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Domain-General Abilities and Mathematical Achievement: A Developmental Perspective on the Roles of Executive Functions, Visual-Spatial Skills, and Emotion Regulation

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ABSTRACT

Mathematics, like reading and writing, is a core skill in children's academic development and has great impact on their later lives. Mathematical achievement depends not only on domain-specific skills but also on domain-general abilities, such as executive functions, visual-spatial skills, and emotion regulation. The current dissertation author investigates the associations between these domain-general abilities and mathematical achievement to increase in-depth understanding of their roles over the course of child development. Until now, it has been unclear whether associations of executive functions, visual-spatial skills, and mathematics change or remain constant across age, whether core components of executive functions and visual-spatial skills are bidirectionally associated with mathematical achievement in primary school students, and whether there are compensating effects on working memory through emotion regulation in children and adolescents. The author conducted three studies to answer these questions and assessed the variables using a broad test battery. Studies 1 and 3 used regression and moderation analyses; study 2 used a cross-lagged model. All statistical models accounted for verbal reasoning, sex, and socio-economic status. Study 1 indicated a significant and age-invariant association between executive functions and mathematical achievement and a significant and age-dependent association between visual-spatial skills and mathematical achievement. Study 2 demonstrated unidirectional associations between visual-spatial skills and mathematical achievement, but bidirectional association between working memory, as a component of executive functions, and mathematical achievement. Study 3 revealed that emotion regulation skills compensate for low working memory skills in children's and adolescents' mathematical achievement. In sum, this cumulative dissertation offers a differentiated view of the interrelations among three domain-general abilities and their association with mathematical achievement across child development. The outlook highlights the theoretical and practical value of the present dissertation and implications for future research.

1. Introduction

Mathematical achievement plays a crucial role in children's academic development at school and is also associated with important areas later in life. From an academic perspective, it has been shown that early mastery of mathematical concepts at school is predictive of later learning and academic abilities, and that predictive strength competes with that of early language, reading, and attention skills (Duncan et al., 2007). Furthermore, an association between mathematical achievement and student life satisfaction is already present in children in their early school years and remains stable across school life up to adolescence (Huebner et al., 2014; Lyons & Huebner, 2016). Studies also indicate that mathematical achievement has long-term relevance in our modern society, as children's mathematical achievement is associated with later employability and socio-economic status (SES) in adulthood (Bynner, 2004; Ritchie & Bates, 2013; Rivera-Batiz, 1992). Mathematical competences are diverse and comprise a variety of different skills across a student's academic development, including basic number knowledge, arithmetic facts, and the ability to follow procedures (Dowker, 2019).

Considering the importance of mathematical understanding, investigating mathematical skills and the variables that hinder or support mathematical development is of topical interest in developmental and school psychology. A typical approach in research on mathematical achievement is to differentiate between two categories of variables that are essential for mathematical development and achievement, domain-specific and domain-general abilities (Geary, 1994). Domain-specific abilities include quantity understanding, number recognition, and counting, which predict mathematical achievement (Bailey et al., 2014). Domain-general abilities are typically defined as underlying cognitive skills such as processing speed, executive functions, and visual-spatial skills. These variables predict not only mathematical achievement, but also other outcome variables, such as reading and writing (Fuchs et al., 2006; Geary et al., 2017; Passolunghi et al., 2008).

Previously, models of mathematical skill development have been predominantly based on theories of numerical cognition and have focused particularly on the importance of domain-specific abilities and mathematics. Pioneering findings, such as studies on early numerical abilities in infants (Wynn, 1992), the approximate number system (ANS: Cantlon et al., 2009; Dehaene et al., 1990), and models of numerical processing (e.g., Dehaene, 1992), have influenced and determined research on mathematical development to a great extent over the last 30 years. Research has shown that mathematical skills develop very early in life and that numbers and magnitudes are processed through different types of mental representations (e.g., verbally).

At the same time, research has shown that domain-general abilities are another crucial pillar of mathematical development (Bailey et al., 2014; Geary et al., 2017). A theoretical framework for domain-general abilities and mathematical achievement is important to provide orientation for research (Geary, 2004). So far, however, this framework has been vague regarding the relations and distinctions between domain-general abilities and mathematical achievement, and has not included developmental aspects. Furthermore, studies have mainly focused on investigating associations between *cognitive* domain-general abilities and mathematics. There is growing evidence, however, that *non*-cognitive domain-general abilities such as children's (mathematical) self-concept, motivation, and emotion-related abilities play important roles in the development and acquisition of mathematical skills (Watts et al., 2015).

Research on the interplay between cognitive and non-cognitive domain-general abilities and how they act and interact with mathematical achievement in child development has been rather neglected in the field (Gilmore et al., 2020). However, such research is relevant given that domain-general abilities can be interconnected, meaning that one ability might be beneficial to another (Carey et al., 2015). This holistic rationale represents the perspective on child development that children are endowed with various competences and resources (Grob & Hagmann-von Arx, 2018). The present dissertation author aims to address these considerations based on Geary's (2004) theoretical framework for investigating mathematical achievement. The focus is on two cognitive domain-general abilities, executive functions and visual-spatial skills, that have prominent roles in mathematical achievement, and one non-cognitive domain-general ability, emotion regulation, that widens the dissertation framework.

Research has shown that each of these domain-general abilities is strongly associated with mathematical achievement in various age groups. However, most of the studies in this field have investigated these abilities separately and in restricted age ranges. The majority have followed a rather static approach, so little is known about how executive functions, visual-spatial skills, and emotion regulation act or interact with mathematical achievement at different points in children's development. Therefore, the author aims to shed light on these relations.

The current studies investigate whether (a) the associations between domain-general abilities and mathematical achievement change across age, (b) domain-general abilities and mathematical achievement are reciprocally associated over time, and (c) one ability can compensate for deficits in another ability in children's mathematical achievement. Consequently, this work contributes to theory building in developmental psychology and provides relevant practical implications for school psychological research and intervention by striving to increase understanding of how domain-general abilities and mathematical achievement are associated at different points in child development.

The dissertation is structured as follows: Section 2 presents the theoretical background and current state of research on domain-general abilities and their association with mathematical achievement. Section 3 summarizes the research questions. Section 4 gives an overview of the present studies' methodologies, and Section 5 presents a synopsis of the results. A general discussion concludes the dissertation in Section 6.

2. Theoretical Background

2.1 Domain-General Abilities and Mathematical Achievement

Theories of domain-general learning in developmental psychology state that children are equipped with abilities that facilitate learning on a broad level independent of the content to be learned (Sloutsky, 2010). The initial interest in investigating different domains in developmental psychology and their connection to academic achievement, such as learning mathematics, emerged from a school-related context. Researchers sought to investigate the factors that underlie differences in children's academic achievement to understand how to give children the most appropriate support (Binet & Simon, 1905). Specific knowledge and methodology expanded and advanced rapidly during the 20th century; developmental psychology became increasingly divided into different domains considered fundamental for child development, such as cognition, emotion, and motivation. Research in the field has focused particularly on the cognitive domain, which is thought to be especially crucial for mathematical achievement.

Geary (2004) developed a theoretical framework to facilitate research on domain-general abilities and mathematical achievement. It is a hierarchical, conceptual scheme that identifies the most relevant cognitive domain-general abilities for the development of mathematical knowledge and achievement. It includes the central executive, language, and visual-spatial systems. Even though the framework uses empirical evidence to highlight the relevance of these three domains, it does not integrate non-cognitive domain-general abilities. However, these abilities, especially emotion-related variables, have a high impact on mathematical achievement. It has often been recommended that they should be investigated more closely and combined with other cognitive domain-general abilities in mathematics research (e.g., Diamond & Ling, 2016; Geary, 2011a; Gilmore et al., 2020).

A non-cognitive domain-general ability that is essential in children's emotional and academic development is emotion regulation. Children are faced with heterogenous demands in (mathematics) education, implying the need for continuous self-regulation, not only on a cognitive level but also on an emotional level (Calkins & Williford, 2009). Based on these considerations and the afore-mentioned theoretical framework (Geary, 2004), the present dissertation is focused on three domain-general abilities: executive function, visual-spatial skill, and emotion regulation. Together, they play a key role in mathematical development and are strongly associated with mathematical achievement in various age groups (executive function overviews: Bull & Lee, 2014; Cragg & Gilmore, 2014; visual-spatial skills overviews: Hawes & Ansari, 2020, McCrink & Opfer, 2014; emotion regulation overviews: Eisenberg et al., 2005; Valiente et al., 2012).

2.1.1 Executive Functions and Mathematical Achievement

Executive functions are typically defined as "a set of general-purpose control processes that regulate one's thoughts and behaviors" (Miyake & Friedman, 2012, p. 8). Several studies have indicated that executive functions are multi-component constructs consisting of three distinct core components (working memory, inhibition, and cognitive flexibility) that are distinct but closely related to each other (e.g., Diamond, 2013; Miyake et al., 2000). Mathematics is a school subject that makes high demands on children, as they need to regulate their thoughts, select appropriate strategies, and update knowledge continuously (Geary, 2000). These cognitive demands require executive functions, which are predictive of mathematical achievement (overview: Bull & Scerif, 2001). Moreover, a large body of research has shown that each of the three executive function components is important for the development of mathematical skills in various age groups (working memory meta-analysis: Friso-Van den Bos et al., 2013; working memory review: Raghubar et al., 2010; inhibition meta-analysis: Allan et al., 2014; cognitive flexibility meta-analysis: Yeniad et al., 2013).

Performing mathematical operations requires working memory, the ability to store and manipulate information in the mind (Baddeley, 1996). Working memory abilities come into play when children need to keep intermediate results, or other relevant information, in mind while solving mathematical problems. It seems to be one of the key cognitive abilities for mathematical learning (Geary et al., 2017). Inhibition, another core component of executive functions, enables children to control their attention or behavior by suppressing external or internal distractions (Diamond, 2013). It is important in mathematics for suppressing automatic number facts or procedures while for instance, performing calculations. The third core component, cognitive flexibility, is the ability to adjust to changes (Diamond, 2013) and is relevant when changing tasks or strategies in mathematics.

Studies on training executive functions and mathematical achievement have focused especially on working memory and far-transfer effects on mathematical achievement. For example, several studies have shown that children who train their working memory skills increase their mathematical achievement as compared to a control group (Kuhn & Holling, 2014; Layes et al., 2018; Passolunghi & Costa, 2016; St. Clair-Thompson et al., 2010; Witt, 2011; meta-analysis: Sala & Gobet, 2017). However, other studies have shown that working memory does not generalize to mathematical achievement (Alloway, 2012; Dunning et al., 2013; Holmes & Gathercole, 2013; Karbach et al., 2015; meta-analysis: Melby-Lervåg & Hulme, 2013). The inconsistent findings may be explained by: (a) different characteristics of the samples such as age, IQ, or inclusion of children with learning disabilities; (b) measures used to operationalize the criteria for assessing mathematical achievement; or (c) characteristics of the training itself, such as frequency of trainings, time points of assessments, and types of training (Diamond & Ling, 2016; Titz & Karbach, 2014).

2.1.2 Visual-Spatial Skills and Mathematical Achievement

Visual-spatial skills are defined as abilities to reason about space by generating, retrieving, and transforming visual images (cf., Casey et al., 1995). Research has shown that mathematical learning is embedded in spatial representations required for spatial-numerical mappings, such as solving tasks of quantity, time, and geometry, or even tasks of pure arithmetic and counting (Hawes & Ansari, 2020). Similar to executive functions, research has suggested different types of visual-spatial skills, but to date, researchers have not agreed on a widely accepted taxonomy of visual-spatial skills (cf. Newcombe & Shipley, 2015). However, mathematical achievement in different age groups has been associated with subcomponents such as spatial visualization and spatial perception (e.g., Geer et al., 2019; Markey, 2010; McCrink & Opfer, 2014; Mix & Cheng, 2012; Newcombe & Frick, 2010; meta-analysis: Xie et al., 2020). Spatial visualization is the ability to mentally transform and assemble different pieces of objects into more complex patterns (Newcombe & Shipley, 2015), whereas spatial perception is the ability to perceive and understand complex visual configurations and the relationships among objects (Uttal et al., 2013).

The mechanisms that explain the relationship between visual-spatial skills and mathematical achievement are not yet agreed upon. Visual-spatial skills might help to represent numbers spatially along a mental number line, which in turn plays a crucial role in many mathematical operations and concepts, including ordinality, ordering, fractions and arithmetic (Gunderson et al., 2012; meta-analysis: Schneider et al., 2018). In addition, visual-spatial skills play a crucial role in numerical reasoning through conceptualizing or modeling (novel) mathematical problems (Mix et al., 2016; Uttal & Cohen, 2012). Spatial modeling can be applied to nearly every mathematical task (Hawes & Ansari, 2020) and is carried out by spatial visualization in particular (De Hevia et al., 2008; Mix & Cheng, 2012). A third mechanism can be found in specific mathematical areas in which the conceptual nature of items requires spatial skills such as geometry (McCrink & Opfer, 2014). A fourth mechanism might be due to working memory effects that are found together with spatial skills (Hawes & Ansari, 2020).

Training studies with randomized control trials show mixed results for whether visual-spatial skills generalize to mathematical achievement (Cheng & Mix, 2014; but Hawes et al., 2015). Possible reasons for this heterogenous evidence may be similar to those mentioned above for working memory (Hawes & Ansari, 2020). So far, the training studies that have integrated spatial and mathematical contents in school-based settings, especially in primary schools, seem more successful than other approaches (Hawes et al., 2017; Lowrie et al., 2017; but Cornu et al., 2017).

2.1.3 Emotion Regulation and Mathematical Achievement

Emotion regulation is defined as the ability to monitor, evaluate, and modify emotional responses to achieve a certain goal (Thompson, 1994). From a functional perspective, emotion regulation can be divided into adaptive and maladaptive emotion regulation strategies (Grob & Smolenski, 2009; meta-analysis: Aldao et al., 2010). Adaptive strategies enable children and adolescents to cope with situations that evoke negative emotion in a solution-oriented manner and help to maintain their social relationships and self-esteem (Grob & Smolenski, 2009). In contrast, maladaptive emotion regulation strategies such as avoidance, rumination, and suppression might exacerbate the situation that evoked the negative emotion (Aldao et al., 2010).

Research on mathematics has indicated that many students show negative emotions when confronted with mathematical problems (Dowker et al., 2016) and evidence has shown that an adaptive way of regulating negative emotions is positively associated with different developmental outcomes and academic competencies (Eisenberg et al., 2005). Research has also shown that emotion regulation skills are specifically related to higher mathematical achievement in children (reviews: Garner, 2010; Koole, 2009; Valiente et al., 2012; meta-analysis: Durlak et al., 2011). Two lines of research explore possible mechanisms that might explain this relation. One proposed mechanism is more general in nature and focuses on the child's environment. It has been postulated that students with more adaptive, emotion-regulation skills show more positive teacher-child relationships and peer-dynamics. They also show more pro-active participation in the classroom, which in turn has a positive effect on their mathematical learning (Eisenberg et al., 2005; Graziano et al., 2007). The other possible mechanism is cognitive in nature. According to the cognitive-load theory (Sweller, 1994), children's working memory capacities are limited and are required in mathematical tasks to follow instructions and solve problems. However, studies show that negative emotions tax these cognitive (working memory) resources which causes an affective drop in mathematical performance (overview: Ashcraft & Moore, 2009). At the same time, an adaptive way of coping with negative emotions, such as reappraisal, has been associated with less ruminating and more keeping a cool head (Richards & Gross, 2000). Adaptive emotion regulation would liberate, or at least not tax, cognitive resources that could then be used to solve mathematical problems (Ashcraft & Moore, 2009; Sweller, 1994; review: Ashcraft & Krause, 2007).

Only a small number of training studies have investigated the far-transfer effects of emotion regulation on mathematical achievement in children or adolescents. However, individual training studies have indicated that training emotion regulation may generalize to mathematical achievement (Ramirez & Beilock, 2011; Rozek et al., 2019). Other training studies on emotion regulation have also shown effects on mathematical outcomes in adults (Costa et al., 2019) and children with math anxiety (e.g., Brunyé et al., 2013; Pizzie & Kramer, 2020).

2.1.4 Relations among Executive Functions, Visual-Spatial Skills, Emotion Regulation, and Mathematical Achievement

Relations between domain-general abilities and mathematical achievement may depend on the domains of child development and may be complex with respect to their dynamics when investigated simultaneously. Following is the rationale for simultaneously investigating associations between various domain-general abilities and mathematical achievement across various domains in children's development. The focus is on three different aspects relevant for child development that lead to the author's research questions.

Relations across age. Research has shown that executive functions and visual-spatial skills seem to be strongly related to mathematical achievement in different age groups. However, few studies have investigated whether both abilities and mathematical achievement change or remain constant across development and the results so far have been heterogenous. One line of research has assumed the *fade out hypothesis*, which states that executive functions and visual-spatial skills become less important for mathematical achievement with age (executive functions: Stipek & Valentino, 2015; visual-spatial skills: Mix et al., 2016). The researchers have claimed that executive functions and visual-spatial skills are particularly necessary in the initial and early mathematical learning phase and that this early knowledge becomes increasingly automatic and procedural with age. Another line of research has indicated that both abilities become more important as students get older (executive functions: Geary, 2011b; visual-spatial skills: Mix & Cheng, 2012). It has assumed that more complex and abstract mathematical problems require more executive resources and a greater need to visualize them. Other studies have indicated that the relations between both types of domain-general abilities and mathematical achievement remain stable across age (Cragg et al., 2017; Hawes et al., 2019).

No study to date, however, has used a wide age range to investigate the associations between both domain-general abilities and mathematical achievement across development. Furthermore, executive functions and visual-spatial skills, as domain-general abilities of cognitive functions, have rarely been investigated simultaneously, even though both skills share variance, as retrieving and manipulating visual images require executive resources, especially working memory (Hawes & Ansari, 2020; Verdine et al. 2014). In sum, it is unclear whether each domain-general ability makes a distinct contribution to mathematical achievement when accounting for the respective other domain-general ability and when these associations are particularly strong across age. A better understanding of these associations across child development is not only relevant for theory, but also for practical implications.

Bidirectional patterns. The idea that cognitive abilities represent the basis for emerging scholastic skills (e.g., Cattell, 1987; discussion: Peng & Kievit, 2020) is a conventional view based of decades of empirical evidence. It has also influenced many longitudinal investigations and their research designs. The majority of studies have concluded that *early* executive functions and visual-spatial skills predict *later* mathematical achievement, which has strengthened the concept of unidirectional associations between cognitive domain-general abilities and mathematical achievement (Cameron et al., 2019). However, the *theory of mutualism* states that the relations between domain-general abilities and mathematical skills may actually follow bidirectional patterns and co-develop to mutual benefit across childhood (review: Peng & Kievit, 2020). Investigating the bidirectional relations between domain-general abilities and mathematical achievement, is a rather new field of research into children's academic development. Hence, only a few studies have investigated such bidirectional associations and results have been mixed (e.g., Fuhs et al., 2014; Geer et al., 2019; but Fung et al., 2020). Importantly, no study has investigated the mutuality of these relations in primary school students. There is an urgent need for studies in this age range given that the primary school period is extremely important for emerging mathematical and cognitive skills (Geary, 2000).

Compensating effects of emotion regulation. Only a few studies have investigated emotion regulation and its associations with children's *and* adolescents' mathematical achievement (e.g., Oberle et al., 2014). Research has shown that children with high social-emotional skills, including adaptive emotion regulation skills, can compensate for their low fluid intelligence in terms of their mathematical achievement (Gut, et al., 2012). There is reason to think that such a compensation effect might also be found for other cognitive variables such as working memory. Children's working memory may be easily overtaxed by ruminating thoughts when confronted with negative emotions. Adaptive emotion regulation skills may free up these cognitive resources and make them available for solving mathematical tasks (Richards & Gross, 2000).

