) *is violated, the bank pools all the borrower types at the contract $(I_h, 0, 0)$.*
(ii) In case $I_l < I_h < I_m$ holds and (C.5) *is violated, the bank pools all the borrower types at the*

contract $(I_l, 0, 0)$.

Proof. See appendix. QED

This proposition says that the bank will pool all the types, should the conditions of separation and partial pooling are not met, at the first best contract for the bottom type. In this case we will not observe any VC or collateralised credit.

So far, we have focused only on the violation of condition (C.1) in proposition 1. In case only this condition is violated but conditions (C.2) and (C.3) are met, we showed the bank will still serve all the types, although it may not separate them always. However, if either the cost of venture capital or cost of collaterlization is too high we might see that the bank does not serve some entrepreneurs and that leads to an additional inefficiency resulting from the shut-down of the entrepreneurs from the financial market.

4.3 Banning VC activities, shutdown of entrepreneurs and inefficiency

In this section, rather than completely characterizing all the possible equilibria we will highlight one extreme example to underline the possibility of inefficient shut down of a type from the market. We will assume that cost of VC is prohibitively high, such that the bank cannot offer any VC contract. This can also be thought of as a case when the bank is not permitted to undertake any VC investment. A parallel analysis could be done by assuming that the cost of collateralization is prohibitively high (or the entrepreneurs do not have collateral to pledge). In either case there might be inefficiency due to limited market access: in the first case the e_h type might get shut down and in the second case the e_l type. We will focus on the shut down of the former kind. Conditions for shutdown of e_l types when (C.2) is violated can be derived following similar arguments and are available on request.

In case complete separation is rendered unviable due to high cost of VC, condition (C.3) is violated for any $B > 0$. Therefore, with the lack of a screening device at its disposal, the bank might find it worthwhile sometimes to not serve the e_h types.

Take the example of figure 4 again. Here type e_h is the bottom type. If condition (C.2) is not met and even if (C.1) is satisfied, the bank will no longer separate the highly innovative type type. It is because if it did, it would incur a negative profit on these types. Indeed, in order to serve e_h types, the bank needs to pool them with at least one other type. Therefore, unless the e_h types are close enough to other types or they are present in high numbers relative to other types, they might not be served.

Further, note the only pooling involving e_h types that is consistent with profit maximization in this case is where the bank pools all the types i.e., complete pooling. A partial pooling of e_h types with any other type is not incentive compatible. However, note the complete pooling will occur at the contract $(I_h, 0, 0)$ and at this contract the bank gives the e_m and e_l types an informational rent. We can imagine if this joint informational rent turns out to be higher than what the bank gets by serving the e_h types, viz. their social surplus (net of any cost in alternative scenarios), then the bank will not serve e_h types. Thus, an inefficient equilibrium ensues with the high type shut down from the market. The exact conditions for these are summarized in the proposition below.

Proposition 5 *If $I_h < I_l < I_m$,* (C.2) *is violated for all $B > 0$ and* (C.3) *holds then if*
$f_h S_h \geq \max\left\{ f_m(1-e_m)(I_m - I_h) + f_l(1-e_l)(I_l - I_h) - (1-\beta)e_l \left[\frac{I_m - I_l}{\Delta_{ml}}\right], [f_m(1-e_m) + f_l(1-e_l)](I_l - I_h) \right\}$
The bank pools all the entrepreneurs at $(I_h, 0, 0)$.
If the above condition is not satisfied and if
(i) $\frac{1}{f_m}\left[(1-\beta)e_l f_l \left(\frac{1-e_l}{e_m - e_l}\right)\right] \leq 1$*, then the bank shuts down the e_h type entrepreneurs and separates the e_m and e_l type entrepreneurs offering contracts:*

$$
\begin{aligned}
R_m^{sds} &= I_m, & C_m^{sds} &= 0, & B_m^{sds} &= 0 \\
R_l^{sds} &= I_l - \frac{e_l}{1-e_l} C_l^{sds}, & C_l^{sds} &= \frac{I_m - I_l}{\Delta_{ml}}, & B_l^{sds} &= 0
\end{aligned}
\tag{14}
$$

(ii) $\frac{1}{f_m}\left[(1-\beta)e_l f_l \left(\frac{1-e_l}{e_m - e_l}\right)\right] > 1$*, then the bank shuts down the e_h type entrepreneurs and pools the e_m and e_l type entrepreneurs offering the contract:*

$$
R_m^{sdp} = R_l^{sdp} = I_l, \quad C_m^{sdp} = C_l^{sdp} = 0, \qquad B_m^{sdp} = B_l^{sdp} = 0 \tag{15}
$$

Proof. See appendix. QED

Here, the superscript $'sds'$ stands for shutdown with separation and $'sdp'$ stands for shutdown with pooling.

The conditions in the above proposition are intuitive. The first condition has on the left side the gains from serving the e_h types viz. the surplus produced by e_h types. The right side contains the cost of serving the e_h types viz. the informational rents the bank gives to the other types when it pools them. However this cost of serving e_h type has to be adjusted with the costs that the bank incurs in cases when it decides not to serve the e_h type. That is the cost corresponding to screening in $'sds'$ and cost of pooling in $'sdp'$. Only when the gain by serving e_h types is higher than the net costs in the two cases when e_h types are kept out of the market, that the bank chooses to pool all. In case the bank decides not to serve e_h type (i.e. not to pool all the types) it chooses between pooling or separating the rest of the types and conditions (i) and (ii) in the Proposition 5 determine that. The interpretation of conditions in (i) and (ii) is similar to the interpretation of condition (C.1) in the case of complete separation in Section 4.1.1.

To see diagrammatically, the consequence of prohibitively high cost of VC bank we can refer to Figure 4 again keeping in mind that no point in B–R plane is achievable any more. Focusing on the C–R plane, note that collateral alone can not help separate the three types. Why? Starting from the full information contract for the e_h type, denoted by the point I_h, the bank can in no way distort this contract to dissuade the other types from mimicking it. Any attempt to offer a contract with more collateral combined with lower interest rate will make the contract even more desirable to less innovative types with lower failure probability. Therefore, the bank might decide not to serve the e_h types if it finds its profits to be lower when pooling all than serving only the e_m and e_l types. This would lead to inefficient exclusion of the high types from accessing credit and a loss in social welfare. No surprise that a similar shut down may occur even when e_l is the bottom type. Consider the same scenario with prohibitively high cost of VC with $I_l < I_h < I_m$. The situation, however, is slightly benign here. Now, the bank finds it harder to exclude the e_h types because now the bank also has the option of involving the e_h types in a partial pooling equilibrium – which the bank didn't have in the previous case. The following proposition sums up the results for this case:

Proposition 6 *If $I_l < I_h < I_m$,* (C.2) *is violated for all $B > 0$ and* (C.3) *holds then if*
$$f_h S_h \leq \min\left\{[I_m - I_l](1-e_m)f_m + [I_h - I_l]\left((1-e_h)f_h - \frac{(1-\beta)f_l e_l}{\Delta_{ml}}\right), [I_m - I_h]\left((1-e_m)f_m - \frac{(1-\beta)f_l e_l}{\Delta_{ml}}\right)\right\}$$
The bank shuts down the e_h type entrepreneurs and separates the e_m types with e_l types offering the contracts in (14). *If the above condition is not satisfied and*

(i). Condition (C.5) *holds, then the bank pools e_h with e_m types and separates e_l type entrepreneurs offering contracts in* (13)

(ii). (C.5) *does not hold then the bank pools all the entrepreneurs at the contract* $(I_l, 0, 0)$.

Proof. See appendix. QED

4.4 Numerical examples

In this section I present a few numerical examples to elaborate on the results in the previous sections. We first start with understanding the role of heterogeneity and cost of screening in determining the type of equilibrium. To do that we make some normalizations. First is that for this section we assume that the heterogeneity across the three types is uniform. That is the parameter, $d := (e_h - e_m) = (e_m - e_l)$ determines the heterogeneity in the economy. We fix the parameters as in the table below for both the examples in this section.