These assumptions are in accordance with the *cognitive load theory* (Ashcraft, 2001; Paas et al., 2004) and reflect a mechanism rather cognitive in nature. It is important to shed light on the reasons *why* emotion regulation is related to mathematical achievement to increase our understanding of the possibility that strengths in affective domain-general abilities can possibly compensate for deficits in cognitive domain-general abilities. This is relevant for theory, interventions in the field of school psychology, and the training of mathematical skills. A step in this direction would be to investigate whether emotion regulation can compensate for cognitive deficits in children's and adolescents' mathematical achievement, and whether these effects are specific to working memory or more general in nature, as compensation effects may exist for all executive function components.

Summary. Most of the studies that have investigated the associations of executive functions, visual-spatial skills, and emotion regulation with mathematical achievement, have considered these abilities separately and investigated only short periods of child development focused on restricted age ranges. The present dissertation increases our understanding of (a) *when* domain-general abilities might be related to children's mathematical achievement during their school lives, (b) *whether and how* domain-general abilities might be mutually related across primary school ages, and (c) whether deficits in one ability can be *compensated for* by strengths in another. This is important to generate relevant knowledge for research, theory, and intervention in developmental and school psychology. In addition, not all the studies in this field considered other essential variables related to mathematical achievement, such as sex, SES, and especially verbal reasoning. This is surprising, as research has indicated their impact on mathematical achievement (e.g., Geary, 2006; Hoff, 2013; Lindberg et al., 2010; for meta-analyses, Kuncel et al., 2004; Peng et al., 2019). By investigating the roles of executive functions, visual-spatial skills, and emotion regulation in mathematical achievement, the author addresses the following research questions; individual studies account for sex, SES, and verbal reasoning.

3. Research Questions

Study 1. *Do associations between executive functions, visual-spatial skills, and mathematical achievement change or remain constant across age?* (Kahl Grob, Segerer, & Möhring, 2019)

Study 2. *Are associations among executive functions, visual-spatial skills, and children's mathematical achievement unidirectional or bidirectional across time?* (Kahl, Segerer, Grob, & Möhring, 2020)

Study 3. *Does emotion regulation compensate for deficits in working memory and other executive functions in children's and adolescents' mathematical achievement?* (Kahl, Grob, & Möhring, 2020)

4. Methods

4.1 Overview of the Samples and Procedures

Participants of all studies were drawn from the standardization sample of the Intelligence and Development Scales-2 (IDS-2, Grob & Hagmann-von Arx, 2018). The IDS-2 is a broad test battery that enables the assessment of cognitive functions (intelligence and executive functions) and developmental functions (psychomotor skills, socioemotional competence, scholastic skills, and attitude toward work) in 5- to 20-year-olds. The standardization sample (N = 2030) consisted of German-speaking participants from Switzerland, Germany, Austria, and Liechtenstein. Specific subsamples of this standardization sample were used for the studies in the present dissertation, as specified below.

Study 1. The final sample for study 1 consisted of 1,754 participants (52.5% female) aged between 5 and 20 years (M = 12.33 years, SD = 4.44), after excluding participants with missing data in the variables executive functions, visual-spatial skills, mathematical achievement, and the control variables.

Study 2. This longitudinal study was funded by the Suzanne and Hans Biaesch Foundation for the Enhancement of Applied Psychology via an award to the author of this dissertation. The study targeted 6- to 7-year-old children who had participated in the IDS-2 standardization study (T1) and who lived in Switzerland at the time. In total, 182 participants of the original sample met these criteria, of which 78 participants (48.7% female) were re-assessed three years later (T2). Selected IDS-2 subtests to measure executive functions, visual-spatial skills, and mathematical achievement were used, as well as another standardized test on mathematics. Importantly, the time between T1 and T2 was an average of three years but varied for individual participants (M = 3.09, SD = 0.49, $diff_{min}$ = 2.35, $diff_{max}$ = 4.28). This was caused by different lengths of data collection periods: the extensive data collection for the IDS-2 standardization study was carried out over two years; the re-assessments in the current study were carried out over eight months.

Study 3. Children and adolescents were included in this study if they were 7 to 15 years of age and if they attended 2^{nd} to 9^{th} grade (N = 1077), the participants' mandatory school time. Students in the first grade were not included as children's emotional responses to their transition from kindergarten to school can vary widely. This could influence the association between emotion regulation and mathematics for this age group (Morrison et al., 2010). Participants missing data in the variables executive functions, emotion regulation, mathematical achievement, or control (*n* = 85) were excluded. This resulted in a final sample of 992 participants (M_{age} =11.29 years; *SD* = 2.37; 49.3% female).

4.2 Overview of the Measures

The main variables were executive functions, visual-spatial skills, emotion regulation, and mathematical achievement, whereas the control variables were sex, SES, and verbal reasoning. All variables were assessed using standardized tests, and almost every variable was measured with the IDS-2 test battery (Grob & Hagmann-von Arx, 2018). An exception was study 2, in which the variable *mathematical achievement* was measured using the IDS-2 as well as an additional standardized test (The German Test of Mathematical Abilities; DEMAT 2+: Krajewski et al., 2004; DEMAT 3+: Roick et al., 2004; DEMAT 4: Gölitz et al., 2006; DEMAT 5+: Götz et al., 2013) at T2 to use two approaches to operationalize mathematical achievement. Typically, to measure variables with large test batteries (e.g., Wechsler Intelligence Scale for Children, WISC-IV, Wechsler, 2003), participants begin most subtests at different item difficulties, depending on their age. Furthermore, the end of the subtests usually depends on participant performance. All variables under investigation consisted of various subtests or items. Both tests to measure mathematical achievement were based on the school curriculum and covered various topics the students were confronted with at school. Executive functions were operationalized by three subtests, each of which measured one of the core components: working memory, inhibition, and cognitive flexibility. Visual-spatial skills were operationalized by two subtests that measured spatial visualization and spatial perception. To assess emotion regulation strategies, participants were exposed to a realistic, age-appropriate scenario in which they had to explain how they would cope with negative emotions. Participants' reported strategies were classified as adaptive or maladaptive. The control variable, verbal reasoning, was measured by two subtests, whereas SES was operationalized by maternal education level. The reliability of all measures used in the studies was satisfying, good or even high (Cronbach's αs: .70 - .97).

4.3 Overview of the Data Analyses

Statistical analyses were performed by controlling for the variables sex, SES, and verbal reasoning. The cross-sectional data from studies 1 and 3 were analyzed using IBM SPSS Statistics 23.0. Data for study 2 was analyzed using Mplus 7.0 (Muthén & Muthén, 2012).

Study 1. A hierarchical linear regression analysis was conducted to investigate the association between executive functions, visual-spatial skills, and mathematical achievement. All the respective indicators for the main variables, executive functions, visual-spatial skills, and verbal reasoning, were aggregated to give a single value for each construct, whereas the sum of correct answers (raw score) was used to operationalize mathematical achievement. Quadratic main effects (e.g., visual-spatial skills* visual-spatial skills) and interaction effects between executive functions and age, and visual-spatial skill and age, were also added to the analyses (Darlington & Hayes, 2016). Significant interactions were analyzed and visualized using simple slopes analyses (cf. Aiken et al., 1991).

Study 2. To investigate the bidirectional and longitudinal relations among executive functions, visual-spatial skills, and mathematical achievement, a cross-lagged panel model (CLPM) was computed with within-construct variance controlled by autoregressive paths. A manifest approach defined the CLPM as a differentiated model by using the manifest variables of each subtest of the respective domain-general abilities (i.e., executive functions and visual-spatial skills). A full information maximum likelihood (FIML) approach was used to handle missing values so that the available information could be used for the analysis. All variables were age-standardized to address the time differences between assessment time points. The time difference and the respective interaction terms with all main variables were entered into the model to investigate whether the effects were dependent on the time difference.

Study 3. Linear regression analyses were computed to investigate whether there was an independent association between the core components of executive functions and emotion regulation, and mathematical achievement. All variables were age-standardized so that the achievements of children and adolescents could be compared. Furthermore, the moderation effects between each core component of executive functions and emotion regulation were also examined (e.g., working memory*emotion regulation) with mathematical achievement as a dependent variable and accounting for the respective other components of executive functions (e.g., inhibition and cognitive flexibility). The moderation analyses were conducted with PROCESS, a modeling tool for SPSS (Hayes, 2018). The nature of the significant interactions was analyzed and visualized using simple slopes analyses (cf. Aiken et al., 1991).

5. Synopsis of Results

Study 1. The key results were that executive functions and visual-spatial skills both related distinctly to mathematical achievement. The relation was linear for executive functions and did not vary with age, while the relation for visual-spatial skills was curvilinear and age-dependent, steeper for those with lower visual-spatial skills and stronger in adolescents than in children. These effects held after controlling for SES, sex, and verbal reasoning. The whole model, including the control variables (sex, SES, and verbal reasoning) and age, explained 86% of the variance in mathematical achievement.

Study 2. One subcomponent of visual-spatial skills, spatial visualization, predicted mathematical achievement three years later across primary school. However, earlier mathematical achievement did not predict children’s later visual-spatial skills, which indicates a unidirectional relation between spatial visualization and mathematical achievement. Furthermore, an interaction effect on mathematical achievement was found between working memory and the time difference between T1 and T2. This effect indicated that working memory predicted mathematical achievement only if the time length between assessments was large and children’s working memory scores were high. Regarding reversed effects, it was found that earlier mathematical achievement predicted later working memory. Therefore, the results indicate partial bidirectional effects between working memory and mathematical achievement in primary school students. The effects held, even after accounting for sex, SES, verbal reasoning, and importantly, also for children’s previous achievement in mathematics, executive functions, and visual-spatial skills at T1.

Study 3. The results reveal that all components of executive functions and emotion regulation were significantly associated with children’s mathematical achievement across their mandatory school time. The higher participants scored on emotion regulation, working memory, inhibition, or cognitive flexibility, the higher they scored on mathematical achievement. In the context of emotion regulation, the children and adolescents who described more adaptive regulation strategies scored higher in the mathematical achievement test. This relation holds after accounting for each core component of executive functions, sex, SES, and verbal reasoning. Furthermore, the association between working memory and mathematical achievement was moderated by emotion regulation. Importantly, this significant interaction effect was not found between inhibition and emotion regulation or cognitive flexibility and emotion regulation, which indicates that it is specific to working memory. Each of these models (including control variables) was able to explain approximately 40% of the variance in children's and adolescents' mathematical achievement.

6. General Discussion

The author has investigated the relations among children's executive functions, visual-spatial skills, emotion regulation and mathematical achievement in child development, focusing on relations across a wide age span, mutual relations between abilities, and compensatory effects of abilities. The results show that executive functions, visual-spatial skills, and emotion regulation are separable constructs that contribute independently to mathematical achievement across a wide age span in child development. The effect appears to be constant for executive functions and age dependent for visual-spatial skills. Furthermore, the results indicate partially bidirectional relations between working memory and mathematical achievement in primary school students. Finally, children and adolescents with high emotion regulation skills can compensate for working memory deficits in their mathematical achievement.

The discussion starts by describing new theoretical insights, embedding the results for each study into the current state of research in the field. This is followed by identifying the practical implications for developmental psychology and school psychology. General strengths and limitations of the research follow, along with future directions for research in the field. The discussion closes with an overall conclusion.

6.1 New Theoretical Insights

Study 1. The existing research had shown mixed results for the question of whether associations among executive functions, visual-spatial skills, and mathematical achievement change or if they remain constant across development and could only provide information at specific time points in children's development. The present study clarified this point by showing that these associations appear to be asymmetrical across age, more specifically, across a student's whole school career. Regarding executive functions, the linear relation indicates that the higher the participants scored on executive functions, the higher they scored on mathematical achievement. This effect remained for children *and* adolescents. This result supports neither the *fade out hypothesis* nor the hypothesis that the association between executive functions and mathematical achievement increases as students get older. The results are in accordance with those of Cragg and colleagues (2017) that indicate executive functions, as overall cognitive control processes, seem to be a solid and continuous basis for solving mathematical tasks across age. In contrast, the results for visual-spatial skills indicate that they play an especially important role in adolescence. This might be due to the rising complexity of mathematics in older age groups, in line with the hypothesis that visual-spatial skills might be helpful for visualizing more abstract mathematical problems (Hawes & Ansari, 2020). However, this remains rather speculative as the mechanisms are not completely clear (Hawes & Ansari, 2020). Furthermore, and in contrast to executive functions, the association between visual-spatial skills and mathematical achievement did not follow a linear pattern. The curvilinear effect of visual-spatial skills on mathematical achievement showed that the relation with visual-spatial skills was strong for participants with a low level of visual-spatial skills. When a certain high level of visual-spatial skills had been reached, the relation with mathematics became very small.

Study 2. In contrast to the first study, study 2 distinguished between different core components of executive functions and visual-spatial skills. This might be important, as research has shown that different components of these domain-general abilities seem to follow different mechanisms in their relation to mathematics and can, therefore, relate differently to mathematical achievement. The results of this longitudinal study highlight the roles of working memory and spatial visualization in primary school students. The unidirectional association between spatial visualization and mathematical achievement is in accordance with the findings of Fung and colleagues (2020) and might indicate its importance in mathematical learning. The (partially) bidirectional relation between working memory and mathematical achievement are in line with previous findings. Bidirectional associations between executive functions, including working memory, and mathematical achievement have been found in these studies with kindergarten children (e.g., Fuhs et al., 2014; Miller-Cotto & Byrnes, 2019). However, these results qualify the former findings, as they indicate that children with high working memory abilities benefit from them in the long term across primary school. Conversely, results indicate that mathematical achievement might be an important factor for the development of working memory skills during primary school. Importantly, this study was the first to investigate these relations in primary school students—when students are exposed continuously to formal mathematics for the first time. This early educational phase is crucial as it provides the basis for later emerging scholastic abilities (Duncan et al., 2007).

Study 3. For the first time, the relation between emotion regulation and mathematical achievement has been investigated in a large sample of children *and* adolescents, in contrast to previous studies which often focused on young children only. Combining cognitive and affective domain-general abilities qualifies previous research, as the current study shows a possible compensation effect of emotion regulation on mathematical achievement. The results indicate that children and adolescents with low working memory capacities benefited the most from having higher emotion regulation skills that allowed them to attain a higher level of mathematical achievement. As this effect was working memory specific, the results may shed light on possible mechanisms to explain *why* emotion regulation is related to mathematical achievement. Adaptive emotion regulation might keep working memory demands at a low level so that resources can be applied to solving mathematical problems, which supports the cognitive load theory in a mathematical context (Paas et al., 2004; Sweller, 1994). This is in accordance with the results of a recent training study that showed adaptive emotion regulation led to less rumination and, therefore, to higher mathematical achievement (Costa et al., 2019).

These new insights can inform and qualify future theoretical models. The theoretical framework by Geary (2004) can be seen as a rather static and hierarchical model in which cognitive domain-general abilities are related to mathematical achievement. The present findings suggest that the relations between domain-general abilities and mathematical achievement can be complex, dynamic, and can vary in different domains of child development. Furthermore, future frameworks should include non-cognitive abilities, such as emotion regulation, that are related to mathematical achievement and can be interconnected with other cognitive domain-general abilities.

6.2 Practical Implications

The results have practical implications for school psychology and mathematics curriculum, as they reveal information that is important for intervention and training (studies) on mathematical skills. The potential for intervention rests on the empirical evidence that all three domain-general abilities are malleable and can be trained across age (executive functions meta-analysis: Diamond, 2012; visual-spatial skills meta-analysis: Uttal et al., 2013; emotion regulation: e.g., Durlak et al., 2011). A general and overall insight based on the present data is that associations with mathematical achievement emerge early in childhood and these relations increase or at least remain constant across development or mandatory school time. This implies the need for early interventions to prevent possible gaps or scissor effects across development or school life. As studies 1 and 2 used large, representative samples with wide age ranges, this might increase the generality of this recommendation. However, training studies investigating their far-transfer effects to mathematical achievement have reported inconsistent results for each domain-general ability. One possible reason is the heterogeneity of participant characteristics, such as age. The developmental perspective presented here, therefore, leads to suggestions for future training (studies). These recommendations for each ability, with cross-links to the other domain-general abilities when indicated, follow, along with practical implications for children with difficulties in mathematics.

Executive functions. In the context of executive functions and mathematical achievement, the results highlight the prominent role of one specific core component in mathematical achievement, working memory. This finding is in accordance with earlier research (overview: Raghubar et al., 2010) and is highly relevant, as deficits in working memory are associated with mathematical learning disabilities (Geary et al., 2007; metanalysis: David, 2012). Study 2 shows that working memory is predictive for mathematical achievement, but only when working memory skills are high at T1 and time between assessments is long. These empirical findings are in line with Diamond and Ling (2016) in a review on the training of executive functions and their transfer effects where they make suggestions for future training studies. They suggest long-term studies to investigate the far-transfer effects of executive functions on mathematical achievement, since there might be cumulative effects that require more time to generalize to mathematical skills. Surprisingly, there are only few studies on the long-term effects covering more than one year (Diamond & Ling, 2016), even though executive functions are beneficial to children’s learning processes and represent a cognitive basis for the acquisition of new knowledge. Furthermore, the authors suggest a high intensive training of executive functions so that generalizing effects may be seen (Diamond & Ling, 2016). The data from study 2 shows, additionally, that mathematical achievement predicts working memory at a later point. Therefore, it could be considered to use possible bidirectional relations and co-developmental processes in interventions by combining the training of mathematical skills and working memory. These combined approaches might be particularly promising based on the results on school-based trainings of visual-spatial skills (Hawes & Ansari, 2020).

Emotion regulation. Diamond and Ling (2016) specifically highlighted the need to consider the importance of abilities, such as self-control, self-efficacy and (positive) emotions, for the effectiveness of interventions on executive functions. Results of the present dissertation research on synergistic relations between cognitive and non-cognitive domain-general abilities and mathematical achievement can provide implications relevant to this point. The results of study 3 show that emotion regulation seems promising as an ability that could be trained in interventions on mathematics. Firstly, it is related with mathematical achievement across mandatory school time, and secondly, it might compensate for deficits in students' working memory in favor of mathematical achievement. Some studies have shown generalizing effects of emotion regulation on mathematical achievement (e.g., Pizzie & Kramer, 2020). However, the number of training studies on emotion regulation to promote mathematical achievement is still very limited. Interestingly, some studies have shown that improving working memory skills also resulted in higher emotion regulation abilities (Schweizer et al., 2013; Xiu et al., 2016). Hence, reciprocal relations are possible. Future training (studies) are, thus, recommended to train working memory and emotion regulation in a combined design to investigate their possible cumulative or reciprocal effects on mathematical achievement. In addition to the practical implication, this would also provide new knowledge on the mechanisms underlying the relation between working memory and emotion regulation.

Visual-spatial skills. The author provides three main practical aspects regarding visual-spatial skills that might be helpful for designing future interventions. First, the results of study 1 showed curvilinear effects between visual-spatial skills and mathematical achievement. As relations were found to be stronger in participants with poor skills, it indicates that those students may profit more from training their visual-spatial skills than students who show more sophisticated visual-spatial skills. For the school curriculum, this could mean that it is important to train children to a certain "basic" level of visual-spatial skills early in their school careers. Integrating a normative measure might be promising, since school-based interventions have shown the highest efficacy in comparison to other approaches (e.g., Hawes et al., 2017). A second consideration might be to integrate executive functions, especially working memory, into the training studies of visual-spatial skills. This is important since, as discussed, a mechanism that might explain the relation between visual-spatial skills and mathematical achievement is working memory effects (Hawes & Ansari, 2020). The results of study 1 showed that effects of visual-spatial skills were distinctly related to mathematical achievement across age. A third aspect might be the important role of spatial visualization in mathematical learning, also discussed in the literature as a possible mechanism to explain the relationship between visual-spatial skills and mathematical achievement (Hawes & Ansari, 2020). Training studies focusing on this component could bear promising results for children's mathematical achievement.

Children with mathematics difficulties. Investigating the selected domain-general abilities might also be informative for children with difficulties in mathematics, such as children with dyscalculia or mathematics anxiety. Considering these groups is relevant for school psychology given that the prevalence of dyscalculia ranges between 3-6 % (Kuhn, 2015) and studies on children with math anxiety have shown an even higher prevalence, above 10% (Ashcraft & Moore, 2009). Research on these specific groups requires knowledge about typical development, especially across age, due to persistence and remission of specific characteristics, such as the relation between working memory and mathematical achievement. According to a model by Kaufmann and von Aster (2012), executive functions, visual-spatial skills, and emotion-related variables correspond to the identified potentially causative factors of dyscalculia on a domain-general level. Investigations into how deficits might be compensated for could be highly relevant since research indicates that some dyscalculic children show persistent deficits in working memory (Kucian & von Aster, 2015) and children with math anxiety are vulnerable to an affective drop in performance (Ashcraft & Moore, 2009). A possible approach for interventions could be to foster children's emotion regulation skills, which could help them improve in mathematics and handle these situations in an adaptive way. There is preliminary evidence that emotion regulation training could help children with math anxiety improve their mathematical achievement (e.g., Pizzie & Kramer, 2020) and that emotion regulation training could help dyscalculic children control high-risk behaviors, such as aggression (Narimani et al., 2013).

6.3 Strengths, Limitations, and Future Directions for Research

Strengths. The current approach to focus on the roles of executive functions, visual-spatial skills, and emotion regulation on mathematical achievement combines cognitive and non-cognitive domain-general abilities under a developmental perspective. It is an advantage that the samples in all the studies covered time points in children's development that are very relevant for practice and theory, and some had not been investigated before. Studies 1 and 3 include large representative samples with wide age ranges that allow conclusions to be derived for a broad spectrum of child development in school-aged children. Whereas these two studies used cross-sectional data, study 2 is built upon longitudinal data. One of the strengths of study 2 is that it investigated bidirectional associations in primary school students, the subjects of most basic mathematical interventions and trainings across children's academic life. Other strong predictors of mathematical achievement such as sex, SES, and verbal reasoning were controlled for in all studies, as research shows that they are related to mathematical achievement (meta-analyses: Kuncel et al., 2004; Peng et al., 2019). Furthermore, it is an advantage that all main variables, executive functions, visual-spatial skills, emotion regulation, and mathematical achievement were measured with the same standardized instrument, the IDS-2, so that every construct under investigation was assessed in a conceptually equivalent manner.

Limitations. Due to the use of either cross-sectional or longitudinal data in the studies included in the present dissertation, it was not possible to reveal any causal relations between the domain-general abilities under investigation and mathematical achievement. Three important domain-general abilities were chosen as the scope of this dissertation. It does not, therefore, reflect the entire complexity of domain-general abilities, including self-efficacy and mathematical self-concept (Parker et al., 2013) and student-teacher relationships (McKinnon & Blair, 2019), and their relations with mathematical achievement. An additional limitation, relevant to practical implications, is that the studies in the current dissertation focus only on typically developing children. This limits the generalizability of results to specific groups such as dyscalculic children and children with math anxiety. It should be noted that the findings described might be different in these groups (Geary, 2011a; Kucian & von Aster, 2015). Another point is that the mathematical achievement tests are based on the school curriculum. Consequently, mathematical achievement was operationalized by one factor and it was only possible to distinguish between two different mathematical areas at T2 of study 2.

Future directions for research. The present research yields new insights into bases for future research. With the aim of augmenting former assumptions on theoretical models and interventions, the results lead compellingly to the need for experimental designs. Hence, possible underlying mechanisms of the relations among the variables might be investigated. It would be important to also integrate other relevant cognitive and non-cognitive domain-general abilities, such as self-efficacy or (mathematical) self-concept, into future research. Furthermore, it would be highly relevant for assessments, interventions, and counselling to investigate how the described relations appear in specific groups that show difficulties with learning or performing mathematics, such as dyscalculic children or children with mathematics anxiety. Another highly relevant aspect to be considered in future research is to address a clear differentiation between different operationalizations of the variables that form part of these studies. For example, a recent meta-analysis by Xie and colleagues (2020) has shown that the relation between visual-spatial skills and mathematics clearly depends on the specific operationalization of both variables. For different studies to be comparable, it is important to work on a taxonomy to make the operationalization of variables clear cut.

There is also growing interest in research linking domain-general abilities and domain-specific abilities (Geary, et al. 2017). However, there is still neither an overarching model that provides an overview of relevant domain-general abilities and their mechanisms towards mathematical achievement across child development, nor a model that combines domain-general and domain-specific abilities within a single, unified model. A developmental perspective on both categories of relevant variables for mathematical achievement could be an overall goal for research and for theory building with relevant consequences for intervention and curricula.

6.4 Conclusion

The present dissertation provides new insights for developmental psychology, school psychology, and research on mathematics by confirming and expanding previous research on domain-general abilities and mathematical achievement. The implications given by the present dissertation lead to three overarching conclusions that can be drawn for all of the above-mentioned disciplines.

Firstly, integrating a developmental perspective into investigating relations between domain-general abilities and mathematical achievement is of high relevance. The results of the present research show that the associations may vary depending on age or different sections of child development when variables are investigated simultaneously. These findings represent an important step forward in our understanding of how and when these abilities relate to mathematical achievement. This research approach is not only important for theoretical models, but also for practical reasons (Cragg & Gilmore,

2014) as it may lead to a better understanding of the heterogenous results from (training) studies. In addition to factors such as different (training) approaches, the developmental period in which students are assessed is important as mechanisms between domain-general abilities and mathematics may change across age, and likewise, the needs of students of different ages and contexts change. This dissertation shows that it is indeed important to consider these aspects and represents a plea for integrating a developmental perspective into research in this field in order to assure that conclusions of studies can be derived correctly for age groups of interest.

Secondly, the partially bidirectional relations between working memory and mathematical achievement shed light on possible dynamic and complex relations between domain-general abilities and mathematical achievement in primary school students. These results extend and strengthen a new line of research, that cognitive domain-general abilities and academic achievement, or even other domains of child development, can be reciprocally related to each other (McClelland & Cameron, 2019; Peng & Kievit, 2020). The results support the idea to follow this line of research in developmental psychology and school psychology, as it may contribute to grasping co-developmental processes between cognitive abilities and academic abilities. Even though there is still a lot to learn and much research needed in this area, the consequences for curricula, assessment, and trainings could be large.

Thirdly, the results are also an important step in integrating emotional and cognitive abilities into investigations of children’s mathematical achievement. This approach follows the rationale of an integrative perspective of academic development that emotion and cognition are closely connected (reviews, Blair, 2002; Koole, 2009). This might be promising given that research has shown that regulatory skills in both affective and cognitive domains are highly relevant for later academic achievement (Liew, 2012). This latter point may provide new insights into domain-general theories of learning in the field of mathematical achievement. It might change the initial motivation of developmental psychology from identifying the underlying factors of mathematical achievement to identifying the *beneficial interconnectedness* between the different domains of child development and mathematical achievement. This supports a more resource-oriented approach in future studies by highlighting how the different domains act and interact dynamically, and even beneficially, with each other across development.

7. References

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APPENDIX A: STUDY 1

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Executive Functions and Visual-Spatial Skills Predict Mathematical Achievement –

Asymmetrical Associations Across Age

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Abstract

Children's mathematical achievement depends on their domain-specific abilities and their domain-general skills such as executive functions (EFs) and visual-spatial skills (VSS). Research indicates that these two domain-general skills predict mathematical achievement. However, it is unclear whether these skills are differently associated with mathematical achievement across a large age range. The current cross-sectional study answered this question using a large, representative sample aged 5-20 years (*N* = 1754). EFs, VSS, and mathematical achievement were assessed using the Intelligence and Development Scales–2. Hierarchical regression analyses were computed with EFs and VSS as predictor variables and mathematical achievement as dependent variable. We examined (non-) linear effects and interactions of EFs and VSS with age. Results indicated that EFs and VSS were distinctly associated with mathematical achievement above and beyond effects of age, sex, maternal education, and verbal reasoning. Effects of EFs were linear and age-invariant. Effects of VSS were curvilinear and stronger in adolescents than in children. Our results indicated that EFs and VSS related differently to mathematical proficiency across age, suggesting a varying impact on mathematics across age.

Keywords

executive functions, visual-spatial skills, mathematical achievement

Executive Functions and Visual-Spatial Skills Predict Mathematical Achievement – Asymmetrical Associations Across Age

Introduction

Mathematical achievement is essential for success in modern society, resulting in better job prospects and quality of life (Geary, 2011a; Gross, Hudson, & Price, 2009). Although precursors such as early quantitative competencies emerge long before entering school, mathematical skills are developed considerably in primary and secondary school (Geary, 2000). However, which abilities and factors advance children's and adolescents' mathematical skills? On the one hand, research has indicated that children are predisposed to represent and acquire numerical knowledge (e.g., Dehaene, 1992; Gelman, 1990; Wynn, 1992). Domain-specific abilities such as an understanding of numerosity or ordinality, and the approximate number sense are present early in life and provide the basis for subsequent, more complex mathematical skills (e.g., Geary, 2000). On the other hand, general cognitive processes play a key role for mathematical skills (LeFevre et al., 2010) with particularly executive functions (EFs) and visual-spatial skills (VSS) predicting mathematical achievement (for overviews, see Cragg & Gilmore, 2014; Mix & Cheng, 2012). The present study aims at extending previous research by investigating the *distinct* association of these two domain-general skills with mathematical achievement using a large, representative sample even when accounting for the respective other skill and other control variables. Importantly, we explore whether the relations of these two domain-general skills with mathematics vary with age from preschool to young adulthood.

Executive Functions and Mathematics

Miyake and Friedman (2012) define EFs as "a set of general-purpose control processes that regulate one's thoughts and behaviors" (p. 8). Several studies indicated that this construct consists of highly related but separable components such as working memory, inhibition, and cognitive flexibility (e.g., Diamond, 2013; Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003; Miyake et al., 2000). Working memory enables to hold and manipulate information in mind (Baddeley, 1996). Several mathematical operations require working memory, for example when a person keeps intermediate results in mind while continuing an operation. Research yielded strong evidence for a link between working memory and mathematics in various age groups (e.g., Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; for overviews, see Bull & Lee, 2014; Cragg & Gilmore, 2014). Fewer studies have investigated inhibition and cognitive flexibility. Inhibition involves controlling attention or behavior to suppress a dominant response (Diamond, 2013) and comes into play when incorrect, but more automatic number facts should be suppressed (e.g., inhibiting six when being asked to multiply three by three). Several studies have shown that inhibitory skills predict mathematical achievement (e.g., Bull & Scerif, 2001; Gilmore et al., 2013). Cognitive flexibility is based on working memory and inhibition and is defined as being flexible and being able to change perspectives (Diamond, 2013). In mathematics, this ability is essential when children have to quickly change strategies or switch between different types of tasks. In line with results on other components, it was found that cognitive flexibility predicted mathematical achievement (for a meta-analysis, Yeniad, Malda, Mesman, van IJzendoorn, & Pieper, 2013).

Overall, several components of EFs seem to be strongly related to mathematical achievement at different ages. However, so far, it is unclear whether a) EFs as a unified construct is related to mathematical achievement and b) whether the association of EFs with mathematical achievement differs systematically across age. With respect to the first question, a recent study indicated no relation between mathematical achievement and EFs at the latent variable level (Hawes, Moss, Caswell, Seo, & Ansari, 2018) which seems surprising in light of studies about separate components. With respect to age-related changes in this association, results are heterogenous so far. Using a longitudinal approach with children from first to fifth grade, Geary (2011b) showed that the contribution of EFs to mathematics increased with age. Contrary to this result, another longitudinal study (Stipek & Valentino, 2015) supported the “fade-out” hypothesis, which proposes that EFs become less important with age. This latter study indicated that EFs were vital for younger children’s mathematical achievement but less so for older children. A third possibility was suggested by another recent study providing evidence for an age-invariant relation between EFs and mathematics (Cragg et al., 2017). More concretely, using several cross-sectional samples between 8-25 years, the authors found stable relations between mathematics and in particular working memory across the age groups tested. Overall, these mixed results indicate that the question of how the association between EFs and mathematics changes across childhood, adolescence, and young adulthood is insufficiently answered as of today.

Visual-Spatial Skills and Mathematics

VSS are defined as the ability to generate, retrieve, and transform structured visual images (cf., Casey, Nuttal, & Benbow, 1995). Like EFs, VSS comprise different subcomponents, but researchers are discordant what the components are (cf. Newcombe & Shipley, 2015). Research revealed that different types of VSS relate to mathematical achievement (e.g., Frick, 2018; Lauer & Lourenco, 2016; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014; for reviews, see McCrink & Opfer, 2014; Mix & Cheng, 2012), with links found between mathematics and spatial assembly competencies (Kyttälä, Aunio, Lehto, Van Luit, & Hautamaki, 2003; Markey, 2010), mental rotation (Gunderson, Ramirez, Beilock, & Levine, 2012), and spatial scaling (Möhring, Newcombe, & Frick, 2015).

With respect to developmental changes in the magnitude of these relations, it has often been proposed that VSS becomes increasingly associated with mathematics across development (Mix & Cheng, 2012). In line with this conclusion, some studies have shown that relations between spatial skills and mathematics became indeed stronger with increasing age (e.g., Li & Geary, 2013; Wolfgang, Stannard, & Jones, 2001). However, other studies showed contrary results. For example, Holmes and Adams (2006) found that spatial skills predicted unique variance in third graders’ mathematics performance, but less variance in fifth graders’ performance. Again, other studies indicated stable relations between spatial and mathematical ability in 4- to 11-year-olds (Hawes et al., 2018), 6- to 12-year-olds (Mix et al., 2016), and in 8- to 25-year-olds (Cragg et al., 2017). Overall, analogously to the link between EFs and mathematics, it remains poorly understood whether space-math relations change or remain stable across a student’s career.

The Present Study

The predictive power of EFs and VSS on mathematical achievement has been repeatedly shown in several studies (Cragg & Gilmore, 2014; Mix & Cheng, 2012). However, so far, it remains unanswered whether these skills are distinctly associated with mathematical achievement and whether these associations change across age. Prior studies have revealed mixed results with respect to this latter question. Possible explanations for these mixed results can be found in methodological differences among these studies. Whereas some studies have used a longitudinal approach (e.g., Geary, 2011b), most of these studies used a cross-sectional design (e.g., Cragg et al., 2017; Mix et al., 2016). Furthermore, depending on the study, VSS, EFs, and mathematical achievement have been differently operationalized and different control variables were included in the analyses. Moreover, these studies have often used varying age groups, with sometimes small age ranges that are not necessarily comparable (e.g., Hawes et al., 2018; Verdine et al., 2014; for reviews, see Cragg & Gilmore, 2014; Mix & Cheng, 2012). Importantly, no study to our knowledge has analyzed effects of EFs and VSS concurrently on mathematics across a wide age range. Such an endeavor may be crucial as working on a VSS task, such as mentally manipulating spatial images, requires executive resources. That is, controlling for EFs seems essential as it may clarify the predictive power of VSS *per se* on mathematical performance.

Building on this theoretical background, the aims of the present study were two-fold. First, we investigated whether EFs and VSS are distinctly associated with mathematical achievement. To this end, we used a large, cross-sectional sample aged 5-20 years. Second, we explored whether associations of EFs and VSS with mathematical achievement differ in strength across age. We controlled for sex, socioeconomic status, and verbal reasoning in the statistical analyses given that these variables have been shown to relate to the variables of interest and such relations were beyond the scope of the present study (e.g., mathematical performance: Baird, 2011; Hoff, 2013; Lindberg, Hyde, Petersen, & Linn, 2010; for a meta-analysis: Kuncel, Hezlett, & Ones, 2004; Peng, Wang, Wang, & Lin, 2019; spatial ability: Levine, Foley, Lourenco, Ehrlich, & Ratliff, 2016; Levine, Huttenlocher, Taylor, & Langrock, 1999; Levine, Vasilyeva, Lourenco, Newcombe, & Huttenlocher, 2005).

Method

Participants

Participants (N = 2030, 51.8% female) were drawn from the standardization sample of the Intelligence and Development Scales-2 (IDS-2, Grob & Hagmann-von Arx, 2018). Participants with missing data in the variables EFs, VSS, mathematical achievement, or in the control variables were excluded (n = 236). Furthermore, we excluded participants with z-scores beyond ±2.58 on EFs, VSS, or mathematical achievement as these participants scored beyond 99% of the remaining sample and their scores may bias the statistical procedures (cf. Field, 2013, n = 40). This resulted in a final sample of 1754 participants (52.5% female) aged 5 to 20 years (M = 12.33 years, SD = 4.44). The distribution of the sample across age can be seen in Supplementary Table 1. To see whether excluded participants differed on their SES or verbal reasoning skills, a MANOVA was conducted. Excluded participants showed a lower SES, $F(1, 1965) = 8.11$, $p < .05$, $\eta^2 = .004$, and lower scores on verbal reasoning, $F(1, 1965) = 30.62$, $p < .001$, $\eta^2 = .015$, compared to participants who remained in the study.

Participants were assessed in Switzerland (56.2%), Germany (39.2%), Austria, and Lichtenstein (4.7%). A total of 76.0% were monolingual native German speakers, 15.3% reported being bilingual, and 8.5% reported another language being their dominant language. However, participants' German level was sufficient to comprehend the task instructions and to answer accordingly. The Ethics Committee of [BLINDED FOR REVIEW] approved the current study, which was performed in accordance with the rules laid down in the 1964 Declaration of Helsinki and its later amendments. Participants and families provided consent for participation.

Measures

All variables of the present study were assessed with the same test battery, namely the IDS-2 (Grob & Hagmann-von Arx, 2018). The IDS-2 is an extensive test battery that enables to measure a wide range of developmentally relevant competencies of children, adolescents, and young adults. More concrete, this instrument allows to measure participants' intelligence, executive functions, psychomotor skills, social-emotional competencies, scholastic skills, and attitudes towards work. However, for the present study, only particular subtests of the constructs intelligence, executive functions, and scholastic skills were included (for details, see below). The IDS-2 is a revised version of the Intelligence and Development Scales (IDS, Grob, Meyer, & Hagmann-von Arx, 2009) and is currently validated in a number of studies (for initial evidence on high construct validity, cf. Grieder & Grob, 2019). The previous version of the IDS-2 was shown to be highly reliable and valid (Hagmann-von Arx, Grob, Petermann, & Daseking, 2012; Hagmann-von Arx, Meyer, & Grob, 2008; Hagmann-von Arx, Petermann, & Grob, 2013).

Testing took place either at the laboratory of the respective University or at participants' homes. As is customary in intelligence testing, children started at different points of the test depending on their age and finished the test depending on their performance.