Table 1: Parameter values for the numerical examples

Parameter	Value
κ	7
γ	2
v_0	2
v_1	4
e_m	0.5
λ	1
δ	0

Given these parameters, $S(e) > 0, \forall e \leq 0.78$. Therefore, a social planner would finance any firm with failure probability less than 0.78. Keeping our assumption that all firms are socially desirable and assuming that $e_m = 0.5$, we will, therefore, allow a maximum d equal to 0.27.

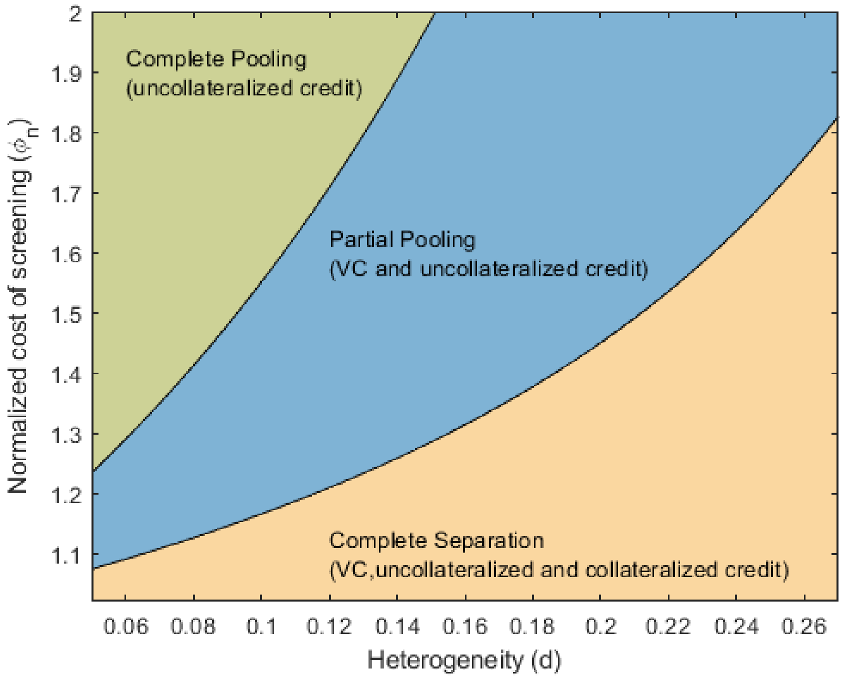
Example 1: Separation is viable

In this example we will work in an environment where complete separation is a viable option for the bank in the sense that conditions C.2 and C.3 hold throughout. The idea is to find which combinations of ϕ, β and d give rise to which type of equilibrium – complete separation, partial pooling, complete pooling. To reduce the dimensionality of the problem in this example, we assume a symmetric cost of credit and venture capital, so that both types of cost can be represented by one parameter. We define cost of screening parameter (ϕ_n) as the cost of VC which is tied to the cost of collateral such that $\phi = \phi_n := \frac{1}{\beta}$. This says the higher the cost of collateralization $(1-\beta)$, the higher the cost of VC (ϕ).

Note, given e_m, each value of d fixes values of e_l and e_h. Given parameters in table 1, this gives us symmetric information interest rates, $I_i \, \forall i \in \{l, m, h\}$ and that determines the bottom and top type. Continuing with our assumption, all our parameters ensure that e_m is the top type. Finally, the distribution of types, and the cost of screening parameter ϕ_n pins down the equilibrium and the optimal contracts in the equilibrium.

Assuming uniform distribution of types, figure 5 draws which kind of equilibrium arise and what type of contracts are offered in that equilibrium. A higher heterogeneity and lower screening cost support a separating equilibrium. Further, starting from a separating equilibrium, and holding the cost of screening fixed, as heterogeneity decreases first partial pooling and then complete pooling emerge as equilibrium. This is so because when the types are closer to each other they produce surpluses close to each other and it becomes less attractive to separate them as it incurs a cost. For a given level of heterogeneity and again starting from a separating equilibrium, a higher screening cost first shifts the equilibrium to be partial pooling and then to pooling as the cost become higher.

Figure 5: Equilibria with varying heterogeneity and cost of screening

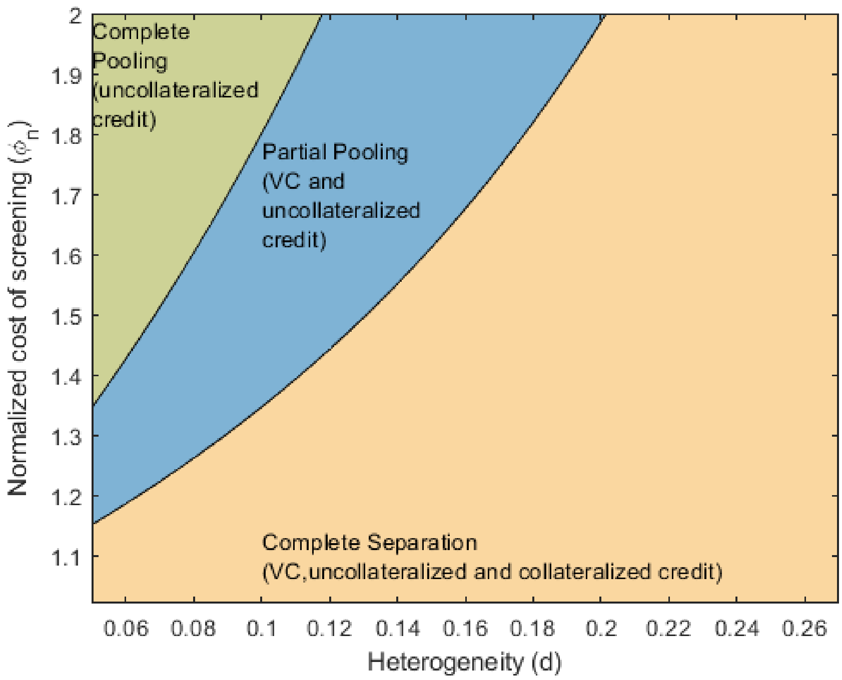


Note on the figure: Uniform distribution with $f_l = f_m = f_h = 1/3$ and $I_l > I_h$

The parameters chosen in this example are such that they give rise to $I_h < I_l < I_m$, just like the way figure 4 is drawn. Therefore, from Proposition 2 we know that in figure 5 under partial pooling, VC is offered alongside uncollateralized credit. If we had $I_l < I_h < I_m$, then according to Proposition 3, under partial pooling collateralized credit will be offered alongside uncollateralized credit.

To see the effect of distribution on the possible equilibria, let us deviate from the uniform distribution and consider a uni-modal distribution with higher proportion of e_m types. With a bigger population of e_m types, as discussed in earlier, the bank has more leeway in separating the types. Thus, with the distribution skewed towards the e_m types, the separation region in the figure 5 should expand at the expense of the partial pooling and complete pooling regions. Figure 6 is drawn with $f_m = \frac{1}{2}$ and $f_l = f_h = \frac{1}{4}$ to demonstrate this.

Figure 6: Equilibria with varying heterogeneity and cost of screening



Note on the figure: Uni-modal distribution – $f_m = \frac{1}{2}$, $f_l = f_h = \frac{1}{4}$ and $I_l > I_h$

Example 2: Separation is not viable

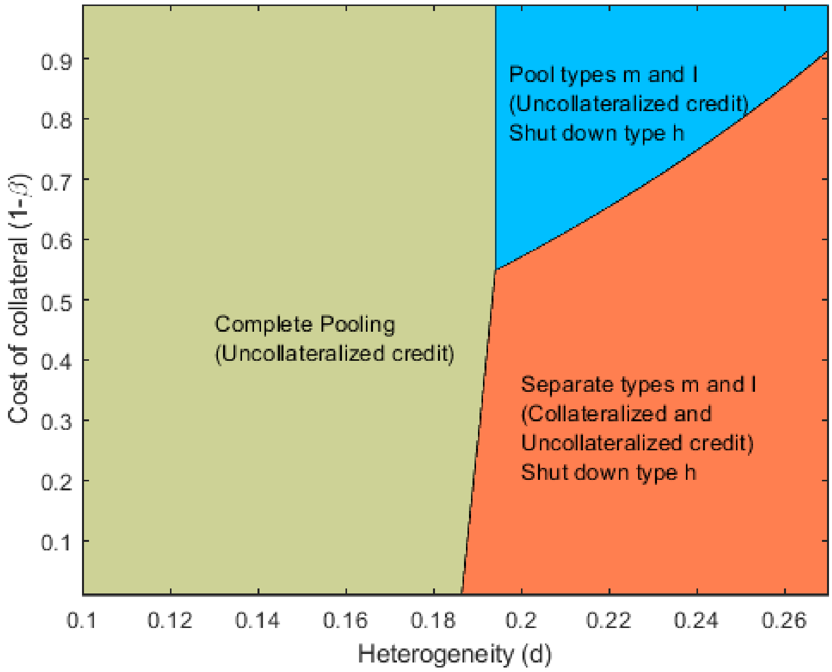
Let us now look at an example where cost of separation is too high such that condition (C.2) is not met and that ensures that the bank will never choose to separate the types.