Mathematical achievement

This task measured a broad range of mathematical achievement using 19 sections, with each section consisting of several items. Content was based on the Swiss school curriculum (Erziehungsdirektoren-Konferenz, D-EDK, 2013). Five- to 7-year-olds started with tasks assessing mathematical precursor skills, for example counting, magnitude understanding, or knowledge about invariance (e.g., children were asked to count small wooden cubes in accordance to a prior presented number). Eight- to 10-year-olds started with problems on equations, proportions, or mental addition (e.g., "There are 15 dices in a box. If I would add three – how many dices would be in the box?"). Participants aged 11 years and older started with items measuring their knowledge about advanced proportional reasoning, geometry, algebra, or fractions (e.g., participants were asked to compare the magnitude of fractions and decimal numbers). Based on extensive pilot studies and as is common in other test batteries (e.g. Wechsler Intelligence Scale for Children, WISC-IV, Wechsler, 2003), participants were assumed to solve the items before their starting point correctly. Difficulty increased within every section. The task was stopped when participants answered incorrectly in five subsequent items. The first eight sections were untimed whereas items of the following sections had a time limit of each 90 seconds. The number of correct answers served as dependent variable. This subtest showed a high reliability (Cronbach's α = .97). External validity of this subtest was indicated by significant correlations with student's math grades (*r* = .37, *N* = 478) and their mathematical competencies based on parents' report (*r* = .44, *N* = 726; Grob & Hagmann-von Arx, 2018). Furthermore, it was found that mathematically gifted students performed significantly better in this math test as compared to a control group with average competencies in mathematics (Grob & Hagmann-von Arx, 2018), which suggests a high differential validity of this subtest.

Executive functions

Three tasks measuring participants' inhibition, cognitive flexibility, and updating skills were used. The inhibition task was an adapted Stroop task (Stroop, 1935) in which participants were asked to name the correct color of four different animals (dolphin, chick, frog, ladybug). Similar Stroop tasks with for instance fruits have been used in other recent studies (cf. Neuenschwander, Röthlisberger, Cimeli, & Roebers, 2012; Röthlisberger, Neuenschwander, Cimeli, & Roebers, 2013). The task consisted of three conditions. In the first condition, children were asked to name the correct color of each animal in a set of 36 congruently colored animals as quickly and accurately as possible. In the second condition, participants were asked to name the correct color of each animal in a set of 36 black-white presented animals. In the final condition, they were asked to name the correct color of each animal in a set of 36 incongruently colored animals (see Figure 1a). The times of completion for each condition (t1, t2, and t3) were measured. Following previous work in this area (Archibald & Kerns, 1999; Roethlisberger, Neuenschwander, Michel, & Roebers, 2010) an inhibition score was created using the following formula (t3 – [t1xt2]/[t1+t2]). The formula represents a measure of interference with lower values representing higher inhibition skills.

To assess cognitive flexibility, participants had to name as many examples as possible in a given category within 90 seconds. Such verbal fluency tasks have often been used as a measure of children's cognitive flexibility in previous studies (van der Elst, Hurks, Wassenberg, Meijs, & Jolles, 2011; for a summary, cf. Diamond, 2013). Five- to 9-year-olds were asked to name examples within the categories "animals" and "food". Ten- to 20-year-olds were additionally presented with the categories "words beginning with E" and "words beginning with alternatingly S or L". The number of correct words across target categories served as cognitive flexibility score.

To measure updating, participants had to recall different letters in a backward order as has been typically done in other intelligence tests (e.g., WISC-IV, Wechsler, 2003). The task consisted of 10 trials. Participants started with two letters and one additional letter was added stepwise to increase difficulty. Participants were presented with a maximum of nine letters. The task was stopped when participants produced three subsequent false recalls. The number of correctly recalled series was used as an updating score. The subtests measuring EFs showed satisfying reliabilities (Cronbach's αs: .71 - .75; Schmitt, 1996).

Visual-spatial skills

Two different assembly tasks measuring VSS were used. Such assembly tasks have often been used to operationalize spatial skills (e.g., in Jirout & Newcombe, 2015; Kyttäla et al., 2003; Markey, 2010). In the first task, participants were asked to replicate two-dimensional figures by using triangular or rectangular cutouts (see Figure 1b). The task consisted of 20 items. The figures became more complex by either increasing the number of the required cutouts or by blurring the edges of the required cutouts in the to-be-copied two-dimensional pattern (similar to the block design subtest in the WISC-IV). The task was stopped when participants produced three subsequent false replications. Time to assemble the configuration was limited for each item (ranging from 30-120 seconds). The number of correctly replicated configurations was used as dependent variable.

Similarly, in the second assembly task, participants were asked to reproduce a pattern (see Figure 1c). They were presented with a printed pattern of circles and asked to copy this pattern using thin plates on a separate sheet. Participants started with the first pattern using a time limit of 60 seconds. The task consisted of four trials with increasing complexity. Accuracy was assessed by comparing the produced pattern to a stencil. Participants received points for accurate performance ranging between zero (i.e., no overlap with the target) to two points (i.e., exact overlap) for every circle. The number of points served as dependent variable. These assembly tests showed high reliabilities (Cronbach's αs: .92 - .95)

Verbal reasoning

Verbal reasoning was assessed using two tasks, each consisting of 34 items. In one task, participants were asked to name the opposite of words (e.g., poor). In the other task, participants were presented with a set of three pictures of one category (i.e., showing different cars) and asked to name the category. Difficulty increased and the task was stopped when participants gave three subsequent false answers. Neither task used a time limit. In both tasks, the number of correct answers served as dependent variables. Reliabilities for both tasks were high (Cronbach's αs: .91 - .94).

Socioeconomic status

SES was estimated based on the mother's educational level as assessed in parent questionnaires (cf. Hoff, 2013). These educational levels ranged from obligatory school education to university degrees (on a five-point Likert scale).

Data analytic procedure

Cross-sectional data were analyzed using IBM SPSS Statistics 23.0. To investigate whether EFs and VSS independently predicted mathematical achievement after accounting for age, sex, SES, and verbal reasoning, a hierarchical linear regression analysis was conducted. To examine whether associations of EFs and VSS with mathematical achievement changed across the different age groups, interaction terms involving age (e.g., age*VSS) were entered into the model. As relations of mathematical achievement with age, EFs, or VSS may follow non-linear patterns (cf. age: Geary, 2000; EFs: Stipek & Valentino, 2015; VSS: Li & Geary, 2013), quadratic terms of age, EFs, and VSS were also entered into the model. By including these terms, we were able to distinguish between quadratic main effects (e.g., VSS*VSS) and interaction effects between the variables (Darlington & Hayes, 2016). Significant interactions were analyzed and visualized by simple slopes analyses (cf. Aiken, West, & Reno, 1991).

Results

Preliminary analyses

As the constructs under investigation (EFs, VSS, and verbal reasoning) were operationalized with several tasks, we aggregated these indicators to obtain a single value for each construct (see Supplementary Table 2 for descriptive statistics of each single task and Supplementary Table 3 for bivariate and partial correlations between the separate tasks). To obtain single scores for these constructs, we estimated principle-axis regression factor scores, each based on a one-factor solution (Di Stefano, Zhu, & Mîndrila, 2009). These extracted factor scores are z-standardized and comprise the shared variance of the tasks. Thus, factor scores are considered to be a measure of error-free estimates of the latent abilities of individuals.

It was found that these factor scores accounted for large portions of variance in the indicators of VSS (87.94%), EFs (76.92%), and verbal reasoning (94.35%), thus reflecting large homogeneity across the tasks used. Factor scores and the other variables under investigation were moderately correlated with each other, when controlling for age (see Table 1). Multicollinearity analyses at the factor score level showed acceptable tolerance and VIF statistics (EFs: tolerance = 0.214, VIF = 4.664: VSS: tolerance = 0.221, VIF = 4.533). A concerning level of multicollinearity is reached by a VIF value beyond 10 and a tolerance value below 0.10 (e.g., O'brien, 2007).

Relations of EFs and VSS with mathematical achievement across age

The regression model revealed distinct relations between EFs and VSS with mathematical achievement above and beyond effects of age, sex, SES, and verbal reasoning (see Table 2). That is, the higher participants scored on tasks assessing EFs and VSS, the higher they scored in the mathematical task. The initial model at step 1 included the control variables (age, sex, SES, and verbal reasoning) and explained 79% of the variance in mathematical achievement. Adding EFs and VSS in a second step, increased the explained variance significantly (6%) as did the interaction and quadratic effect terms (1%) in a third step. With respect to the interaction and quadratic effects, it was found that the quadratic terms of VSS were significant and moderated by age. Effects of EFs were neither quadratic nor dependent on age. As the interaction terms EF*EF*age and VSS*VSS*age did not explain any further variance, these terms were excluded from the model.

Follow-up single slope analyses (Aiken et al., 1991) were conducted to reveal the nature of the interaction effect VSS*age and to illustrate this effect graphically. As is common in research investigating interaction effects, the relation between VSS and mathematical achievement is displayed for one standard deviation above (i.e., adolescents) or below the mean age in our sample (i.e., children; see Figure 2a). The full regression models for the estimated effects of VSS on mathematical achievement for children and adolescents are shown in Supplementary Table 4 and 5, respectively. As can be seen in Figure 2a, associations between VSS and mathematical achievement are stronger in adolescents than in children. However, the quadratic effect shows that with increasing scores on VSS tasks, the relation with mathematical achievement becomes smaller. Contrary to VSS, the effect of EFs on mathematical achievement was linear and unconditional on age. As can be seen in Figure 2b, the higher participants scored on tasks assessing EFs, the higher they scored in the mathematical task, which remained similar for children as well as adolescents.

Given that the current test of mathematical achievement was based on the curriculum, it was the case that older children were more likely asked to solve geometry items as compared to younger children. Therefore, it is possible that the above-mentioned effects indicating that VSS and mathematics are more strongly related in adolescents as compared to children reflect adolescents' increased likelihood of solving geometry items. To examine this possibility, we computed an additional analysis. In a first step, two independent raters categorized items of the mathematics task as assessing geometry or not (showing a high inter-rater reliability of *IRR* = 1). Eight out of 64 items were rated to assess geometry knowledge. In a next step, all analyses described above were conducted again with a new mathematics score excluding these eight items assessing geometry knowledge. Results showed identical effects (see Table 3).[1] Even though the beta coefficients of the interaction VSS*age became smaller as compared to the analysis with the full mathematics score ($\beta_{\text{without geometry}}$ = .059 vs. $\beta_{\text{with geometry}}$ = .070), the interaction was still significant. Therefore, it seems that – even after excluding geometry items - the association between VSS and mathematics seems stronger in adolescents as compared to children.

Discussion

The current study investigated relations among EFs, VSS, and mathematical achievement in children, adolescents, and young adults, and assessed whether associations between these variables changed in quality across this age range. Our findings indicate that EFs and VSS predict mathematics independently, which is in line with several studies showing similar predictions in various age groups (for overviews, see Cragg & Gilmore, 2014; Mix & Cheng, 2012). Our results also extend and qualify these previous findings. Using a large, representative sample aged 5-20 years, our findings indicate for the first time that relations between EFs and mathematical achievement are age-invariant whereas effects of VSS on mathematics are age-dependent and more pronounced for adolescents than for children.

Research has repeatedly shown that EFs and VSS are associated with mathematical achievement (for reviews, see Cragg & Gilmore, 2014; Mix & Cheng, 2012). However, the respective studies mainly focused on specific time points in development and have often used small sample sizes. Furthermore, previous studies rarely examined VSS and EFs simultaneously and thus, accounted for

[1] According to the instructions, the task was stopped when participants answered incorrectly in five subsequent items. Consequently, in some subjects, the termination of the test could also have been caused by geometry items. In order to achieve a complete correction of the test results with regard to the contributions of the geometry items, we estimated latent mathematical ability scores based on a two-parametric item-response model with the data of the completed math items only. We excluded all geometry items from the model. In this latent variable approach, the interaction term VSS*age showed a tendency (*p* = .057). The size of the effect, however, appears only slightly reduced as compared to the analyses with geometry items ($\beta_{\text{without geometry}}$ = .057 vs. $\beta_{\text{with geometry}}$ = .070).

the influence of the respective other skill. Moreover, there is some heterogeneity with respect to how authors define constructs such as visual-spatial working memory (VSWM). Whereas some authors interpreted outcomes of this measure as indicating executive functioning (e.g. Cragg et al., 2017; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015), others used VSWM as a marker for spatial skills (for an overview, cf. Mix & Cheng, 2012). A recent study from Hawes and colleagues (2018) indicated that VSWM is better characterized as indicating EFs; however, more research is needed to more clearly differentiate spatial measures from those indicating executive functioning.

Our study allowed to address those previous methodological considerations as our large, cross-sectional sample spanned development from 5 to 20 years. Furthermore, we used reliable subtests and refrained from including VSWM. In the statistical analyses, we entered EFs and VSS simultaneously while accounting for a number of control variables in order to reveal the *relative* contribution of each domain-general skill to mathematical achievement. Our results revealed that EFs and VSS are significantly associated with mathematical achievement, even after other strong predictors such as verbal reasoning were accounted for. Especially age and verbal reasoning correlated highly with mathematical achievement and together with other control variables explained approximately 80% of the variance in our sample's mathematical thinking. This goes in line with results shown by Verdine et al. (2014), in which age, vocabulary knowledge, and SES explained already 61% of the variance in a sample of 44 three- to four-year-olds. By contrast to this latter study, effects of the present study were shown for the entire period of children's school career. Our findings provide valuable insights for students' mathematical development. This information is central to progress in psychological theory, indicating that EFs and VSS need to be considered in theoretical models that define the importance of domain-general skills on mathematical achievement. Furthermore, our results are essential for training and measuring mathematical achievement considering that EFs and VSS are often neglected in intervention programs and diagnostics.

Few studies investigated whether these associations of EFs and VSS on mathematics change across age and results to date are heterogeneous. That is, for EFs (e.g., Cragg et al., 2017; Geary, 2011b; Stipek & Valentino, 2015) and for VSS (e.g., Cragg et al., 2017; Hawes et al., 1018; Mix et al., 2016; Wolfgang et al., 2011), there is no clear evidence whether the magnitudes of relations to mathematics change or remain stable across age. The current findings demonstrate that relations between EFs and mathematics are age-invariant (for similar results, see Cragg et al., 2017). Thus, it seems that EFs are continuously important across childhood, adolescence, and young adulthood. Contrary to that finding, the association of VSS with mathematical achievement was age-dependent with stronger relations of VSS and mathematics for adolescents as compared to children. One reason for this finding could be that adolescents are more likely solving geometry items as compared to younger participants as they advance to higher mathematics. To investigate this explanation, we conducted an additional analysis and excluded geometry items. It was found that even after removing these geometry items from the mathematics score, effects remained similar. The beta coefficients of the VSS*age interaction decreased slightly indicating that geometry items did have an impact on the results. However, the interaction between VSS and age was still significant, highlighting that this interaction does not simply persist because adolescents solve more geometry items. Overall, our results indicate that spatial ability seems particularly crucial when learning and coping with more complex mathematics as opposed to basic number concepts or operations (cf. Mix & Cheng, 2012).

Beyond clarifying the associations of EFs and VSS with mathematics, the current study also investigated whether these associations across age follow a linear or a non-linear pattern. Effects of EFs on mathematical achievement followed a linear pattern across age, implying that higher EFs are constantly related to higher mathematical achievement. On the contrary, effects of VSS on mathematics followed a curvilinear pattern. Our data suggested that the relation between VSS and mathematics did not follow a proportional association with “the more [VSS] the better” for mathematical achievement. Instead, it seemed that this effect reached a ceiling at a certain high level of VSS at which point the association of VSS with mathematics became small to non-existent. These results bear important implications for intervention programs and school curricula. One conclusion with respect to EFs may be that even improving EFs on a high level may make a difference on mathematical achievement. With respect to VSS, results implicate that children and adolescents with poor VSS may profit more from training VSS than participants with high VSS.

In the current study, we consider it an advantage that participants were tested with the same standardized instrument, covering every construct under investigation in a conceptually equivalent manner. In case of the mathematical assessment tool, the task was age-specific and based on the school curriculum. Thus, this measure covers what students are confronted with at school and items measured performance in a variety of mathematical topics and operations. Consequently, our study shows proximity to how mathematical achievement is defined in a school context and thus, bears high ecological validity.

At the same time, our study shows some limitations. The mathematical achievement test was developed to measure mathematical knowledge on topics of a student’s entire school career between kindergarten until high school. Consequently, results do not provide information about the relations between *specific* mathematical topics such as arithmetic, geometry, or word problems and EFs and VSS that hold for the entire sample. Future studies may focus on isolated mathematical skills and their specific associations to EFs and VSS across age (as for example in Cragg et al., 2017). Furthermore, our pattern of relations between EFs, VSS, and math is restricted to one specific type of spatial skills and may differ with other spatial abilities (cf. Mix et al., 2016). In addition, it needs to be noted that our design was cross-sectional, precluding conclusions about the causal effects of EFs and VSS on mathematics. Future studies with longitudinal designs and training studies should investigate potential causal relations.

Using a large, representative sample, the current study provides evidence that EFs and VSS are distinctly related to mathematical achievement. Our study widens our in-depth understanding of domain-general skills and their link to mathematical achievement by clarifying the age-invariant or age-specific association across a wide age range.

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Table 1. Partial Correlations Between Executive Functions, Visual-Spatial Skills, Mathematical Achievement, and the Control Variables Sex, SES, and Verbal Reasoning (n = 1754) After Accounting for Age.

Variable	1	2	3	4	5	6
1 Sex	_					
2 SES	.03	_				
3 Verbal reasoning	.03	.22**	_			
4 EFs	.12*	.17**	.54**	_		
5 VSS	-.13	.16**	.46**	.47**	_	
6 Mathematical achievement	-.10*	.17**	.58**	.59**	.57**	_

Note: Factor scores were used for EFs, VSS, and verbal reasoning. EFs = executive functions; VSS = visual-spatial skills; SES = socioeconomic status; Sex: 0 = male, 1 = female.
**p* < .05, ***p* < .001.

Table 2. Relations Among EFs, VSS, and Mathematical Achievement: Hierarchical Regression Analysis

	Variables	*b*	β	*T*	Sign. *T*	R^2	*F* (*df1, df2)*	Sign. *F*
Step 1						.79	1677	< .001
	Age	.973	.316	14.911	< .001		(4, 1749)	
	Sex	-1.725	-.063	-5.773	< .001			
	SES	.252	.029	2.571	.010			
	Verbal reasoning	8.327	.608	28.740	< .001			
Step 2						.85	1642	< .001
	EFs	4.465	.325	16.236	< .001		(6, 1747)	
	VSS	3.716	.271	13.688	< .001			
Step 3						.86	964	<. 001
	EFs*Age	.143	.043	1.344	.179		(11, 1742)	
	VSS*Age	.255	.070	2.441	.015			
	EFs*EFs	-.033	-.003	-.131	.896			
	VSS*VSS	-0.893	-.066	-3.543	< .001			
	Age*Age	-.122	-.170	-7.100	< .001			

Note: EFs = executive functions; VSS = visual-spatial skills; SES = socioeconomic status
Age = mean-centered age; Sex: 0 = male, 1 = female.

Table 3. Relations Among EFs, VSS, and Mathematical Achievement with Excluded Geometry Items: Hierarchical Regression Analysis

	Variables	*b*	β	*T*	Sign. *T*	R^2	*F* (*df1, df2)*	Sign. *F*
Step 1						.79	1597	< .001
	Age	.873	.312	14.477	< .001		(4, 1749)	
	Sex	-1.335	-.054	-4.831	< .001			
	SES	.191	.024	2.112	.035			
	Verbal reasoning	7.535	.606	28.131	< .001			
Step 2						.85	1639	< .001
	EFs	4.419	.355	17.712	< .001		(6, 1747)	
	VSS	3.488	.281	14.161	< .001			
Step 3						.87	1057	<. 001
	EFs*Age	.080	.026	.857	.391		(11, 1742)	
	VSS*Age	.195	.059	2.144	.032			
	EFs*EFs	-.195	-.018	-.894	.371			
	VSS*VSS	-1.106	-.090	-5.035	< .001			
	Age*Age	-.115	-.177	-7.694	< .001			

Note: EFs = executive functions; VSS = visual-spatial skills; SES = socioeconomic status
Age = mean-centered age; Sex: 0 = male, 1 = female.