Suppose conducting VC business for the bank is prohibitively expensive (ϕ is too large). As discussed in section 4.3 this can also be thought of as a case where the bank is prohibited from conducting VC business. Therefore, the bank will serve the high type only when it finds the conditions optimal for pooling all the types, in other cases it will set the terms of the contracts such that the high innovative types do not find funding attractive enough to participate. This entails a loss of welfare, for a socially desirable project is left unfunded.

In a similar exercise to the previous example, we analyze the role of heterogeneity and cost of screening on equilibrium outcome. However, this time the cost of VC and collateral are not tied to each other. However, the dimensionality this time is reduced because we do not need to care about the role of cost of VC, once it is set prohibitively high. So, we analyze the problem with cost of collateral $(1-\beta)$ and heterogeneity (d).

We see in figure 7 the bank can shut down the high default probability types for higher heterogeneity. The intuition is similar to above, that with highly heterogeneous entrepreneurs, the incentive to separate is higher as the bank can extract surplus from each type. With higher d,

Figure 7: Equilibria with varying heterogeneity and cost of collateral



Note on the figure: $f_l = 0.5, f_m = 0.3, f_h = 0.2$, cost of VC is set to be prohibitively high and $I_l > I_h$

e_h is higher as well and the surplus it produces is smaller (though positive) and any attempt to separate it will not be worth given the high cost of VC. Therefore, instead of pooling it with the others, the bank chooses to not serve the high type. These are the two shut-down regions in the right. As discussed earlier these regions generate social loss due to exclusion of socially desirable projects.

5 Alternative assumptions

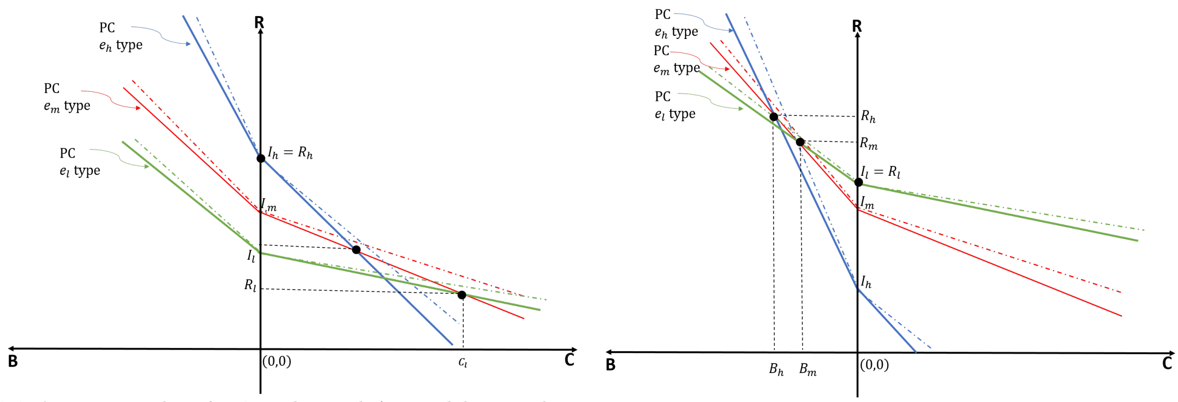
In this section we discuss the alternative assumptions to the ones made in the previous section.

5.1 If $I_l < I_m < I_h$ or $I_h < I_m < I_l$

Let us start with the violation of assumption 4. Suppose instead of the two possibilities discussed in section 4 where e_m is the top type, we have the es such that either $I_l < I_m < I_h$ or $I_h < I_m < I_l$ holds. We can analyse these two scenarios with the graphical tool developed in the previous section. Using the fact that the bank will never use both VC investment and Collateral in a contract we can restrict our attention again to the 2-dimensional planes as in figure 4.

Let us consider the case when $I_l < I_m < I_h$. The low type is the bottom type and under asymmetric information all the other types would mimic it should the bank offer the symmetric information contracts. How does the bank distort the contract of the e_l type to prevent that? Offering a VC contract will not help as the higher innovative types would prefer such a contract even more. So, the bank can offer the low type, a collateralized credit, asking it to put up a collateral and pay a lower interest rate – something that the more innovative types would not find attractive. However, there is still a threat of e_h type selecting the first best contract of the e_m type. To address that the bank needs to distort it as well. Again VC contract will not be effective, but requiring a collateral will still work as the high risk type would prefer not to put up a collateral (figure 8a).

Figure 8: Contracts when e_m is not the top type



(a) An example of a 'credit-only' equilibria when $I_l < I_m < I_h$

(b) An example of a 'VC-only' equilibria when $I_h < I_m < I_l$

Thus we see, in such a scenario no VC contract is offered. Of course whether the bank ultimately separates all the types or engages in some form of pooling would depend on other parameters such as heterogeneity and cost of collateral $(1-\beta)$ but what is true regardless is that the bank will never offer a VC contract in this scenario.

A symmetric line of arguments in case of $I_h < I_m < I_l$, will establish that we will observe only VC market as collateralized credit will never be effective in sorting the entrepreneurs (figure 8b).

5.2 If $\kappa \leq \gamma + v_0$ or $\gamma + v_0 + v_1 \leq \kappa$

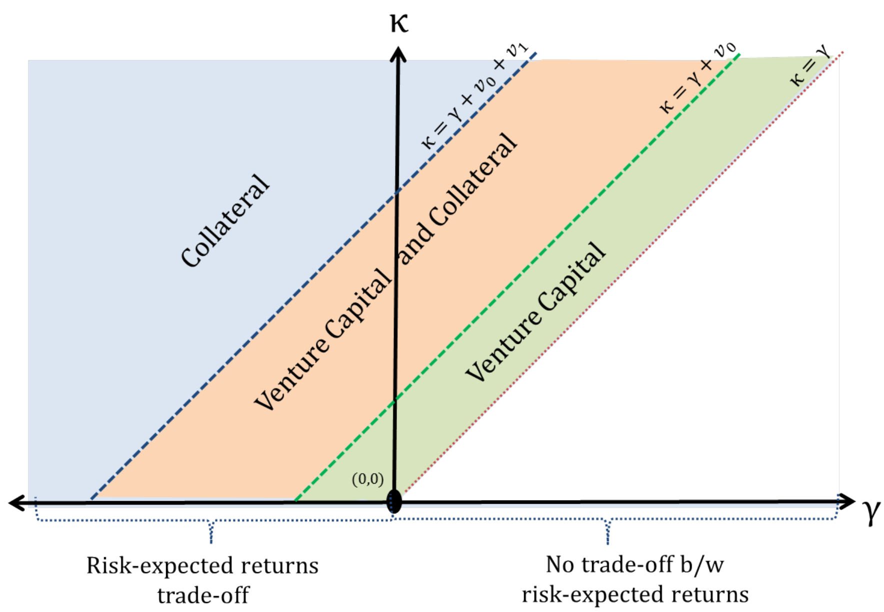
Let us discuss what happens when either of the two inequalities in assumption 2 is not met. Suppose $\kappa \leq \gamma + v_0$, then using (8) we can see $I'(e) < 0, \forall e \in (0,1)$. This implies $I_h < I_m < I_l$ and we just showed above in such case we shall observe only VC contracts as collateral is not an effective screening device. This result is related to Lengwiler and Rishabh (2018) where they show for $\kappa = \gamma$

and $v_1 = 0$ but for any arbitrary type distribution that bank will not find collateral any useful. Besanko and Thakor (1987) show the same in two type borrower case. Here I show that indeed collateral will not work as a screening device but the bank can offer VC contracts to make up for that.