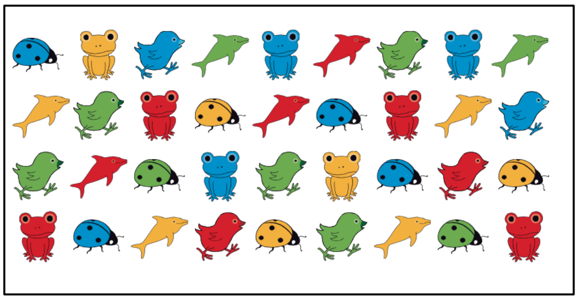
Figure Captions

Figure 1. A selection of three tasks (©Hogrefe AG, Bern: With kind permission of Hogrefe AG) used to measure inhibition (A) and visual-spatial skills (B & C). In the inhibition task (A), participants were asked to name the correct color of each presented animal. In the first assembly task (B), participants were presented with 2-dimensional geometrical figures and asked to copy them using triangular or rectangular cutouts. In the second assembly task (C), participants were presented with a printed pattern of circles and were asked to copy this pattern using thin plates.

Figure 2. Participants' mathematical achievement scores as a function of individual differences of their visual-spatial skills (A) or executive functions (B). Separate curves are shown for participants with one standard deviation above (i.e., adolescents) or below the mean age (i.e., children).

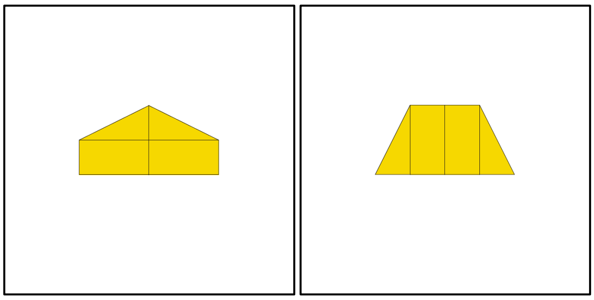
Figure 1.

A)



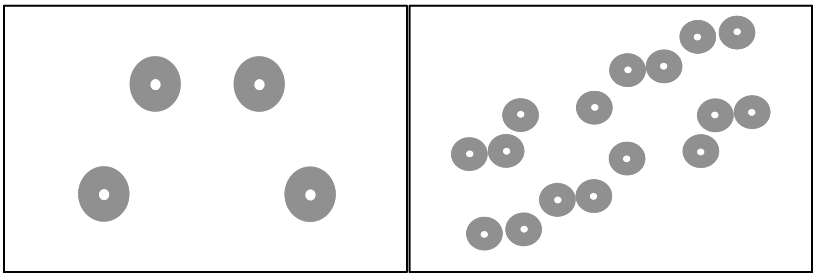
Incongruent condition of the inhibition task.

B)



Two items of the first assembly task.

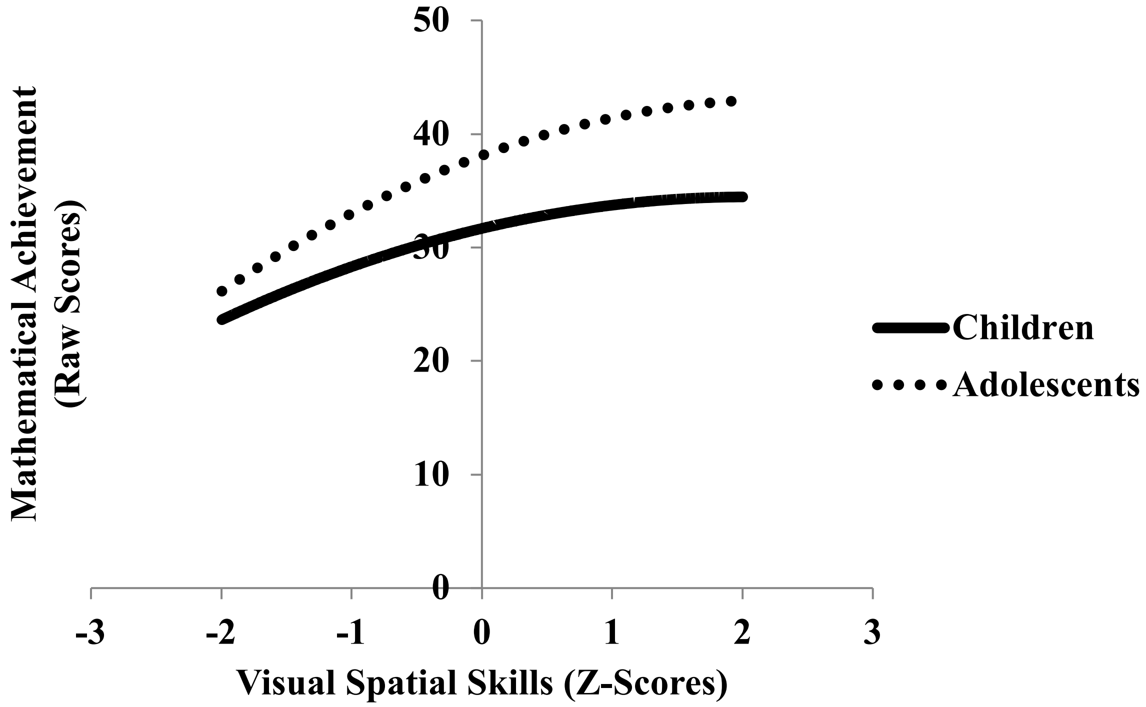
C)



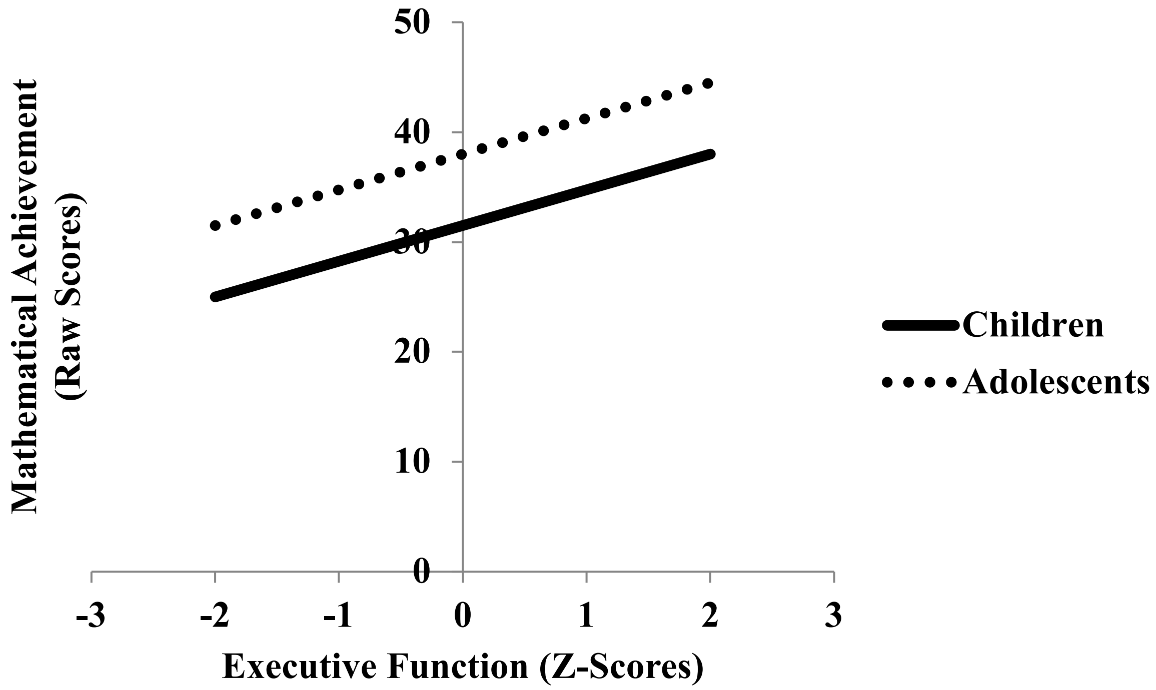
Two items of the second assembly task with different difficulty levels.

Figure 2.

A)


50
40
30
20
10
0
-3
-2
-1
0
1
2
3
Mathematical Achievement (Raw Scores)
Visual Spatial Skills (Z-Scores)
Children
Adolescents

B)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-3
-2
-1
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1
2
3
Mathematical Achievement (Raw Scores)
Executive Function (Z-Scores)
Children
Adolescents

Supporting Information

Supplementary Table 1. Age Distribution of the Sample

Age group	Age range	M_{age}	n_{total}	n_{final}
5-6	5;0-6;11	6;0	280	223
7-8	7;0-8;11	8;0	309	281
9-10	9;0-10;11	10;0	305	268
11-12	11;0-12;11	11;9	270	241
13-14	13;0-14;11	14;0	242	211
15-16	15;0-16;11	15;9	214	198
17-18	17;0-18;11	18;0	163	150
19-20	19;0-20;11	12;9	204	182

Supplementary Table 2. Descriptive Statistics of the Separate Tasks (n = 1754)

Task	*M*	*SD*	Min	Max	Score limit
Executive functions					
Inhibition task	34.32	18.45	8	129	_
Cognitive flexibility task	63.13	28.60	8	148	_
Updating task	4.04	1.70	0	10	10
Visual-spatial skills					
Assembly task (figures)	39.99	14.71	0	80	80
Assembly task (plates)	28.94	14.66	0	76	76
Verbal reasoning					
Find-Opposite task	18.04	5.69	2	34	34
Find-Category task	20.28	7.04	1	34	34
Mathematical achievement	34.66	13.70	1	64	64

Supplementary Table 3. Correlations for the Separate Tasks

	1	2	3	4	5	6	7
1 Inhibition task	_	.69	.59	.67	.64	.65	.67
2 Cognitive flexibility task	.32	_	.68	.75	.75	.82	.83
3 Updating task	.26	.36	_	.65	.61	.70	.70
4 Assembly task (figures)	.33	.36	.33	_	.76	.77	.79
5 Assembly task (plates)	.21	.28	.21	.40	_	.75	.76
6 Find-Opposite task	.20	.44	.39	.39	.25	_	.89
7 Find-Category task	.27	.50	.38	.45	.29	.65	_

Note: Bivariate correlations are shown above the diagonal. Partial correlations controlling for age are shown below the diagonal. Values of the inhibition task were reversely transformed so that higher values indicate higher inhibitory skills.
All correlations are significant at $p < .001$ level.

Supplementary Table 4. Relations Among EFs, VSS, and Mathematical Achievement: Hierarchical Regression Analysis with Estimated Effects for Children

	Variables	*b*	β	*T*	Sign. *T*	R^2	*F* (*df1, df2*)	Sign. *F*
Step 1						.79	1677	< .001
	Age	.973	.316	14.911	< .001		(4, 1749)	
	Sex	-1.725	-.063	-5.773	< .001			
	SES	.252	.029	2.571	.010			
	Verbal reasoning	8.327	.608	28.740	< .001			
Step 2						.85	1642	< .001
	Age	.293	.095	4.716	< .001		(6, 1747)	
	Sex	-1.607	-.059	-6.139	< .001			
	SES	.077	.009	.923	.356			
	Verbal reasoning	3.971	.290	13.204	< .001			
	EFs	4.465	.325	16.236	< .001			
	VSS	3.716	.271	13.688	< .001			
Step 3						.86	964	< .001
	Age	1.810	.587	8.568	< .001		(11, 1742)	
	Sex	-1.481	-.054	-5.804	< .001			
	SES	.191	.022	2.311	.021			
	Verbal reasoning	3.918	.286	13.232	< .001			
	EFs	2.937	.214	5.073	< .001			
	VSS	1.933	.141	3.231	< .001			
	EFs*Age	.143	.057	1.344	.179			
	VSS*Age	.255	.107	2.441	.015			
	EFs*EFs	-.033	-.003	-.131	.896			
	VSS*VSS	-.6893	-.066	-3.543	< .001			
	Age*Age	-.122	-.431	-7.100	< .001			

Note: EFs = executive functions; VSS = visual-spatial skills; Age = mean-centered age – 1 *SD;* Sex: 0 = male, 1 = female; SES = socioeconomic status.

Supplementary Table 5. Relations Among EFs, VSS, and Mathematical Achievement: Hierarchical Regression Analysis with Estimated Effects for Adolescents

	Variables	*b*	β	*T*	Sign. *T*	R^2	*F* *(df1, df2)*	Sign. *F*
Step 1						.79	1677	< .001
	Age	.973	.316	14.911	< .001		(4, 1749)	
	Sex	-1.725	-.063	-5.773	< .001			
	SES	.252	.029	2.571	.010			
	Verbal reasoning	8.327	.608	28.740	< .001			
Step 2						.85	1642	< .001
	Age	.293	.095	4.716	< .001		(6, 1747)	
	Sex	-1.607	-.059	-6.139	< .001			
	SES	.077	.009	.923	.356			
	Verbal reasoning	3.971	.290	13.204	< .001			
	EFs	4.465	.325	16.236	< .001			
	VSS	3.716	.271	13.688	< .001			
Step 3						.86	963	< .001
	Age	-.367	-.119	-3.022	.003		(11, 1742)	
	Sex	-1.481	-.054	-5.804	< .001			
	SES	.191	.022	2.311	.021			
	Verbal reasoning	3.918	.286	13.232	< .001			
	EFs	4.212	.307	8.015	< .001			
	VSS	4.208	.307	8.346	< .001			
	EFs*Age	-.143	.069	1.344	.179			
	VSS*Age	.255	.110	2.441	.015			
	EFs*EFs	-.033	-.003	-.131	.896			
	VSS*VSS	-0.893	-.066	-3.543	< .001			
	Age*Age	-.122	-.348	-7.100	< .001			

Note: EFs = executive functions; VSS = visual-spatial skills; Age = mean-centered age + 1 *SD;* Sex: 0 = male, 1 = female; SES = socioeconomic status.

APPENDIX B: STUDY 2

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Bidirectional Associations Among Executive Functions, Visual-Spatial Skills, and Mathematical Achievement in Primary School Students: Insights from a Longitudinal Study

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Abstract (148)

The present longitudinal study investigated the bidirectional associations among mathematical achievement, visual-spatial skills, and executive functions (EF) consisting of working memory, inhibition, and cognitive flexibility. Six- to 7-year-olds (*N* = 182) were examined at the beginning of primary school and three years later using the Intelligence and Development Scales-2. Using a cross-lagged panel model, our results identified asymmetrical patterns. Whereas early visual-spatial skills predicted children's mathematical achievement, it was found that working memory predicted mathematical achievement depending on the time difference between the assessments. Children with higher initial working memory skills showed accelerated gains in mathematical performance when the time difference between assessments was large. Importantly, early mathematical achievement predicted changes in later working memory, pointing to a bidirectional relation between working memory and mathematical achievement. Our findings widen the understanding of the dynamic and complex interplay among EF, visual-spatial skills, and mathematical achievement in primary school students.

Introduction

Mathematical knowledge is an important pillar for children's academic development and influences later employability, mental health, and life satisfaction even after accounting for reading ability and intelligence (Lyons & Huebner, 2016; Paglin & Rufolo, 1990; Rivera-Batiz, 1992). Due to the important role of numerical abilities and probabilistic thinking in modern societies, there is an increasing awareness in investigating the development of mathematical skills and its predicting variables. Research has indicated that children seem predisposed to numerical processing (Wynn, 1992) and that domain-specific skills such as a basic, numerical understanding (e.g., the number sense, Dehaene, 1992) are linked to mathematical outcomes (Butterworth, 2005). Furthermore, several studies show that general cognitive processes as for example processing speed, working memory or visual-spatial skills, play a key role in the development of mathematical skills (Chu et al., 2016). It was found that especially executive functions (EF) and visual-spatial skills are strongly associated with children's mathematical achievement (for reviews, see Cragg & Gilmore, 2014; Mix & Cheng, 2012; Newcombe et al., 2018). So far, relations among EF, visual-spatial skills, and mathematical understanding were predominantly investigated using cross-sectional samples indicating concurrent relations among these skills (e.g., Blair & Razza, 2007; Cragg et al., 2017; Lee et al., 2012; Monette et al., 2011). However, recent evidence from longitudinal studies shows that EF and visual-spatial skills predict later mathematical performance (e.g., Gunderson et al., 2012; Lee & Bull, 2016).

Whereas these studies have improved our understanding of associations among those variables, the majority of these studies has measured EF and visual-spatial skills as foundational abilities that help developing later mathematical thinking. This idea reflects a conventional view about cognitive abilities being the basis for emerging scholastic skills (e.g., Cattell, 1987; Sternberg et al., 2008; for a discussion, see Peng & Kieviet, 2019). As a consequence, results of these studies suggest an unidirectional association, with *early* EF and visual-spatial skills predicting *later* mathematical performance. However, relations between domain-general abilities and mathematical skills may be more mutually connected as illustrated by the following example: several mathematical operations require working memory when, for example, a person keeps an intermediate result in mind while solving an arithmetical problem. Therefore, while children solve many mathematical problems, they improve their working memory skills at the same time. In turn, an increased working memory capacity facilitates holding mathematical information in mind which is again helpful for solving subsequent arithmetical problems. This example indicates that relations among cognitive and academic variables may be more dynamic and complex than reflected in most of the current research. In line with this conclusion, there is a growing number of studies indicating reciprocal relations between cognitive abilities and academic achievement (van der Maas et al., 2006; for a review, Peng & Kieviet, 2019).

Uni- and bidirectional relations between executive functions and mathematical achievement

EF is a set of regulation and control processes that enables goal-oriented actions and manages one's thoughts and behaviors (Miyake & Friedman, 2012). Typically, EF are distinguished into three components: working memory, inhibition, and cognitive flexibility (e.g. Diamond, 2013). Working memory is a cognitive store with a limited capacity that holds information in mind and enables to manipulate this information (Baddeley, 1996), inhibition enables to control attention or behavior to suppress a dominant response (Diamond, 2013), and cognitive flexibility is the ability to change perspectives (Diamond, 2013). All components have been shown to relate to mathematical achievement in various age groups across childhood and adolescence (e.g., Bull & Lee, 2014; Bull & Scerif, 2001; Cragg & Gilmore, 2014; Gilmore et al., 2013; Yeniad, et al. 2013), with in particular, working memory being strongly related to mathematical achievement (e.g., Geary et al., 2007).

Longitudinal studies have shown that the single components of EF and also a unified EF construct predict mathematical achievement at a later point (Clark et al., 2010; Geary, 2011; Lee & Bull, 2016; Vandenbroucke et al., 2017). However, the vast majority did not investigate the reverse effect, namely whether earlier mathematical achievement may predict later EF. This lack of knowledge is predominantly based on the idea that academic achievement must have a foundation in basic cognitive skills (Cattell, 1987), which is, for example, evident in interventions that focus on EF to improve mathematical achievement (e.g., Dunning et al., 2013; Kubesch, 2016). However, recent research provides increasing evidence that speaks for bidirectional associations between cognitive and mathematical skills (for a review, Peng & Kieviet, 2019). This bidirectionality is explained by the theory of mutualism (e.g., van der Maas et al., 2006), which states that cognitive and academic skills develop in tandem and interact mutually and beneficially with each other across child development. Bidirectional relations can be examined by looking at how these skills change over time, with cross-lagged models as a typical methodological approach. Such an approach requires an assessment of every skill at each measurement time point in a longitudinal examination (Selig & Little, 2012). However, few studies have implemented this methodological consideration to investigate mutuality among executive functions, visual-spatial skills, and mathematical achievement (for exceptions, e.g., Fuhs et al., 2014; Fung et al., 2020; Geer et al., 2019; Miller-Cotto & Byrnes, 2019; Schmitt et al., 2017).

Overall, these latter studies have shown that core components of EF predict later mathematical achievement while at the same time, early mathematical achievement predicts later EF (for an exception, see Fung et al., 2020). In one of these studies, it was found that in particular working memory and mathematical achievement predicted each other from the beginning of kindergarten throughout second grade (Miller-Cotto & Byrnes, 2019). Results of another study indicated bidirectional relations among every core component of EF (working memory, inhibition, and cognitive flexibility) and mathematical achievement as well as a significant relation between children's rate of growth in EF components and their growth in mathematical achievement over the preschool period (Schmitt, et al., 2017). In a sample of 4-year-olds, Fuhs and colleagues (2014) examined bidirectional relations between EF and mathematical achievement across the kindergarten years. Results from this study show that EF and mathematical achievement are in fact mutually associated across kindergarten, even though EF predicted mathematical achievement to a stronger extent than vice versa. However, these results are contrasted by Fung et al. (2020) who investigated the bidirectional association among EF, visual-spatial skills, and mathematical achievement in Chinese kindergarten children in their transition to first grade in primary school. Regarding EF, they neither found EF predicting later mathematical achievement nor vice versa. Summing up these studies, it seems that bidirectional relations between EF and mathematical achievement are likely—at least in children prior to primary school.

Uni- and bidirectional relations between visual-spatial skills and mathematical achievement

Visual-spatial skills are defined as the ability to create, retrieve, and transform visual images (Casey, et al., 1995). Visual-spatial skills comprise different subcomponents, but there is not yet a typology that the scientific community has agreed upon (cf. Linn & Peterson, 1985; Newcombe & Shipley, 2015). Even though there is no consensus on the factorial structure yet, it has been repeatedly shown that several types of visual-spatial skills, such as mental rotation, spatial visualization, or spatial perception (Newcombe & Shipley, 2015; Uttal et al., 2013) relate to mathematical achievement. Whereas mental rotation refers particularly to recognizing rotated objects in 2D or 3D, spatial visualization is often described as the ability to assemble different pieces of objects into a more complex configuration (Newcombe & Shipley, 2015). Spatial perception in turn is seen as the ability to understand the spatial relationships among objects in visual images (Uttal et al., 2013). Each of these types of visual-spatial skills is associated with mathematical achievement in various studies (e.g., Kahl et al. 2019; McCrink & Opfer, 2014; Markey, 2010; Mix & Cheng, 2012; Möhring et al., 2018; Newcombe & Frick, 2010; for a meta-analysis, Xie et al., 2020).

As is the case in studies investigating associations between EF and mathematics, the vast majority of studies on visual-spatial skills and mathematical performance has investigated relations concurrently or in a longitudinal, unidirectional manner (e.g., Frick, 2019; Gunderson et al., 2012; Verdine et al., 2016). To our knowledge, there are only two studies investigating bidirectional associations between visual-spatial skills and mathematical achievement so far. Fung et al. (2020) studied visual-spatial skills in Chinese kindergarten children and found unidirectional patterns between early mental rotation, spatial disembedding and later math, but not vice versa. In the second study, Geer et al. (2019) investigated students' spatial visualization, spatial perception, and mathematical skills during primary school in first- through third-graders. Contrary to findings from Fung et al. (2020), the authors showed bidirectional relations with earlier spatial visualization predicting later math skills as well as early math skills predicting later spatial visualization skills. Importantly, no bidirectional relations were found for spatial perception. Therefore, the few studies investigating bidirectional relations revealed mixed findings so far and more studies are clearly needed.

The present study

As of today, the majority of studies investigated bidirectional relations between domain-general skills and mathematical achievement in kindergarten or preschool children (Fuhs et al., 2014; Fung et al., 2020; Miller-Cotto & Byrnes, 2019; Schmitt et al., 2017). Given that the time period in primary school is highly important for developing (later) mathematical and cognitive skills (Duncan et al., 2007; Geary, 2000), studies in this age range are urgently needed. Increasing our knowledge about the interplay between cognitive skills and mathematical learning during this period is crucial for improving instruction, intervention, and psychological assessments. Moreover, the few studies on bidirectionality have often examined only one of the two domain-general skills with mathematical achievement. Thus, the aims of the present study were twofold: First, we aimed to examine the bidirectional associations among both domain-general skills (EF and visual-spatial skills) with mathematical achievement while accounting for SES, sex, and verbal reasoning. Second, we aimed to investigate these relations in primary-school student and thus, during a period when students are continuously confronted with formal mathematics.

To this end, we used a cross-lagged panel model (CLPM) with two assessment time points. At the first assessment time point (T1), participants took part in the standardization study of a test battery, the Intelligence and Development Scales-2 (*N* = 2030 participants aged 5-20 years, IDS-2, Grob & Hagmann-von Arx, 2018). From this sample, we recruited a sub-sample of 182 primary-school students who were 6-7 years old at T1 and in first grade. These participants were invited to take part at a second assessment time point (T2) using the same measurement instruments. Consequently, EF, visual-spatial skills, and mathematical achievement were tested with the same standardized instrument at both time points. The current study expands on findings of a previous study that was carried out by our research group (Author, Year). In this respective study, we investigated associations among EF, visual-spatial skills, and mathematical achievement in 1754 participants aged 5-20 years, using a cross-sectional design. Participants were similarly drawn from the standardization sample of the IDS-2 (Grob & Hagmann-von Arx, 2018). We were able to show that EF and visual-spatial skills independently explained variance in participants' mathematical achievement. However, the cross-sectional design did not allow to assess longitudinal, bidirectional relations among the variables of interest. Therefore, in the current study, we aimed to extend this earlier study by examining the bidirectional effects among EF, visual-spatial skills, and children's mathematical achievement in primary-school children.

Method

Participants and Study Procedure

Participants of the current study were recruited from a pool of children who took part in the standardization of the IDS-2 test battery (Grob & Hagmann-von Arx, 2018). The full standardization sample included 2030 children, adolescents, and young adults aged 5 to 20 years, examined in Switzerland, Germany, Austria, and Liechtenstein. After an average of three years, a subsample of this standardization sample was invited to participate in the present study. Participants were re-assessed using selected subtests on EF, visual-spatial skills, and mathematical achievement as well as another standardized test on mathematics. This latter test session was considered as the second assessment time point (T2) whereas the assessment in the standardization phase of IDS-2 was considered to be the first assessment time point (T1). This longitudinal study included children living in [BLINDED] who were 6 to 7 years old (6;00 - 7;11) at the time of their first assessment in the IDS-2 standardization study. Of the 182 participants who met these criteria, 78 former participants participated in the second assessment (48.7% female). The assessments were conducted by trained experimenters and took place in individual face-to-face settings.

Drop-out analyses showed that children from families who agreed for another assessment demonstrated significantly higher scores on verbal reasoning, $F(1, 180) = 14.69$, $p < 0.001$, $\eta2 = 0.075$, working memory, $F(1, 180) = 19.76$, $p < 0.001$, $\eta2 = 0.099$, cognitive flexibility, $F(1, 180) = 4.17$, $p < 0.05$, $\eta2 = 0.023$, inhibition ($F(1, 180) = 4.15$, $p < 0.05$, $\eta2 = 0.023$, and mathematical achievement, $F(1, 180) = 197.64$, $p < 0.001$, $\eta2 = 0.11$, than children from families who did not agree for the follow-up assessment.

The time between T1 and T2 was three years on average but differed for individual children ($M = 3.09$, $SD = 0.49$, $diff_{min} = 2.35$, $diff_{max} = 4.28$). Due to the comprehensive and time-consuming individual test sessions, the test period of the original IDS-2 standardization study (T1) extended over a period of two years (2015-2017). The assessments at T2 were carried out over a period of eight months (June 2019 - February 2020).

A total of 74.3% of the subsample were monolingual native German speakers, 17.6% reported being balanced bilingual, and 8.1% reported another language being their dominant language. Care was taken that every participant showed a German language level that allowed understanding and following the task instructions. Assessments took place individually at participants' homes or at the laboratory of the respective University. Parents and children provided written and oral consent for participation. Participation was compensated with a voucher for toys to the value of 25 Swiss franc (approximately 28 US-Dollars). Furthermore, parents received feedback on their children's mathematical performance. The Ethics Committee of [BLINDED FOR REVIEW] approved the current study which was performed in accordance with the rules laid down in the 1964 Declaration of Helsinki and its later amendments.

Measures

Variables of interest were assessed with standardized tests, with almost every variable being measured with the IDS-2 test battery (Grob & Hagmann-von Arx, 2018) at both assessment time points. An exception is the variable *mathematical achievement* which was additionally measured with another standardized test (The German Test of Mathematical Abilities; DEMAT 2+: Krajewski et al., 2004; DEMAT 3+: Roick et al., 2004; DEMAT 4: Gölitz et al., 2006; DEMAT 5+: Götz et al., 2013) at T2 to ensure the generalizability of the findings with respect to the prediction of mathematical achievement (for details, see below). The entire test session took approximately 2.5 hours, including breaks.

Intelligence and Development Scales-2 (IDS-2)

The IDS-2 is a test battery that covers a wide range of core competencies in development and intelligence (e.g., executive functions, psychomotor skills, social-emotional competencies). The age norms on subtest and scale levels are identified on the continuous norming technique (Timmerman et al., 2020). Selected scales of this broad instrument were used for the present study. As is customary in intelligence tests covering a wide age range and levels of proficiency (e.g. Wechsler Intelligence Scale for Children, WISC-IV, Wechsler, 2003), participants started at different items of the scales depending on their age in most subtests. Furthermore, they terminated the test session on sub-test levels depending on their performance. For our analyses, we used age-standardized values to reduce age-specific variance and to account for the different time intervals between the assessment time points in our sample.

Mathematical achievement (IDS-2)

This subtest assessed mathematical achievement based on the Swiss school curriculum (Erziehungsdirektoren-Konferenz, D-EDK, 2013). In total, this task comprised 19 items with every item consisting of different sub-items. Items covered different mathematical topics with difficulty increasing within every item. Younger children (five to seven years) were asked to start with mathematical precursor skills (e.g., counting or knowledge about invariance), followed by items on equations, proportions, or mental addition for children aged eight to ten years. Older participants (11 years and older) started with items on proportional reasoning, geometrics, algebra, or fractions. The task was stopped when participants answered incorrectly in five subsequent items. The first eight items were untimed whereas the following items had a time limit of 90 seconds. The number of correct answers was summed up and served as a mathematical achievement score. The authors reported high reliability scores in different age groups (Cronbach's αs: .88-.92). External validity of the scale was demonstrated by significant correlations with student's math grades (r = .37, N = 478) and with parents' reports on their children's mathematical competencies (r = .44, N = 726; Grob & Hagmann-von Arx, 2018).

Mathematical achievement (DEMAT)

The German Test of Mathematical Abilities (DEMAT) comprised a set of different standardized tests in order to measure children's mathematical skills in various grades. In accordance to the IDS-2, this test is also based on the school curriculum. As participants of our study were in different grades at T2 (2nd to 6th grade), we used the respective versions of the DEMAT to measure their mathematical achievement (DEMAT 2+: Krajewski et al., 2004; DEMAT 3+: Roick et al., 2004; DEMAT 4: Gölitz et al., 2006; DEMAT 5+: Götz et al., 2013). The DEMAT was used in several recent studies and showed high reliability and validity (e.g. Paetsch et al., 2015; Schuepbach, 2015; Winkelmann et al., 2012). Each DEMAT version is designed for different grades and includes three sections with (1) arithmetical problems, (2) geometry, and (3) word problems. By contrast to the IDS-2, children did not finish the test depending on their performance but were asked to complete as many items as possible within a given time limit in each section. Correct answers were summed up and this score was standardized with respect to student's grade level and used as variable for our analyses. Reliabilities of these measures were high for each version of the DEMAT (Cronbach's αs = .82-.91).

Executive functions (IDS-2)

The three components of EF, inhibition, cognitive flexibility, and working memory were measured separately by using three tasks of the IDS-2. Inhibitory skills were assessed by using an adapted Stroop task (Stroop, 1935). Participants were asked to name the correct color of four different animals (dolphin, chick, frog, ladybug). This and similar Stroop tasks with for instance fruits have been used in other recent studies (cf. Author, Year; Neuenschwander et al., 2012; Röthlisberger et al., 2013). In three different conditions, participants were asked to name the correct color of each presented animal as quickly and accurately as possible (for a total of 36 animals). The first condition was a set of congruently colored animals; in the second condition, animals were presented in black and white. In the third condition, the set consisted of incongruently colored animals. The times of completion and accuracy for each condition (t1, t2, and t3) were measured. To create an inhibition score, these times were computed according to the following formula (t3 – [t1xt2]/[t1+t2]).

To measure cognitive flexibility, participants were asked to name as many examples as possible in a given category. Previous studies have used such verbal fluency tasks to assess cognitive flexibility in children (van der Elst et al., 2011; for a summary, cf. Diamond, 2013). Children aged five-to-nine years were asked to name as many examples as possible within the categories "animals" and "food". Ten- to 20-year-olds were additionally presented with the categories "words beginning with E" and “words beginning with alternatingly S or L”. Time was limited to 90 seconds for each trial. The number of correct words within target categories served as cognitive flexibility score.

To measure working memory, participants were asked to recall different numbers or letters in forward and backward order as is customary in other intelligence tests (e.g. WISC-IV, Wechsler, 2003). The task consisted of 40 items. Participants started with an item containing two digits or letters. To increase difficulty, additional digits or letters were added stepwise in each trial. Nine items were the maximum number of digits or letters to be recalled. The task was stopped after three subsequent incorrect recalls. The number of correctly recalled trials served as working memory score.

Scores were age-standardized to reduce age-specific variance and to facilitate the comparability of scores in different age groups. Subtests measuring EF showed satisfying reliabilities (Cronbach’s αs: .70-.75; Schmitt, 1996).

Visual-spatial skills (IDS-2)

Visual-spatial skills were measured using two different tasks, assessing children’s spatial visualization and spatial perception (Linn & Peterson, 1985; Newcombe & Shipley, 2015). Such tasks have often been used to operationalize visual-spatial skills in other studies (e.g. Jirout & Newcombe, 2015; Kahl et al., 2019; Kyttäla et al., 2003; Markey, 2010). The task measuring spatial visualization consisted of 20 items with increasing difficulty. Each item had a time limit, ranging from 30-120 seconds. Children were asked to replicate two-dimensional figures by using triangular or rectangular cutouts (see Figure 1, similar to the block design subtest in the WISC-IV). The task was stopped after three subsequent false replications. The number of correctly replicated configurations was age-standardized and used as a variable for our analyses.

In the task measuring spatial perception, children were presented with a printed pattern of circles and asked to copy this pattern using thin plates on a separate sheet. The task consisted of four trials and every pattern had a time limit of 60 seconds. A similar task is used in a test battery assessing professional competences in children and adolescents (HAMET 2; Dieterich et al., 2001). Difficulty increased with every item. Participant’s performance was operationalized as the accuracy of the produced patterns compared to the template. A score ranged between zero (i.e., no overlap with the template) to two points (i.e., exact overlap). The number of points were summed up. Again, age-standardized values were used for our analyses. Both visual-spatial tasks showed satisfying to high reliabilities for different age groups (Cronbach’s αs: .72-.84)

Verbal reasoning

Two tasks measured verbal reasoning with each subtest consisting of 34 items with increasing difficulty. In one subtest, participants had to name the opposite of a given concept (e.g., rich vs. poor). The second subtest assessed how accurate participants could name the category of a set of three different pictures (e.g., train, plane, bicycle: means of transportation). The task was stopped when participants gave three subsequent false answers. Neither task used a time limit. In both tasks, the number of correct answers were summed-up, and a single age-standardized composite score served as variable for our analyses. Reliabilities for different age groups were high (Cronbach’s αs: .86-.89).

Socioeconomic status

We used the maternal educational level as an index for SES (cf. Hoff, 2013), as indicated on a five-point Likert scale ranging from mandatory school education to university degree.

Data analytic procedure

We calculated a CLPM to assess the bidirectional, longitudinal relationships among EF, visual-spatial skills, and mathematical performance (see Figure 2). In such models, autoregressive paths control for within-construct variance. Using this approach, it is possible to test multiple paths and point out the unique longitudinal effects of several variables. Cross-lagged effects between constructs from prior to subsequent assessment time points indicate predictive effects of changes across time. We used a full information maximum likelihood (FIML) approach to handle the missing values, so that all information could be used for our analysis. FIML is considered as a robust method to deal with missing data in structural equation modeling and therefore also with CLPM (Cham et al., 2017, Enders, 2001).

Since EF and visual-spatial skills were operationalized through various subtests, we aimed at modelling latent variables. However, the bivariate correlations of the respective indicators of EF and visual-spatial skills were only moderately related with each other, with one pair being not related at all (see Table 2). Similarly, when we computed principle-axis regression factor scores (Di Stefano et al., 2009) in order to estimate the latent abilities of participants, it was found that these factor scores accounted for rather unsatisfying portions of the variance (< 50%) in EF and visual-spatial skills. Hence, it was not possible to reliably model latent variables and thus, an aggregated view of EF and visual-spatial skills was not supported by the data. Consequently, we used a manifest approach, defining the CLPM by using the manifest variables of each subtest for a differentiated model (see Figure 2). We accounted for the variables sex, SES, and verbal reasoning at T1. Considering that the time difference between individual assessments varied among participants, we used age-standardized values for all variables in our analyses that were measured at T1 and T2. To examine whether effects depended on this time difference (Tdiff), we entered the time difference variable into our analysis as well as the interaction terms for all main variables in a second step (e.g., working memory*Tdiff). Data was analyzed using Mplus 7.0 (Muthén & Muthén, 2012).

Results

Raw scores of the variables of interest at both assessment time points can be seen in Table 1. As expected, participants showed higher average performance in all main variables at the second time point, indicating a developmental progression in each ability tested. The following analyses of the current study were conducted using age-standardized values of the variables. Table 2 depicts bivariate correlations that include correlations among variables within each time point but also between the assessment time points (cross-lagged correlations). Within and between each time point, it was found that mathematical achievement was significantly related to the variables working memory and to spatial visualization with *r*s ranging from .28 - .53. The variables inhibition, cognitive flexibility, and spatial perception showed significant but rather small correlations with mathematical achievement at T1 (all *r*s < .26) and no significant correlations with mathematical achievement at T2.

The differentiated CLPM can be seen in Figure 2. The model is saturated given that there were as many estimated parameters as observed statistics (variances and covariances) in the data set. All variables correlated to each other in this model. We accounted for sex, SES, and verbal reasoning by regressing predictor variables at T1 and criteria variables at T2 on these variables. Autoregressive effects of variables between the two assessment time points indicate the degree of stability of the constructs over time (Selig & Little, 2012). Our CLPM indicated stability across the three-year period for the following variables: mathematical achievement measured with the IDS-2: $\beta = .496$, $p <.001$, mathematical achievement measured with the DEMAT: $\beta = .396$, $p <.001$, working memory: $\beta = .480$, $p < .001$, cognitive flexibility: $\beta = .359$, $p <.001$, and spatial visualization: $\beta = .513$, $p <.001$. Inhibition ($\beta = .147$, $p = .22$) and spatial perception ($\beta = -.041$, $p = .75$) did not indicate stability between the two assessments, even though spatial perception and spatial visualization were strongly related across these time points.