What about when $\gamma + v_0 + v_1 \leq \kappa$? First note $I'(e) = \dfrac{\kappa - \gamma - v_0 - v_1(2e - e^2)}{(1-e)^2}$. Further for $e \in (0,1)$, $0 < 2e - e^2 < 1$. Therefore if $\gamma + v_0 + v_1 \leq \kappa$ then $I'(e) > 0$. This implies $I_l < I_m < I_h$ but for this case we already showed that collateralzed credit will work but not the VC. Therefore, if $\gamma + v_0 + v_1 \leq \kappa$ we will observe only credit contracts and no VC.

This helps us bring the results in section 4 together with the discussion here. We get three distinct regions as shown in figure 9. In the middle region the possible set of contracts in equilibrium support bank as a venture capitalist. In the two adjoining regions, the bank has to take form of a pure credit granting institution or a pure VC. It turns out that the parameter v_1 has a significant role in this trifurcation. As v_1 becomes larger, the region that supports bank as a VC expands.

Figure 9: Set of possible equilibrium contracts



5.3 Alternative V functions

This refers to our assumption 1. We already discussed the assumption about increasing V function in section 4. Here we will talk about the assumption that V is quadratic in e. The idea here is that when the outside opportunity increases at sufficiently high rate with the type then, under symmetric information, in order to serve the higher types, eventually the bank will need to lower the interest rate it charges on its credit. This creates incentives for the medium types to prefer symmetric information contracts of both the lower and higher types. Therefore, the bank need to distort the contracts of the non-medium types in two different directions – one through collateral and one through VC. Any V function that satisfies this property will generate these incentives and therefore a co-existence of collateralized credit with VC financing. Our quadratic V function is one such example. For instance, a cubic or quartic function would also work and in a more extreme example an exponential V function would also generate these incentives.

In contrast to non-linear V function, a linear V function would generate incentives for the higher type to mimic the lower types or lower types to mimic the higher types, but never both. As we showed above that in the former case the bank will employ only collateral and in the latter case only venture capital to screen entrepreneurs.

6 Conclusion

Banks serve the economy by providing a useful screening service. They screen entrepreneurs, who possess more information about the prospects of their projects than any other financier does. In this study, I presented a model of financial intermediation that explains how, to perform its screening role, banks sometimes (need to) act as a venture capitalist.

I show that when the bank is in a situation in which it cannot effectively separate observationally identical entrepreneurs with the use of collateral alone, it resorts to VC financing. I show that this kind of situation may arise when an entrepreneur's project returns, as well as their reservation utility, depend on their entrepreneurial type. The model predicts that from the bank's menu of contracts, the more entrepreneurial (more innovative) types self-select into the VC contract, where a VC contract consists of capital and value-adding investments by the bank against a high repayment in the success state. The less innovative entrepreneurs self-select into the collateralized credit contract. I also discuss the welfare properties of the optimal contracts. I show that if the bank's VC business is prohibited (or is prohibitively costly), then inefficiency due to restricted market access may ensue, as the bank may decide not to serve all the types.

I develop a novel mechanism that leads to a situation where an intermediate type (*medium type*) has the incentive to mimic both the higher and the lower types. This results in a screening device being only partially effective, requiring another screening device to be introduced in the menu of contracts. The mechanism rests on the two opposing forces–one that strengthens the principal's incentive to demand a higher amount from the agents of higher type, and another that works to reduce that effect. In my set-up, the former effect arises due to the agent's (entrepreneur's) success state return being increasing in their type. The latter effect derives from a reservation utility function that is increasing in type. This general mechanism has potential for application in other economic contexts as well.

References

Andrieu, Guillaume, and Alexander Peter Groh. 2012. "Entrepreneurs' financing choice between independent and bank-affiliated venture capital firms." *Journal of Corporate Finance*, 18(5): 1143–1167.

Avery, Robert B, Raphael W Bostic, and Katherine A Samolyk. 1998. "The role of personal wealth in small business finance." *Journal of Banking & Finance*, 22(6-8): 1019–1061.

Barba, Robert, and Tanaya Macheel. 2016. "Banks Play Growing Role in Funding Fintech." *American Banker*.

Barro, Robert J. 1976. "The loan market, collateral, and rates of interest." *Journal of money, Credit and banking*, 8(4): 439–456.

Bengtsson, Ola, and John RM Hand. 2011. "CEO compensation in venture-backed firms." *Journal of Business Venturing*, 26(4): 391–411.

Berlin, Mitchell, Kose John, and Anthony Saunders. 1996. "Bank equity stakes in borrowing firms and financial distress." *Review of Financial Studies*, 9(3): 889–919.

Besanko, David, and Anjan V Thakor. 1987. "Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets." *International Economic Review*, 671–689.

Bottazzi, Laura, Marco Da Rin, and Thomas Hellmann. 2008. "Who are the active investors?: Evidence from venture capital." *Journal of Financial Economics*, 89(3): 488–512.

Bottazzi, Laura, Marco Da Rin, and Thomas Hellmann. 2009. "What is the role of legal systems in financial intermediation? Theory and evidence." *Journal of Financial Intermediation*, 18(4): 559–598.

Boyd, John H, Chun Chang, and Bruce D Smith. 1998. "Moral hazard under commercial and universal banking." *Journal of Money, Credit and Banking*, 426–468.

Burchardt, Jens, Ulrich Hommel, Dzidziso Samuel Kamuriwo, and Carolina Billitteri. 2016. "Venture capital contracting in theory and practice: implications for entrepreneurship research." *Entrepreneurship Theory and Practice*, 40(1): 25–48.

Cumming, Douglas J. 2005*a*. "Agency costs, institutions, learning, and taxation in venture capital contracting." *Journal of Business Venturing*, 20(5): 573–622.

Cumming, Douglas J. 2005*b*. “Capital structure in venture finance.” *Journal of Corporate Finance*, 11(3): 550–585.

Cumming, Douglas J, and Samuele Murtinu. 2016. “On the efficiency of bank-affiliated venture capital.” *SSRN Working Paper*.

Da Rin, Marco, Thomas Hellmann, and Manju Puri. 2013. “A survey of venture capital research.” In *Handbook of the Economics of Finance*. Vol. 2, 573–648. Elsevier.

De Aghion, Beatriz Armendariz, and Christian Gollier. 2000. “Peer group formation in an adverse selection model.” *The Economic Journal*, 110(465): 632–643.

De Bettignies, Jean-Etienne, and James A Brander. 2007. “Financing entrepreneurship: Bank finance versus venture capital.” *Journal of Business Venturing*, 22(6): 808–832.

De Meza, David, and David C Webb. 1987. “Too much investment: a problem of asymmetric information.” *The quarterly journal of economics*, 281–292.

Ewens, Michael, Ramana Nanda, and Christopher T Stanton. 2020. “The evolution of CEO compensation in venture capital backed startups.” *NBER Working Paper*.

Federal Reserve Board. 2020. “Financial regulators modify Volcker rule.” *Federal Reserve Board*.

Ferreira, Miguel A, and Pedro Matos. 2012. “Universal banks and corporate control: Evidence from the global syndicated loan market.” *Review of Financial Studies*, 25(9): 2703–2744.

Freixas, Xavier, and Jean-Charles Rochet. 2008. *Microeconomics of banking.* MIT press.

Ghatak, Maitreesh. 1999. “Group lending, local information and peer selection.” *Journal of Development Economics*, 60(1): 27–50.

Ghatak, Maitreesh. 2000. “Screening by the company you keep: Joint liability lending and the peer selection effect.” *The Economic Journal*, 110(465): 601–631.

Ghatak, Maitreesh, Massimo Morelli, and Tomas Sjöström. 2007. “Entrepreneurial talent, occupational choice, and trickle up policies.” *Journal of Economic Theory*, 137(1): 27–48.

Gilson, Ronald J, and David M Schizer. 2003. “Understanding venture capital structure: a tax explanation for convertible preferred stock.” *Harvard Law Review*, 116(3): 874–916.

Gompers, Paul, Anna Kovner, Josh Lerner, and David Scharfstein. 2010. "Performance persistence in entrepreneurship." *Journal of Financial Economics*, 96(1): 18–32.