With respect to cross-sectional relations, the analyses showed that changes in mathematical achievement (DEMAT) were predicted by spatial visualization, $\beta = .247$, $p = .03$, and by children's earlier mathematical achievement (IDS-2), $\beta = .396$, $p = .002$. Therefore, it seems that early spatial visualization predicts later mathematical performance, even after accounting for earlier mathematical ability, EF, sex, SES, and verbal reasoning. However, given that the mathematical achievement score of the DEMAT consists of some geometry items, we conducted another analysis in which a geometry-adjusted value (i.e., the section "word problems" of the DEMAT) served as indicator of mathematical performance. Results showed an identical predictive effect by spatial visualization with an even stronger beta coefficient, $\beta = .275$, $p = .02$. No other indicator of EF or the other visual-spatial skill (spatial perception) was identified as directly predicting children's later mathematical achievement, although a trend could be observed for working memory ($\beta = .175$, $p = .09$).

With respect to investigating the effects of the time difference between T1 and T2 (Tdiff), results showed that the time difference did not predict later mathematical achievement ($\beta = -.041$, $p = .67$). Therefore, time intervals between assessments were not associated with participant's mathematical achievement. However, it was found that the time difference moderated the relation between working memory at T1 and changes in later mathematical achievement. This was indicated by a significant interaction term Working Memory*Tdiff ($\beta = .285$, $p < .001$), whereas no other interaction term reached significance. Looking at the simple slope analyses (see Figure 3), it was found that the more the assessments were apart, the more children's mathematical achievement changed in those children scoring high on working memory at T1 (slope: $\beta = .37$, $p < .001$). When assessments were closer to each other, Figure 3 may imply that children's mathematical achievement changed more in children scoring low on working memory at T1; however, this effect was non-significant even though it can be seen as a tendency (slope: $\beta = -.23$, $p = .064$).

With respect to reversed directions, it was found that children's earlier mathematical abilities as measured with the IDS-2 predicted changes in working memory at T2, $\beta = .306$, $p = .017$. Therefore, it seems that children's early mathematical ability is predictive of their later working memory capacity, even after accounting for children's earlier working memory, sex, SES, and verbal reasoning. By contrast, children's earlier mathematical abilities were un-related to their visual-spatial skills at T2.

Finally, changes in cognitive flexibility at T2 were predicted by performance on spatial perception at T1, $\beta = .201$, $p = .026$.

Discussion

In the current study, we investigated bidirectional associations among EF (working memory, inhibition, and cognitive flexibility), visual-spatial skills, and mathematical achievement in primary school children. We used a longitudinal design using a sample of 182 children, tested at the age of 6-7 years and three years later. Our results indicate that early spatial visualization predict changes in mathematical achievement three years later and that early working memory predicts changes in mathematical achievement, depending on the time difference between the assessment time points. Furthermore, early mathematical achievement predicted later changes in working memory.

Research has shown that early EF and visual-spatial skills predict mathematical achievement at a later time point (e.g., EF: Clark et al., 2010; visual-spatial skills: Verdine et al., 2016). Investigating such unidirectional associations reflected the predominant line of research so far. Only a few studies investigated bidirectional relations between cognitive skills and academic performance; however, most of these studies investigated EF and visual-spatial skills separately and mainly in preschool or kindergarten children. Results of these studies indicate bidirectional patterns between EF, visual-spatial skills, and math in children prior to primary school (e.g., Fuhs et al., 2014; Geer et al., 2019). One exception to these findings was revealed by a recent study from Fung et al. (2020) which did not confirm these bidirectional patterns. One reason for these contrasting findings may refer to Fung et al. investigating both constructs (EF and visual-spatial skills) simultaneously which is a similar approach as used in the present study. Our study extended this former research, using a cross-lagged design with the goal to investigate bidirectional relations among EF, visual-spatial skills, and mathematical achievement in *primary school* students. Research on this specific group seems especially relevant, as formal mathematics are introduced in primary school and most of the psychological assessments and interventions on math take place during this time period.

We have found that spatial visualization predicted changes in mathematical achievement three years later during primary school. Children who scored higher on spatial visualization in first or second grade, scored also higher in a standardized math test three years later, even after accounting for their previous mathematical achievement at T1, their EF, SES, and verbal reasoning. However, our data did not yield evidence for the reversed relation with earlier mathematical achievement predicting children's later spatial visualization, indicating a unidirectional relationship between these variables. This asymmetrical result goes in line with findings from Fung and colleagues (2020), but contrasts findings of another recent study showing bidirectional relations between spatial visualization and mathematical achievement in first- through third-grade students (Geer et al., 2019). A potential explanation may be that the current study and the one from Fung and colleagues controlled for a number of additional predictors of mathematical achievement such as EF, SES, or verbal intelligence, whereas these variables were not accounted for in the study from Geer et al. (2019). Therefore, it is possible that other co-occurring variables included in the performance of a visual-spatial task (such as EF) may have contributed to this bidirectional result from Geer and colleagues (2019).

Overall, our research highlights that spatial visualization seem to play a specific role in developing mathematical skills. However, the underlying mechanisms of the relation between visual-spatial skills and mathematics are still hotly debated. Some researchers think that visual-spatial skills might help visualizing mathematical problems, which is beneficial to solve them (Hegarty & Kozhevnikov, 1999). This argument is strengthened by a recent review on possible mechanisms of why visual-spatial skills are related with math (Hawes & Ansari, 2020) and highlights the role of spatial visualization in terms of modeling mathematical problems. Other researchers advocate that spatial skills may help imagining numerical magnitude and ordering this magnitude on a mental number line (cf. Gunderson et al., 2012) which is a foundational concept for learning mathematics (Schneider et al., 2018; Ramani & Siegler, 2008). In addition to these ideas about mechanisms, relations may also depend on the kind of visual-spatial skill and mathematical topic measured. Our data implied no association between mathematical achievement and spatial perception—the second measure of visual-spatial skills of our study. This finding is in line with recent results (Geer et al., 2019), showing no relations between spatial perception and mathematical achievement while associations were found for spatial visualization. A recent meta-analysis by Xie et al. (2020) showed that the strength of the association between visual-spatial skills and math depends on the investigated mathematical areas. For instance, logical reasoning showed a stronger relation to visual-spatial skills as opposed to arithmetic. In our study, we showed similar effects for two different areas, namely a curriculum-based measure comprising various school-related topics and the more specific measure of solving “word problems”.

With respect to relations between EF and mathematical achievement, we have found working memory to predict changes in mathematical achievement, depending on the time difference between both assessments. Our findings indicated that children with high working memory skills were at advantage in mathematical achievement when assessments were further apart, whereas a trend indicates that children with low working memory skills showed more improvement in mathematical achievement when assessments were closer to each other. This finding may indicate that initial advantages of children with high working memory at T1 prevail and increase in the long term, whereas they might be relativized in the short term by catch-up effects from children with low working memory skills. One reason may be found in children’s differential usage of strategies. Children with high working memory abilities may use more advanced strategies that also require more working memory (DiStefano &LeFevre, 2004; Geary et al., 2004; Ramirez et al., 2012). The usage of more elaborated strategies (e.g., decomposition, retrieval) demands high working memory and is related to a better conceptual understanding and higher mathematical achievement (Barrouillet & Lépine, 2005; Mazzocco et al., 2008). In contrast, children with lower capacities may use easier heuristics and invest more effort in general (Gear et al., 2004; Ramirez et al., 2016; Fung et al., 2020). This could be a possible explanation why children with high working memory skills improve in their mathematical achievement over a longer time period as more elaborated strategies need more time to pay off. After longer time periods students may be confronted with more complex mathematical content and may benefit from their higher-order strategies. Notably, this effect is specific for working memory and did not appear for any other component of EF. It highlights the important role of working memory for mathematical learning, which is in line with a great body of research (e.g. Cragg & Gilmore, 2014; Geary et al., 2007).

Importantly, our results indicated also reversed effects, suggesting that earlier mathematical achievement predicted changes in working memory three years later. This result goes in line with other studies with children in kindergarten or preschool (e.g. Fuhs et al., 2014; Miller-Cotto & Byrnes, 2019; Schmitt et al., 2017). A possible mechanism for this reversed effect might be the reciprocal and beneficial relation between learning mathematics and working memory capacities. As primary school children are frequently exposed to mathematics, they have longer learning times and this exposure to mathematics may “train” children’s working memory simultaneously (Fuhs et al., 2014). This assumption is strengthened by the fact that working memory trainings often consist of mathematical tasks (for an overview, Diamond, 2012). A theoretical explanation for these bidirectional effects between working memory and mathematical achievement can be found in theories of the transfer of learning (Klauer, 2000). According to this thesis, domain-general skills (e.g., working memory) are automatically „trained“ when specific skills are trained (e.g., mathematical skills) and vice versa.

The current study had strengths and limitations. We consider it a strength that we investigate bidirectional associations between mathematical achievement and two important domain-general skills, namely EF and visual-spatial skills in a single model. Furthermore, we are the first to investigate these relations in primary school students—a group of children who is intensively exposed to formal mathematics. Moreover, all of our participants were assessed using standardized tests, with mathematical achievement being measured using two tests, both based on children’s school curriculum but including slightly different topics and procedures.

At the same time, our study shows some limitations that should be considered when interpreting the findings. Given that no reliable latent variables could be modeled, it was impossible to provide an aggregated view of EF and visual-spatial skills. Furthermore, even though we used a bidirectional approach, a causal perspective on these relations was not possible and the coefficients must be interpreted with caution given that a CLPM with two assessment time points does not allow to distinguish between the within- or between-person level (Bailey et al., 2018; Hamaker et al., 2015). Future research should focus on experimental designs investigating causal relations between these variables. Finally, the drop-out analyses showed that the remaining subsample was rather selective with higher scores on all main variables including SES and verbal reasoning. This selection bias is linked to the recruitment of the participants at T2. Even though the entire subsample (*N* = 182) was invited to participate in the present study, it seems that particularly those families with high SES and children showing high cognitive and mathematical abilities were more interested in participating again. Therefore, the representativity of the findings is limited and should be considered when interpreting the present results.

Until to date, research on influencing variables of mathematical achievement has focused especially on unidirectional assumptions. However, the last decade of research has shown that patterns between cognitive and academic domains seem to interact and develop mutually. Following this new line of research, the present longitudinal study investigated bidirectional associations between domain-general skills and mathematical achievement. Our results indicate that - at least some - cognitive and mathematical skills develop in tandem and interact mutually with each other across child development. Our results entail important information for intervention and training studies for primary school students as they show that academic and cognitive development can be reciprocal and co-develop beneficially.

However, studies examining causal relations are needed to shed light onto these dynamics in order to provide clear and specific suggestions to the mathematics curricula, interventions and assessments.

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Table 1.

Descriptive statistics of the subtests at each measurement time point.

	Time point 1 (*N* = 182)				Time point 2 (*n* = 78)				-------
Subtest	*M*	*SD*	Min	Max	*M*	*SD*	Min	Max	Max Score
Executive functions									
Working memory	13.76	3.10	5	24	18.87	2.98	14	33	40
Inhibition[a]	53.86	17.76	9	120	34.23	8.43	22	61	--
Cognitive flexibility	31.47	9.56	12	57	57.37	16.40	13	92	--
Visual-spatial skills									
Spatial visualization	24.38	7.70	6	49	38.32	7.76	20	60	80
Spatial perception	12.74	4.39	1	23	26.60	6.75	9	41	76
Math. Achievement (IDS-2)	18.15	5.76	7	36	33.88	6.65	22	50	64
Math. Achievement (DEMAT)	--	--	--	--	54.44	9.02	34	80	--

Note. Children were 6 to 7 years old (6;00 - 7;11) at the time of their first assessment (Time point 1) and were re-assessed on average three years later (Time point 2).

[a] Values of the inhibition task are reversed so that lower values indicate higher inhibitory skills.

Table 2.

Bivariate correlations among all variables of interest at both assessment time points (Time point 1: N = 182; Time point 2: n = 78).

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Time point 1																
1 Sex	-															
2 SES	.083	-														
3 Verbal reasoning	.082	.293*	-													
4 Working memory	-.023	.175*	.446**	-												
5 Inhibition	.196	.166	.201*	.303**	-											
6 Cognitive flexibility	.148	.159	.410**	.276**	.222**	-										
7 Spatial visualization	-.158*	.278	.468**	.401*	.265*	.231**	-									
8 Spatial perception	-.047	.028	.229**	.261	.143	.120	.376**	-								
9 Math. achievement (IDS-2)	-.235**	.285**	.512**	.534**	.239**	.264**	.442**	.250**	-							
Time point 2																
10 Working memory	-.113	.262*	.291**	.555**	.103	-.012	.191	.093	.463**	-						
11 Inhibition	-.038	.094	.015	.031	.132	.077	.131	-.079	.113	.171	-					
12 Cognitive flexibility	.207	.208	.188	.078	.069	.414**	.156	.240*	.109	.235*	.253*	-				
13 Spatial visualization	-.240*	.275*	.199	.329**	.196	.078	.605	.346**	.394**	.330**	.187	.161	-			
14 Spatial perception	-.012	-.017	-.098	.071	-.025	.039	.180	.024	.020	.037	.177	.085	.283*	-		
15 Math. achievement (IDS-2)	-.380**	.183	.336**	.293**	.041	.136	.334**	.200	.575**	.385**	.026	.205	.474**	.072	-	
16 Math. achievement (DEMAT)	-.262*	.375**	.142	.282*	-.002	.021	.331**	.161	.452**	.300**	.248*	.247*	.496**	.181	.557**	-

Note. SES = Socioeconomic status; IDS-2 = Intelligence and Development Scales-2; DEMAT = Deutscher Mathematiktest (The German Test of Mathematical Abilities). Age-standardized value were used for these correlations.
**p* < .05, ***p* < .001

Figure Captions

Figure 1. Two selected items of the task (©Hogrefe AG, Bern: With kind permission of Hogrefe AG) to measure spatial visualization. In this task, participants were presented with 2-dimensional geometrical figures and asked to copy them using triangular or rectangular cutouts.

Figure 2. Differentiated cross-lagged panel model (CLPM) of executive functions (EF), visual-spatial skills, and mathematical achievement. Participants (N = 182) were 6 to 7 years old (6;00 - 7;11) at the time of their first assessment (Time point 1) and were re-assessed (n = 78) on average three years later at the second time point (Time point 2). All manifest variables were age-standardized for analyses. We accounted for the variables sex, SES, and verbal reasoning at T1. All variables correlated with each other. For the sake of clarity, the figure is divided into depicting the autoregressive effects (A) and the cross-lagged effects (B); however, both types of effects were computed statistically in one unified model. Furthermore, only significant paths (*$p < .05$, **$p < .001$) and main variables are included in the figure.

Figure 3. Participants' changes in mathematical achievement (standardized residuals) as a function of individual differences in working memory at T1 (z-Scores). Separate graphs are shown for participants being tested one standard deviation above (Tdiff – large) or below (Tdiff – short) average time differences (Tdiff) between T1 and T2.

Figure 1.

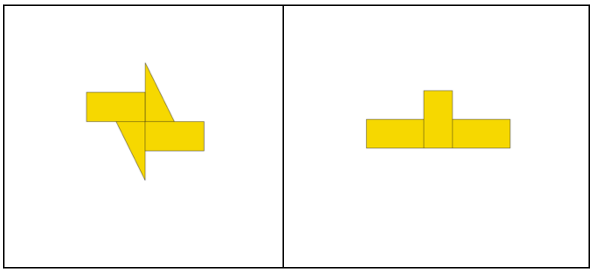


Figure 2.

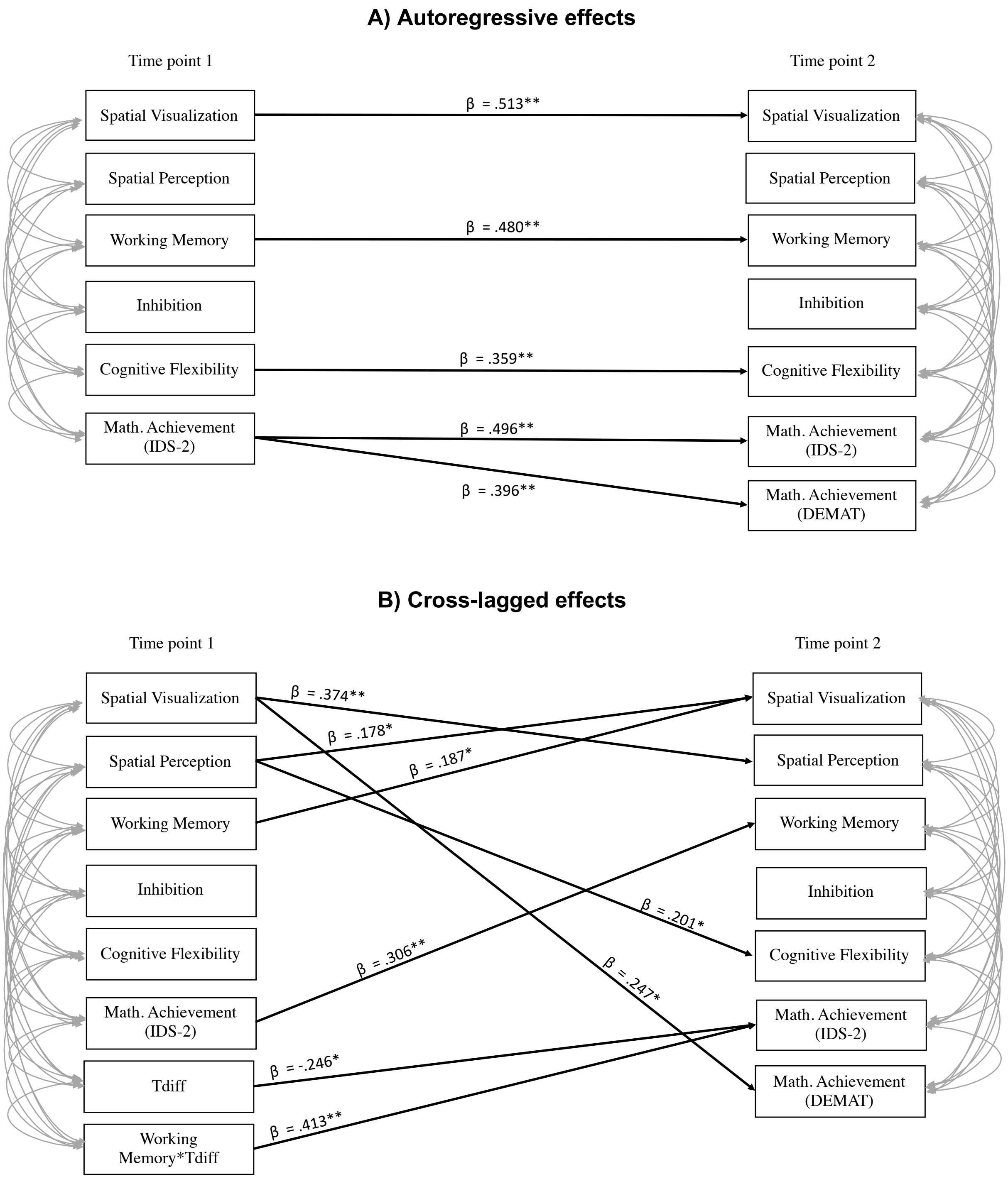
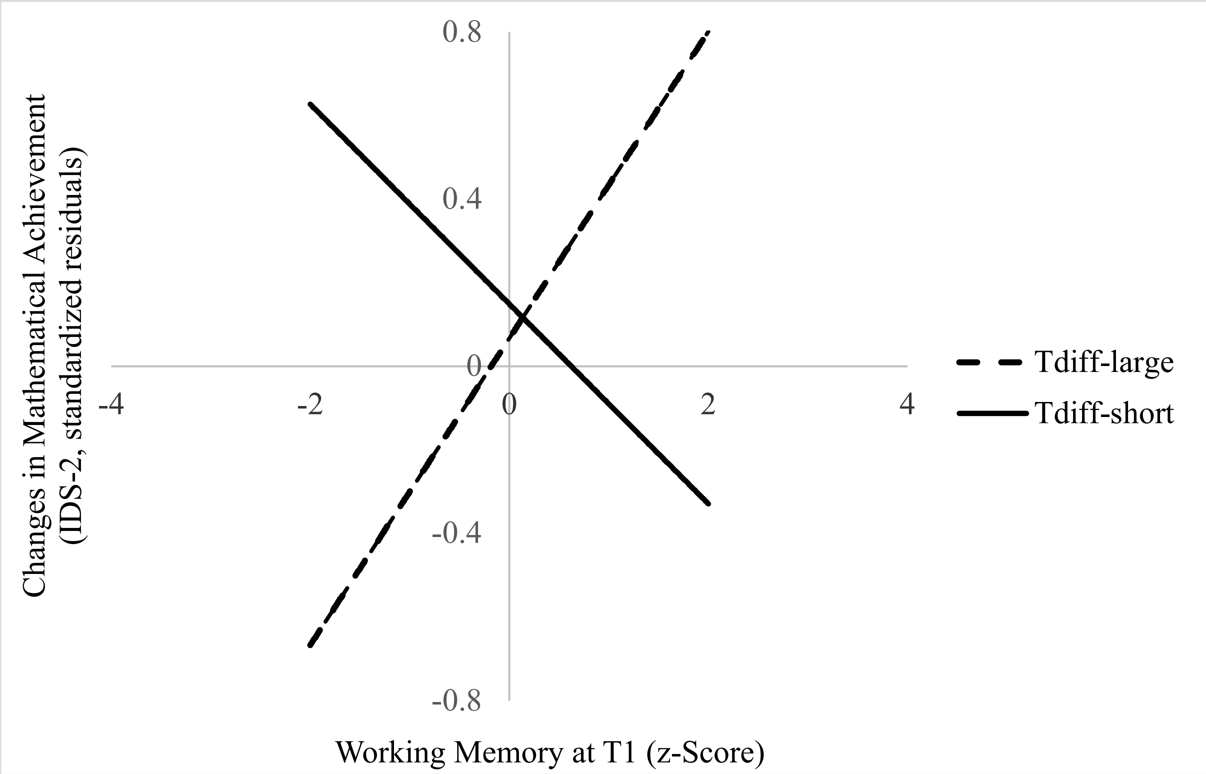

A) Autoregressive effects
Time point 1
Time point 2
Spatial Visualization
Spatial Perception
Working Memory
Inhibition
Cognitive Flexibility
Math. Achievement (IDS-2)
Math. Achievement (DEMAT)
β = .513**
β = .480**
β = .359**
β = .496**
β = .396**
B) Cross-lagged effects
Tdiff
Working Memory*Tdiff
β = .374**
β = .178*
β = .187*
β = .201*
β = .247*
β = .306**
β = -.246*
β = .413**

Figure 3.