Gorman, Michael, and William A Sahlman. 1989. "What do venture capitalists do?" *Journal of business venturing*, 4(4): 231–248.

Hellmann, Thomas. 1997. "Venture capital: A challenge for commercial banks." *The Journal of Private Equity*, 49–55.

Hellmann, Thomas, and Joseph Stiglitz. 2000. "Credit and equity rationing in markets with adverse selection." *European Economic Review*, 44(2): 281–304.

Hellmann, Thomas, and Manju Puri. 2002. "Venture capital and the professionalization of start-up firms: Empirical evidence." *The journal of finance*, 57(1): 169–197.

Hellmann, Thomas, Laura Lindsey, and Manju Puri. 2007. "Building relationships early: Banks in venture capital." *The Review of Financial Studies*, 21(2): 513–541.

Hochberg, Yael V, Carlos J Serrano, and Rosemarie H Ziedonis. 2018. "Patent collateral, investor commitment, and the market for venture lending." *Journal of Financial Economics*, 130(1): 74–94.

House, Christopher L. 2006. "Adverse selection and the financial accelerator." *Journal of Monetary Economics*, 53(6): 1117–1134.

Hsu, David H. 2004. "What do entrepreneurs pay for venture capital affiliation?" *The Journal of Finance*, 59(4): 1805–1844.

Ibrahim, Darian M. 2010. "Debt as venture capital." *University of Illinois Law Review*, 1169.

Invest Europe. 2018. "2017 European Private Equity Activity."

John, Kose, Teresa A John, and Anthony Saunders. 1994. "Universal banking and firm risk-taking." *Journal of Banking & Finance*, 18(2): 307–323.

Kaplan, Steven N, and Per ER Strömberg. 2004. "Characteristics, contracts, and actions: Evidence from venture capitalist analyses." *The Journal of Finance*, 59(5): 2177–2210.

Kaplan, Steven N, Frederic Martel, and Per Strömberg. 2007. "How do legal differences and experience affect financial contracts?" *Journal of financial intermediation*, 16(3): 273–311.

Karlan, Dean, and Jonathan Zinman. 2009. "Observing unobservables: Identifying information asymmetries with a consumer credit field experiment." *Econometrica*, 77(6): 1993–2008.

Kerr, William R, and Ramana Nanda. 2015. "Financing innovation." *Annual Review of Financial Economics*, 7: 445–462.

Lengwiler, Yvan, and Kumar Rishabh. 2018. "Failure of Collateral."

Lorenzetti, Laura. 2014. "Big banks are shunning tradition and turning to tech startups." *Fortune*.

Mahrt-Smith, Jan. 2006. "Should banks own equity stakes in their borrowers? A contractual solution to hold-up problems." *Journal of Banking & Finance*, 30(10): 2911–2929.

Mak, Liz. 2016. "Ten mainland China banks given access to venture capital and private equity business." *South China Morning Post*.

Mann, William. 2018. "Creditor rights and innovation: Evidence from patent collateral." *Journal of Financial Economics*, 130(1): 25–47.

Manso, Gustavo. 2016. "Experimentation and the returns to entrepreneurship." *Review of Financial Studies*, 29(9): 2319–2340.

Minelli, Enrico, and Salvatore Modica. 2009. "Credit market failures and policy." *Journal of Public Economic Theory*, 11(3): 363–382.

Myerson, Roger B. 1979. "Incentive Compatibility and the Bargaining Problem." *Econometrica*, 61–73.

NVCA. 2017. "NVCA Recommends Changes to Volcker Rule to Support VC Investment into Emerging Ecosystems." *National Venture Capital Association*.

Parker, Simon C. 2003. "Asymmetric information, occupational choice and government policy." *The Economic Journal*, 113(490): 861–882.

Reuters. 2016. "China to allow banks to directly invest in high-growth tech firms -sources." *Reuters*.

Robb, Alicia M, and David T Robinson. 2014. "The capital structure decisions of new firms." *Review of Financial Studies*, 27(1): 153–179.

Rothacker, Rick. 2013. "Wells Fargo ramps up private equity despite Volcker Rule." *Reuters*.

Sahlman, William A. 1990. "The structure and governance of venture-capital organizations." *Journal of financial economics*, 27(2): 473–521.

Santos, João AC. 1999. "Bank capital and equity investment regulations." *Journal of Banking & Finance*, 23(7): 1095–1120.

Scheuer, Florian. 2013. "Adverse selection in credit markets and regressive profit taxation." *Journal of Economic Theory*, 148(4): 1333–1360.

Schmalz, Martin C, David A Sraer, and David Thesmar. 2017. "Housing collateral and entrepreneurship." *Journal of Finance*, 72(1): 99–132.

Sengupta, Rajdeep. 2014. "Lending to uncreditworthy borrowers." *Journal of Financial Intermediation*, 23(1): 101–128.

Shukla, Saloni, and Maulik Vyas. 2016. "Private banks keen to play venture capitalists to startups." *The Economic Times*.

Stiglitz, Joseph E, and Andrew Weiss. 1981. "Credit rationing in markets with imperfect information." *The American economic review*, 71(3): 393–410.

The Jakarta Post. 2017. "BRI to Acquire Bahana Artha Vebtura for Rp 71 Billion." *The Jakarta Post*.

Tirole, Jean. 2010. *The Theory of Corporate Finance.* Princeton University Press.

Ueda, Masako. 2004. "Banks versus venture capital: Project evaluation, screening, and expropriation." *The Journal of Finance*, 59(2): 601–621.

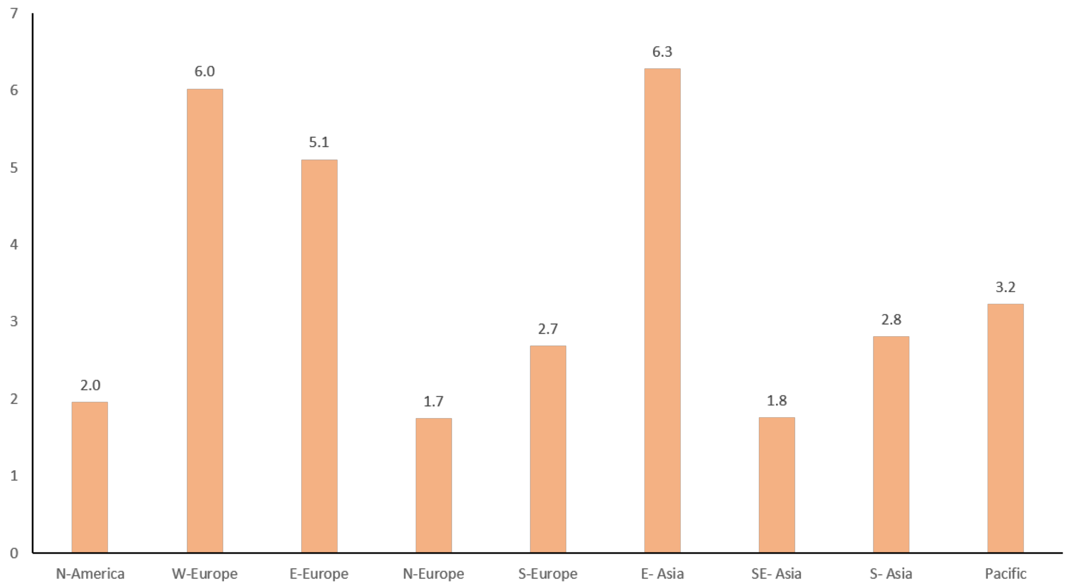
Von Beschwitz, Bastian, and Daniel Foos. 2018. "Banks' equity stakes and lending: Evidence from a tax reform." *Journal of Banking & Finance*, 96: 322–343.

Wasserman, Noam. 2006. "Stewards, agents, and the founder discount: Executive compensation in new ventures." *Academy of Management Journal*, 49(5): 960–976.

Winton, Andrew, and Vijay Yerramilli. 2008. "Entrepreneurial finance: Banks versus venture capital." *Journal of Financial Economics*, 88(1): 51–79.

Appendix A Bank affiliated venture capital investment

Figure A.1: Share of investment by bank affiliated venture capital firms of the total VC investment in the period 2013-17



Notes on the figure: Data is in percent. Source: Thomson One database on private equity.

Appendix B Proofs

Lemma 2 and solution to P-II

Using Lemma 1 and equation (8) we make the following substitutions for R_l and R_h in P-II:

$$R_l = I_l - \frac{e_l}{1-e_l}C_l + \left[\delta + \frac{\lambda}{1-e_l}\right]B_l$$
$$R_h = I_h - \frac{e_h}{1-e_h}C_h + \left[\delta + \frac{\lambda}{1-e_h}\right]B_h$$

Therefore, the Lagrangian for the problem is given by:

$$
\begin{aligned}
\mathscr{L} = & \left[S_l - e_l(1-\beta)C_l - \{\phi - \lambda - \delta(1-e_l)\}B_l\right] f_l + \left[(1-e_m)R_m + e_m\beta C_m - \phi B_m - 1\right] f_m \\
& + \left[S_h - e_h(1-\beta)C_h - \{\phi - \lambda - \delta(1-e_h)\}B_h\right] f_h \\
& - \mu_1 [V_m - (1-e_m)Y_m + (1-e_m)R_m + e_m C_m - \{\lambda + \delta(1-e_m)\}B_m] \\
& - \mu_2 \left[R_m + \frac{e_m}{1-e_m}C_m - \left\{\delta + \frac{\lambda}{1-e_m}\right\}B_m - I_l - \Delta_{ml}C_l + \lambda\Delta_{ml}B_l\right] \\
& - \mu_3 \left[R_m + \frac{e_m}{1-e_m}C_m - \left\{\delta + \frac{\lambda}{1-e_m}\right\}B_m - I_h + \Delta_{hm}C_h - \lambda\Delta_{hm}B_h\right]
\end{aligned}
$$

Where $\Delta_{ij} := \frac{e_i - e_j}{(1-e_i)(1-e_j)} = \left[\frac{e_i}{1-e_i} - \frac{e_j}{1-e_j}\right] = \left[\frac{1}{1-e_i} - \frac{1}{1-e_j}\right]$ and $\Delta_{ml} > 0$, $\Delta_{hm} > 0$. μ_1, μ_2, μ_3 are the Lagrange multipliers.

The Kuhn-Tucker conditions are:

$$-f_l e_l(1-\beta) + \mu_2\Delta_{ml} \le 0, \qquad C_l[-f_l e_l(1-\beta) + \mu_2\Delta_{ml}] = 0 \tag{B.1}$$

$$-f_h e_h(1-\beta) - \mu_3\Delta_{hm} \le 0, \qquad C_h[-f_h e_h(1-\beta) - \mu_3\Delta_{hm}] = 0 \tag{B.2}$$

$$-f_l\{\phi - \lambda - \delta(1-e_l)\} - \mu_2\lambda\Delta_{ml} \le 0, \qquad B_l[-f_l\{\phi - \lambda - \delta(1-e_l)\} - \mu_2\lambda\Delta_{ml}] = 0 \tag{B.3}$$

$$-f_h\{\phi - \lambda - \delta(1-e_h)\} + \mu_3\lambda\Delta_{hm} \le 0, \qquad B_h[-f_h\{\phi - \lambda - \delta(1-e_h)\} + \mu_3\lambda\Delta_{hm}] = 0 \tag{B.4}$$

$$f_m e_m\beta - \mu_1 e_m - \mu_2\frac{e_m}{1-e_m} - \mu_3\frac{e_m}{1-e_m} \le 0, \qquad C_m[f_m e_m\beta - \mu_1 e_m - \mu_2\frac{e_m}{1-e_m} - \mu_3\frac{e_m}{1-e_m}] = 0 \tag{B.5}$$

$$
\begin{aligned}
& -f_m\phi + \mu_1\{\lambda + \delta(1-e_m)\} + \mu_2\left\{\delta + \frac{\lambda}{1-e_m}\right\} + \mu_3\left\{\delta + \frac{\lambda}{1-e_m}\right\} \le 0, \\
& B_m\left[-f_m\phi + \mu_1\{\lambda + \delta(1-e_m)\} + \mu_2\left\{\delta + \frac{\lambda}{1-e_m}\right\} + \mu_3\left\{\delta + \frac{\lambda}{1-e_m}\right\}\right] = 0
\end{aligned} \tag{B.6}
$$

$$f_m(1-e_m) - \mu_1(1-e_m) - \mu_2 - \mu_3 \le 0, \qquad R_m[f_m(1-e_m) - \mu_1(1-e_m) - \mu_2 - \mu_3] = 0 \tag{B.7}$$

In addition, the complementary slackness conditions given the constraints in (P-II) are:

$$
\begin{aligned}
\mu_1 \ge 0, \qquad & \mu_1[V_m - (1-e_m)Y_m + (1-e_m)R_m + e_m C_m - \{\lambda + \delta(1-e_m)\}B_m] = 0 \\
\mu_2 \ge 0, \qquad & \mu_2\left[R_m + \frac{e_m}{1-e_m}C_m - \left\{\delta + \frac{\lambda}{1-e_m}\right\}B_m - I_l - \Delta_{ml}C_l + \lambda\Delta_{ml}B_l\right] = 0 \\
\mu_3 \ge 0, \qquad & \mu_3\left[R_m + \frac{e_m}{1-e_m}C_m - \left\{\delta + \frac{\lambda}{1-e_m}\right\}B_m - I_h + \Delta_{hm}C_h - \lambda\Delta_{hm}B_h\right] = 0
\end{aligned}
$$

Starting with (B.3), Suppose $B_l > 0$, then $\mu_2 = -\frac{f_l\{\phi - \lambda - \delta(1-e_l)\}}{\lambda\Delta_{ml}} < 0$, a contradiction. Therefore,

$B_l = 0$. Similarly, solving (B.2) under the assumption $C_h > 0$, will lead us to a contradiction implying that $C_h = 0$. Note these are true regardless of whether C_l and B_h are non-zero or not.
Now consider if $C_m > 0$, then $f_m\beta = \mu_1 + \frac{\mu_2}{1-e_m} + \frac{\mu_3}{1-e_m}$,
if $B_m > 0$ then $f_m\phi = [\lambda + \delta(1-e_m)]\left[\mu_1 + \frac{\mu_2}{1-e_m} + \frac{\mu_3}{1-e_m}\right]$ and,
if $R_m > 0$ then $f_m = \mu_1 + \frac{\mu_2}{1-e_m} + \frac{\mu_3}{1-e_m}$.
Comparing any two of the three conditions, pairwise, will lead to contradiction implying $R_mB_m = 0$, $C_mB_m = 0$ and $R_mC_m = 0$. This leaves us with the following possibilities:
(i) $R_m = 0$, $C_m = 0$, $B_m = 0$ (ii) $R_m = 0$, $C_m = 0$, $B_m > 0$ (iii) $R_m = 0$, $C_m > 0$, $B_m = 0$ and (iv) $R_m > 0$, $C_m = 0$, $B_m = 0$.
Offering (i) and (ii) are irrational and therefore are ruled out. Offering (iii) cannot be profit maximizing: suppose to the contrary, it is and (iii) satisfies all the constraints but then by reducing C_m slightly and increasing R_m at the same time such that constraints are still satisfied will lead to an unambiguous increase in profit. Therefore, the only possibility that remains is $R_m > 0$, $C_m = 0$, $B_m = 0$ as a result $f_m = \mu_1 + \frac{\mu_2}{1-e_m} + \frac{\mu_3}{1-e_m}$ holds.

Finally, given that $C_h = 0 = B_l$ under the assumption of complete separation bank must choose positive values of C_l and B_h, otherwise there are not enough devices to screen the entrepreneurs. Thus, under complete separation $C_l > 0$ and $B_h > 0$, that gives us

$$\mu_2 = e_l(1-\beta)\frac{f_l}{\Delta_{ml}} \tag{B.8}$$

$$\mu_3 = \{\phi - \lambda - \delta(1-e_h)\}\frac{f_h}{\lambda\Delta_{hm}} \tag{B.9}$$

Given that $0 < \beta < 1$, $\phi - \lambda - \delta(1-e_h) > 0$ and $\Delta_{ml}, \Delta_{hm} > 0$ we have $\mu_2 > 0$ and $\mu_3 > 0$. Therefore, complementary slackness conditions imply that the IC_{ml} and IC_{mh} in (P-II) bind.

Finally to check whether the PC_m binds or not, we look at the optimization problem again with above results. Given that $B_m = 0 = C_m$, $C_h = 0$, $B_l = 0$ and IC_{mh}, IC_{ml}, PC_l and PC_h bind, the problem simplifies to the bank choosing R_m to maximize

$$\left[S_l - e_l(1-\beta)\frac{R_m - I_l}{\Delta_{ml}}\right] f_l + [(1-e_m)R_m - 1] f_m + \left[S_h - \{\phi - \lambda - \delta(1-e_h)\}\frac{R_m - I_h}{\lambda\Delta_{hm}} B_h\right] f_h$$

subject to PC_m i.e. subject to $R_m \le I_m$. Thus, the bank chooses largest possible value of R_m if

$$-e_l(1-\beta)\frac{f_l}{\Delta_{ml}} + f_m(1-e_m) - \{\phi - \lambda - \delta(1-e_h)\}\frac{f_h}{\lambda\Delta_{hm}} \ge 0 \tag{B.10}$$

$$\Leftrightarrow \quad f_m - \frac{\mu_2}{1-e_m} - \frac{\mu_3}{1-e_m} = \mu_3 \ge 0$$

Thus, the bank chooses $R_m = I_m$ when the above holds. If it does not hold, the bank profit is decreasing in R_m and then at least one of the above assumptions will not hold. Specifically, then at least one the two among B_h and C_l will be zero. That will be a case of (partial) pooling. Therefore, (B.10) also becomes a necessary condition for separation, which can be rearranged and written as given in (C.1).

We also require conditions that ensure that by separating the types the bank does not make negative profits on any type. Given the result from Lemma 1 that the types e_h and e_l non-negative profit condition puts upper bounds on C_l and B_h as below:

$$C_l \leq \frac{S_l}{e_l\,(1-\beta)} \quad \text{and} \quad B_h \leq \frac{S_h}{\phi - \lambda - \delta(1-e_h)}$$

With these results, the optimal contracts under complete separation are given in the main text in (11).

Proof of Lemma 3

We show that solution in (11) is also a solution to the original problem, P-I by showing that this solution satisfies all the remaining constraints in P-I. There are four remaining ICs from P-I that are to be checked. These are IC_{lm}, IC_{hm}, IC_{hl} and IC_{lh}.

Proof.
Solution to P-II already satisfies IC_{ml} and IC_{mh}. We consider the remaining four ICs.

IC_{lm} and IC_{hm}:
Consider a $k \in \{l, h\}$

$$\begin{aligned}
& I_k < I_m \quad [\text{By assumption for } k \in \{l,h\}] \\
\Rightarrow \quad & I_k - \frac{e_k}{1-e_k}C_k + \left[\delta + \frac{\lambda}{1-e_k}\right]B_k + \frac{e_k}{1-e_k}C_k - \left[\delta + \frac{\lambda}{1-e_k}\right]B_k < I_m \\
\Rightarrow \quad & R_k + \frac{e_k}{1-e_k}C_k - \left[\delta + \frac{\lambda}{1-e_k}\right]B_k < R_m + \frac{e_k}{1-e_k}C_m - \left[\delta + \frac{\lambda}{1-e_k}\right]B_m
\end{aligned}$$

The last step uses the solution to P-II. This implies IC_{km} is satisfied for $k \in \{l, h\}$ using the definition in (9).

IC_{hl}:
For IC_{hl}, we need to consider the two possible cases: $I_h < I_l$ and $I_h > I_l$. If $I_h < I_l$ then,

$$I_h < I_l$$

Since $\Delta_{hl} > 0$ and $C_l > 0$, the above inequality still holds with the following addition on the right hand side:

$$I_h < I_l + \Delta_{hl} C_l$$

Now noting that $B_l = 0 = C_h$

$$\begin{aligned}
& I_h + \left[\delta + \frac{\lambda}{1-e_h}\right] B_h + \frac{e_h}{1-e_h} C_h - \frac{1}{1-e_h} B_h && < I_l + \Delta_{hl} C_l - \left[\delta + \frac{\lambda}{1-e_h}\right] B_l \\
\Rightarrow\ & R_h + \frac{e_h}{1-e_h} C_h - \left[\delta + \frac{\lambda}{1-e_h}\right] B_h && < I_l + \Delta_{hl} C_l - \left[\delta + \frac{\lambda}{1-e_h}\right] B_l \quad \text{[using solution to P-II]} \\
\Rightarrow\ & R_h + \frac{e_h}{1-e_h} C_h - \left[\delta + \frac{\lambda}{1-e_h}\right] B_h && < I_l - \frac{e_l}{1-e_l} C_l + \frac{e_h}{1-e_h} C_l - \left[\delta + \frac{\lambda}{1-e_h}\right] B_l \\
\Rightarrow\ & R_h + \frac{e_h}{1-e_h} C_h - \left[\delta + \frac{\lambda}{1-e_h}\right] B_h && < R_l + \frac{e_h}{1-e_h} C_l - \left[\delta + \frac{\lambda}{1-e_h}\right] B_l \quad \text{[using solution to P-II]}
\end{aligned}$$

This implies IC_{hl}. For the case when $I_h > I_l$ we have,

$$\begin{aligned}
& I_l < I_h \\
\Rightarrow\ & I_h - I_l < I_m - I_l && \text{[since } I_m > I_h > I_l\text{]} \\
\Rightarrow\ & I_h - I_l < [I_m - I_l] \underbrace{\left(\frac{e_h - e_l}{e_m - e_l}\right)}_{>1} \underbrace{\left(\frac{1-e_m}{1-e_h}\right)}_{>1} \left(\frac{1-e_l}{1-e_l}\right) && \text{[since } e_h > e_m > e_l\text{]} \\
\Leftrightarrow\ & I_h - I_l < [I_m - I_l] \frac{1}{\Delta_{ml}} \Delta_{hl} \\
\Rightarrow\ & I_h < I_l + \Delta_{hl} C_l && \text{[using solution to P-II]}
\end{aligned}$$

Now we are back in the second step of proof in the previous case from where we already showed IC_{hl}.

IC$_{lh}$:

Again there are two possible cases. We start with $I_l < I_h$.

$$I_l < I_h$$

Since $\Delta_{hl} > 0$, $\lambda > 0$ and $B_h > 0$ the above inequality still holds with the following addition on the right hand side:

$$I_l < I_h + \lambda \Delta_{hl} B_h$$

Now noting that $B_l = 0 = C_h$

$$
\begin{aligned}
& I_l - \frac{e_l}{1-e_l}C_l + \frac{e_l}{1-e_l}C_l - \left[\delta + \frac{\lambda}{1-e_l}\right]B_l && < I_h + \lambda\Delta_{hl}B_h + \frac{e_l}{1-e_l}C_h \\
\Rightarrow\ & R_l + \frac{e_l}{1-e_l}C_l - \left[\delta + \frac{\lambda}{1-e_l}\right]B_l && < I_h + \lambda\Delta_{hl}B_h + \frac{e_l}{1-e_l}C_h \qquad \text{[using solution to P-II]} \\
\Leftrightarrow\ & R_l + \frac{e_l}{1-e_l}C_l - \left[\delta + \frac{\lambda}{1-e_l}\right]B_l && < I_h + \frac{\lambda}{1-e_h}B_h - \frac{\lambda}{1-e_l}B_h + \frac{e_l}{1-e_l}C_h \\
\Leftrightarrow\ & R_l + \frac{e_l}{1-e_l}C_l - \left[\delta + \frac{\lambda}{1-e_l}\right]B_l && < I_h + \left[\delta + \frac{\lambda}{1-e_h}\right]B_h - \left[\delta + \frac{\lambda}{1-e_l}\right]B_h + \frac{e_l}{1-e_l}C_h \\
\Rightarrow\ & R_l + \frac{e_l}{1-e_l}C_l - \left[\delta + \frac{\lambda}{1-e_l}\right]B_l && < R_h + \frac{e_l}{1-e_l}C_h - \left[\delta + \frac{\lambda}{1-e_l}\right]B_h \quad \text{[using solution to P-II]}
\end{aligned}
$$

This implies IC_{lh}. In the case $I_h < I_l$ we have,

$$
\begin{aligned}
& I_h < I_l \\
\Rightarrow\ & I_l - I_h < I_m - I_h \qquad\qquad \text{[since } I_m > I_l > I_h\text{]}
\end{aligned}
$$

Noting that the function $\frac{e_h - e}{1-e}$ is decreasing in e, we have,

$$
\begin{aligned}
& I_l - I_h < [I_m - I_h]\underbrace{\left(\frac{\frac{e_h-e_l}{1-e_l}}{\frac{e_h-e_m}{1-e_m}}\right)}_{>1}\left(\frac{1-e_h}{1-e_h}\right) \\
\Leftrightarrow\ & I_l - I_h < [I_m - I_h]\frac{1}{\Delta_{hm}}\Delta_{hl} \\
\Rightarrow\ & I_l < I_h + \lambda\Delta_{hl}B_h \qquad\qquad \text{[using solution to P-II]}
\end{aligned}
$$

Now rest of the proof follows the same steps as in the previous case, giving us IC_{lh} again. QED

Proof of Proposition 2

Following the discussion in section 4.2, $BiCi = 0\ \forall i \in \{l, m, h\}$ will hold. Further, any partial pooling must involve two top types (i.e. e_m with one other type). Since the optimal contracts depend upon who the bank separates, we discuss each case separately.

If $I_h < I_l < Im$, then the bank would pool types e_m and e_l and separate e_h types. As we discussed earlier in order to separate the e_h types, distortion should be through a VC contract that satisfies all the constraints in (P-I).

Proof.

For the pooled types, an undistorted contract that would attract the two top types is the full information contract of e_l type. Therefore the bank would offer

$$R_l = R_m = I_l, \qquad C_l = C_m = 0, \qquad B_l = B_m = 0$$

Clearly this contract, would not be attractive for e_h type and therefore satisfies IC$_{hm}$ and IC$_{hl}$. IC$_{mh}$ and IC$_{lh}$, are the ICs that imply distortion in the first best contract of of the e_h type. That is offering a VC contract to e_h type. How much should the VC investment be? Enough that IC$_{mh}$ is satisfied with equality. Keeping in mind $C_h = 0$ and $B_m = C_m = 0$, IC$_{mh}$ is:

$$\begin{aligned} & R_m = R_h - \left[\delta + \frac{\lambda}{1-e_m}\right] B_h \\ \Rightarrow \quad & I_l = R_h - \left[\delta + \frac{\lambda}{1-e_m}\right] B_h \qquad [\text{since } R_m = I_m = I_l] \end{aligned}$$

Using the PC for e_h.

$$\begin{aligned} & I_l = I_h + \left[\delta + \frac{\lambda}{1-e_h}\right] B_h - \left[\delta + \frac{\lambda}{1-e_m}\right] B_h \\ \Rightarrow \quad & B_h = \frac{I_l - I_h}{\lambda \Delta_{hm}} \end{aligned}$$

And $R_h = I_h + \left[\delta + \frac{\lambda}{1-e_h}\right] B_h$. It can be checked this contract will satisfy IC$_{lh}$ as well.

Therefore, the bank's profit under partial pooling is:

$$\pi^{pp} = \left[f_l\{(1-e_l)I_l - 1\} + f_m\{(1-e_m)I_l - 1\} + f_h\{(1-e_h)R_h - \phi B_h\}\right]$$

and can be expressed as:

$$\pi^{pp} = f_l S_l + f_m S_m - f_m(1-e_m)[I_m - I_l] + f_h S_h - \{\phi - \lambda - \delta(1-e_h)\} f_h \frac{I_l - I_h}{\lambda \Delta_{hm}}$$

Before finding the condition for partial pooling, let us consider the optimal contracts for complete pooling. Since $I_h < I_l < Im$, in the complete pooling equilibrium the bank would offer $R_l = R_m = R_h = I_h$, $C_h = C_m = C_l = 0$, $B_l = B_m = B_h = 0$. Bank's profit under pooling equilibrium is

$$\pi^{pool} = f_l S_l + f_m S_m + f_h S_h - f_l(1-e_l)[I_l - I_h] - f_m(1-e_m)[I_m - I_h]$$

The bank would partially pool if $\pi^{pp} \geq \pi^{pool}$, which gives us:

$$\frac{1}{f_m}\left[\{\phi - \lambda - \delta(1-e_h)\}\frac{f_h}{\lambda}\left(\frac{1-e_h}{e_h - e_m}\right) - f_l\left(\frac{1-e_l}{1-e_m}\right)\right] \leq 1$$

QED

Proof of Proposition 3

Proof.

For the case $I_l < I_h < Im$, the proof follows the same steps as in the proof of the proposition 2. In this case the top two types – e_m and e_h are pooled. The separated type is the low type. Since, the bank will not use VC contract to separate the low type, the bank will set $B_l = 0$. Following the same steps as above we can show that the optimal contracts are:

$$R_h = R_m = I_h, \qquad C_h = C_m = 0, \qquad B_h = B_m = 0$$
$$R_l = I_l - \frac{e_l}{1-e_l}C_l, \qquad C_l = \frac{I_h - I_l}{\Delta_{ml}}, \qquad B_l = 0$$

Further, optimal contracts under complete pooling in the case of $I_l < I_h < Im$ are given by: $R_l = R_m = R_h = I_l$, $C_h = C_m = C_l = 0$, $B_l = B_m = B_h = 0$ Comparing profits in the two cases will gives us:

$$\frac{1}{f_m}\left[(1-\beta)e_l f_l\left(\frac{1-e_l}{e_m - e_l}\right) - f_h\left(\frac{1-e_h}{1-e_m}\right)\right] \leq 1$$

QED

Proof of Proposition 4

Proof. Follows directly from the proofs of propositions 2 and 3. QED

Proof of Proposition 5

Proof. Since the only screening device that is effective in screening the e_h types is too costly to be employed if the bank wants to serve the e_h types it needs to pool them with at least one other type. However, e_h types cannot be partially pooled with any other type given $I_h < I_l < I_m$ because it will not be incentive compatible. Thus, given (C.2) is satisfied, the bank has three options given $I_h < I_l < I_m$ (a) pool all the types, (b) shut down e_h type and separate the remaining two types and (c) shut down e_h type and pool the remaining types. The conditions that govern which of the three options is chosen are derived by comparing the profits under the three options.

Therefore, given $I_h < I_l < I_m$, the only equilibrium consistent with profit maximization where e_h is served must be pooling at the contract that just satisfies the PC of the e_h types. That is at $(I_h, 0, 0)$.

Bank's profit under pooling equilibrium is

$$\pi^{pool} = f_l S_l + f_m S_m + f_h S_h - f_l(1-e_l)[I_l - I_h] - f_m(1-e_m)[I_m - I_h]$$

If the bank instead does not serve the e_h types, the contracts that serve the remaining types should not be attractive to the e_h types. Focusing on the remaining types and considering the separation between e_m and e_l types, the incentive compatible solution can be derived in a similar manner as in the separation problem (P-I). This will give us the contracts in (14). It can be checked that these contracts deliver strictly less utility than V_h to the e_h types. Bank profit in this case is given by:

$$\pi^{sds} = f_l S_l + f_m S_m - f_l e_l (1-\beta) \left[\frac{I_m - I_l}{\Delta_{ml}} \right]$$

Finally, when the bank shuts down the e_h types but pools the remaining types it must involve no collateral as well. Thus, the only profit maximization consistent contract that serves the purpose is the full information contract for e_l type, i.e., $(I_l, 0, 0)$. The bank profit in this case is:

$$\pi^{sdp} = f_l S_l + f_m S_m - f_m(1-e_m)[I_m - I_l]$$

Now we know the bank will pool all the type if: $\pi^{pool} \geq max\{\pi^{sds}, \pi^{sdp}\}$ and this gives us the first condition in the proposition. The bank will shut down e_h type if the above does not hold. It will in turn either separate or pool the other types depending upon if $\pi^{sds} \geq \pi^{sdp}$ and this gives us the two other conditions. QED

Proof of Proposition 6

Proof. Similar to Propositions (3) and (5). QED