0.8
0.4
0
-0.4
-0.8
-4
-2
0
2
4
Changes in Mathematical Achievement
(IDS-2, standardized residuals)
Working Memory at T1 (z-Score)
Tdiff-large
Tdiff-short

APPENDIX C: STUDY 3

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Does Emotion Regulation Compensate Deficits in Various Executive Functions in Children's and Adolescents' Mathematical Achievement?

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Abstract (228)

Many children and adolescents experience negative emotions while learning mathematics and it is likely that children with a more adaptive way of regulating their emotions perform better in mathematics. This conclusion was indeed supported by recent research. A different line of research suggested that emotion regulation could compensate for low fluid intelligence in children's mathematical achievement indicating that a similar compensation effect may exist for other cognitive skills. Building on this research, the aims of the current study were to investigate whether a) emotion regulation is associated with mathematical achievement in children and adolescents and b) emotion regulation can buffer working memory (WM) capacities in the mathematical domain. In an additional set of exploratory analyses, we tested whether these interaction effects hold for other core components of executive functions (EFs) such as inhibition and cognitive flexibility. A large sample of 7- to 15-year-olds (N = 992) was tested using the Intelligence and Development Scales–2. We tested the hypotheses with multiple regression analyses with EFs as predictor variables, emotion regulation as moderator variable, and mathematical achievement as outcome variable. Results showed relations between emotion regulation and children's mathematical achievement across mid-childhood to mid-adolescence. Furthermore, emotion regulation compensates low WM in participants' mathematical achievement, which was not found for inhibitory skills or cognitive flexibility. Our results highlight the dynamic relation between WM and emotional abilities in students' academic success.

Keywords

emotion regulation, executive functions, mathematical achievement

Does Emotion Regulation Compensate Deficits in Various Executive Functions in Children's and Adolescents' Mathematical Achievement?

Mathematical skills are considerably developed and acquired in primary and secondary school (Geary, 2000). Occasionally, learning mathematics becomes a nightmare for some students across this time. Mathematics is perceived as the most stress-evoking subject at school (Beilock, 2008). Feeling lost, anxious, or sad while doing math is more common to students than teachers and parents might expect (Ashcraft, 2002). Evidence indicates that many children and adolescents are faced with negative emotions while doing mathematics (for a review, see Dowker et al., 2016). Research on mathematics anxiety helps understanding the frequency of these negative emotions. For example, Ashcraft and Moore (2009) have found that 17% of the entire population (i.e., including students and adults) are affected by mathematics anxiety.

There is a high interindividual variance in how children cope with such negative emotions and it was found that children who deal more adaptively with negative emotions are at an advantage in school (Eisenberg et al., 2005). Building on this research, the question arises whether there is a *specific* relation between emotion regulation and mathematical achievement. Furthermore, based on theories such as the cognitive load theory (Sweller, 1994), there is reason to assume that children's emotion regulation compensates working memory (WM) skills in children's mathematical performance. In the current study, we aimed to answer these questions and additionally explored whether these compensating effects were specific for WM or more general in nature. To this end, we explored interaction effects for executive functions (EFs) such as inhibition and cognitive flexibility.

Emotion Regulation and Mathematical Achievement

Up to now, most of the studies investigating the association between emotion regulation and math focused on kindergarten children or primary school students (e.g., Graziano et al., 2007). In the majority of these studies, emotion regulation is defined as the ability to manage and change emotional responses (Eisenberg et al., 2005). Children can use several coping strategies to regulate their emotions which are typically distinguished into adaptive and maladaptive strategies. Maladaptive strategies are characterized by avoidance, distorted perception, a lack of impulse control or restlessness (Garnefski et al., 2002). Adaptive coping strategies focus on the stress-triggering situation itself as a solution-oriented approach (e.g., addressing the problem) which enables to maintain people's self-esteem and social relationships (Horowitz & Znoj, 1999). However, it needs to be noted that coping with negative emotions is termed and assessed in different ways across the studies. With respect to terms, researchers called this ability "emotion regulation" (e.g., Eisenberg et al., 2005), "effortful control" (e.g., Valiente et al., 2012b), "emotion competence" (e.g., Trentacosta & Izard, 2007) or "hot executive functions" (e.g., Brock et al., 2009). With respect to measures, studies used knowledge-based instruments, self-reports, teacher-reports, or behavioral observation.

Using these various operationalizations, studies have indicated that coping proactively with negative emotions is related to higher mathematical achievement in young children (Franco et al., 2017; Izard et al., 2001; Trentacosta & Izard, 2007; Valiente et al., 2010; Valiente et al., 2012a; Valiente et al., 2012b; but see Brock et al., 2009). The vast majority of studies revealed this connection in primary school children with fewer studies indicating similar relations in adolescents. To our knowledge, only two studies investigated the relation between emotion regulation and mathematical achievement in adolescence, with both studies yielding evidence for such an association (Gumora & Arsenio, 2002; Oberle et al., 2014).

With respect to potential explanations, Eisenberg et al. (2005) postulates that students with high emotion-regulation skills are more motivated and show higher social competences, which in turn affects their academic performance positively. Empirical evidence for this assumption comes from a study which has found that students with adaptive emotion-regulation strategies showed higher mathematical performance and better relationships with their teachers (Graziano et al., 2007). These latter authors assumed that children with more adaptive emotion-regulation strategies learn better in classroom settings, and are more productive and accurate in completing assignments.

Compensating Effects of Emotion Regulation

In addition to direct links between emotion-regulation skills and mathematical achievement, emotion regulation was also shown to have an *indirect* effect on mathematical achievement. In particular, research has demonstrated that adaptive emotion regulation can compensate low domain-general abilities, which in turn results in higher mathematical achievement (Gut et al., 2012; Konold & Pianta, 2005). For example, Gut et al. (2012) showed that children aged 8-13 years were able to compensate low levels of fluid intelligence with high social-emotional skills (including emotion regulation) in their mathematical performance. This interaction between emotion and cognition in academic performance goes in line with assumptions from Blair (2002), Koole (2009), and LeDoux (2000) pointing out that such interactive effects are important for scholastic success.

There is reason to think that a similar compensation effect may also be found for other cognitive variables such as WM or maybe even other EF components such as inhibition and cognitive flexibility. WM enables to hold and manipulate information in mind (Baddeley, 1996), whereas inhibition is the ability to suppress internal and external sources of distraction (Diamond, 2013). Cognitive flexibility allows to change strategies and perspectives (Diamond, 2013). Research has shown that these components are related to mathematical achievement in various age groups (e.g., Cragg et al., 2017; Bull & Scerif, 2001; Yeniad et al., 2013; for overviews, see Bull & Lee, 2014; Cragg & Gilmore, 2014) and across a large age range (e.g., Kahl et al., 2019). Whereas inhibition and cognitive flexibility are related to mathematical achievement, WM is found to be one of the strongest predictors for mathematical achievement besides intelligence (Geary et al., 2007). Several mathematical operations require WM resources, for example, when children need to keep an intermediate result in mind while going on with the next steps of a mathematical problem.

Besides its strong relation to mathematical achievement, there is reason to believe that specifically WM and emotion regulation are interacting with each other. Coping with negative emotions requires WM capacities (Opitz et al., 2012; Pe et al., 2013) and maladaptive strategies can tax WM resources to a high extent (e.g., Ascraft & Kirk, 2001). Accordingly, adaptive emotion-regulation skills can ensure a “cool mind” and save cognitive resources when for example children are less likely to ruminate while being confronted with negative emotions (Richards & Gross, 2000). Based on assumptions of the *cognitive load theory* (Sweller, 1994, 2016; Paas et al., 2004), it seems reasonable that adaptive emotion-regulation skills can compensate WM by keeping demands of the executive processes to a low level. This theory assumes that the WM capacity is limited and that distracting information occupy WM resources. This is not *per se* specific to a mathematical context but might be relevant for academic achievement in general when different sources might increase the WM load.

Studies on mathematics anxiety give hints about possible indirect effects in a mathematical context. For example, an inadequate way of coping with anxiety (e.g., ruminating) may occupy WM resources (e.g., Beilock & Carr, 2005; Beilock & DeCaro, 2007; Beilock & DeCaro, 2008; Richards & Gross, 2000) which are in turn not available for the mathematical tasks (Ashcraft & Kirk, 2001; LeDoux, 2000). Ashcraft and Moore (2009) describe that mathematics anxiety taxes children's WM resources, resulting in an "affective drop" of math performance. Being able to regulate these negative emotions may free up WM resources and facilitate their usage for mathematical problems. In accordance to this perspective, emotion regulation may work as a buffer or compensator against negative emotions and "protect" the required WM resources to solve mathematical problems. Considering that low WM capacities affect children's mathematical performance (for a review, see Raghubar et al., 2010), it may be that particularly children with low WM capacities are at risk considering that their WM resources are exhausted rapidly (Geary, 2011). On the other hand, it was shown that particularly children with high WM capacities show the strongest relation between mathematics anxiety and mathematical performance (Ramirez et al., 2013), which was mediated by their avoidance to use more advanced problem-solving strategies (Ramirez et al., 2016). Therefore, based on these differing outcomes, it is not entirely clear whether children with high or low WM skills would profit the most from high emotion regulation in the mathematical domain.

The Present Study

The present study followed two goals: First, we were interested in whether emotion regulation is associated with mathematical achievement in our sample of children and adolescents. To this end, we used a large, cross-sectional sample of 7-to 15-year-olds (2nd to 9th grade) from a standardization sample of a recently developed test battery. Compared to previous studies, we are the first to investigate these relations across such a wide age span (Franco et al., 2017; Izard et al., 2001; Valiente et al., 2010; Valiente et al., 2012a; Valiente et al., 2012b). With regards to our sample, we intentionally refrained from including students in 1st grade given that first grade is a transition period as students have just left kindergarten and started with primary school. This new environment may impact students' emotional experiences to a large extent which may influence the interplay between emotion regulation and mathematical achievement (Morrison et al., 2010).

As a second goal, we aimed at investigating whether emotion regulation can compensate WM in children's mathematical achievement. Therefore, we explored whether higher emotion regulation can buffer students' WM capacities. Based on the literature presented above, we explored whether this compensation effect holds particularly for students with high or low WM skills. In this context, we also examined whether these compensating effects hold specifically for WM or whether they can be generalized to other EFs such as inhibition and cognitive flexibility. By doing so, we planned to increase our understanding as to *why* emotion regulation may moderate associations between various cognitive skills and mathematical achievement. That is, does emotion regulation compensate in the same way irrespective of the cognitive ability under investigation, suggesting a general benefit or does emotion regulation compensate specifically WM, indicating a WM-related mechanism. In our statistical analyses, we accounted for age by using age-standardized values, sex, socioeconomic status (SES), and verbal reasoning due to their relation with the main variables (e.g., mathematical performance: Geary, 2006; Hoff, 2013; Lindberg et al., 2010; for meta-analyses: Kuncel et al., 2004; Peng et al., 2019).

Investigating these questions extended a previous study from our research group that used data from the same standardization sample (Author, Year). In this study, we investigated associations among EFs, visual-spatial skills, and mathematical achievement in 1754 participants aged 5-20 years. We were able to show that EFs and mathematical achievement were linearly and constantly related across this large age range whereas associations between spatial skills and mathematics seemed to be non-linear and age-dependent. Importantly, EFs were operationalized as a single factor score that comprised shared variance of the three core components WM, inhibition, and cognitive flexibility. This approach precluded to investigate specific relations among mathematical reasoning and these components. In the current study, we qualified these previous findings by integrating cognitive and affective domain-general skills and investigating their specific direct and indirect relation to mathematics.

Method

Participants

We used a sub-sample (N = 1077) of the standardization sample of the Intelligence and Development Scales-2 (IDS-2, Grob & Hagmann-von Arx, 2018). Children and adolescents were included in the present study if they were 7 to 15 years of age and if they attended 2nd to 9th grade (cf. supplementary Table 1 for an age and grade distribution of our sample). We excluded participants (n = 85) with missing data in the variables WM, inhibition, cognitive flexibility, emotion regulation, mathematical achievement, or in the control variables. This procedure resulted in a final sample of 992 participants (M_{age} =11.29 years; SD = 2.37; 49.3% female). Assessments took place in Switzerland (64.4%), Germany (30.1%), Austria, and Liechtenstein (together 5.4%) at the respective University, at participants' homes, their schools or educational institutions. The assessments took place in individual face-to-face settings with a research assistant in German language. Based on a self-report, the sample consisted of 75.4% monolingual native German speakers and 16% bilingual German speakers. Every participant showed a level of German language skills that allowed him or her to follow the task instructions. The current study was approved by the Ethics Committee of [BLINDED FOR REVIEW] which was performed in accordance with the rules laid down in the 1964 Declaration of Helsinki and its later amendments. Children and adolescents of the study agreed verbally to participate prior to the start of the study; parents gave written consent.

Measures

Variables were assessed using the IDS-2 (Grob & Hagmann-von Arx, 2018). This test battery is a revised version of the Intelligence and Development Scales (IDS, Grob et al., 2009). The IDS showed high reliability and validity (e.g., Hagmann-von Arx et al., 2012; Hagmann-von Arx et al., 2008; Hagmann-von Arx et al., 2013) and the IDS-2 shows a good construct validity (Grieder & Grob, 2019).

The IDS-2 allows measuring intelligence and development in several core functions (e.g., executive functions, psychomotor skills, social-emotional competencies, scholastic skills, and attitudes towards work). For the present study, only particular subtests were included (for details, see below). As is common in intelligence testing, starting points in the test battery depended on participant's age, whereas termination depended on their performance.

Mathematical achievement

Mathematical achievement was measured using a subtest that consisted of 19 items with various sub-items. Mathematical topics of this test were chosen in accordance to the Swiss school curriculum in mathematics (Erziehungsdirektoren-Konferenz, D-EDK, 2013). Seven-year-olds of the current study were presented with items assessing knowledge about counting, invariance, and ordinality. Eight- to 10-year-olds were tested with problems tapping mental addition, equations, and proportions. From the age of 11 years and older, participants started with problems on geometry, advanced proportional reasoning, fractions, and algebra (e.g., participants were asked to solve "56 = 8 x ___ + 4 x ___ "). Each subtest was stopped after the child gave five subsequent incorrect answers. Every item had a time limit of 90 seconds (except of the first eight items). Correct answers were added to a sum-score and this score was age-standardized to reduce age-specific variance. This outcome served as dependent variable. This subtest showed a high reliability (Cronbach's α: .95). Previous research has shown that performance on this subtest was substantially related to students' math grades (r = .37, N = 478) and to parents' estimations of their children's mathematical skills (r = .44, N = 726; Grob & Hagmann-von Arx, 2018). Moreover, this math test showed also good differential validity (Grob & Hagmann-von Arx, 2018), as gifted students in mathematics scored significantly higher as compared to typically developing students.

Working memory

Participants were asked to recall numbers or letters in forward and backward order as is customary in other intelligence tests (e.g., Wechsler Intelligence Scale for Children, WISC-IV, Wechsler, 2003). The maximum number of items (i.e., digits or letters) to be recalled per trial was nine items. Participants started with a trial including two items. To increase difficulty, one additional item was added stepwise in every trial except in trials with two and three items (i.e., trials included 2, 2, 3, 3, 4, 5, 6, 7, 8, 9 items). Each trial was once presented with letters only and once with digits only and presented in either forward or backward order, so that the total number of trials per condition (digit vs. letter and forward vs. backward) amounted to ten trials and the entire task consisted of 40 trials. The task was stopped after three subsequent incorrect recalls. To measure WM and to reduce math-specific variance, we conducted our analyses with only those items that included letters and were performed in backward order (cf. Diamond, 2013). Children's answers for these trials were added to obtain a single score and this variable was z-standardized for every year of age to reduce age-specific variance. However, an additional analysis using all items of this subtest revealed that findings were identical to the findings presented in the result section. Reliability for this subtest was high (Cronbach's α: .88).

Inhibition

Inhibition was measured by an adapted Stroop task in which children and adolescents were presented with four different animals (dolphin, chick, frog, ladybug). Participants were asked to name the correct color of each presented animal. The task consisted of three conditions, with each set consisting of 36 animals. In the first condition, participants were asked to name the correct color of each congruently-colored animal (i.e., a yellow chick and a blue dolphin). In the second condition, participants were presented with the same animals which were now presented in black-white. In the third condition, children and adolescents were presented with a set of incongruently-colored animals (see Figure 1a). Time and accuracy were measured. In accordance to the IDS-2 manual, these times were computed according to the following formula (t3 – [t1xt2]/[t1+t2]) in order to create an inhibition score. This subtest showed satisfying reliabilities (Cronbach's αs: .75; Schmitt, 1996). Similar versions of this task have been used in other studies (cf. Neuenschwander et al., 2012; Roebers et al., 2014; Röthlisberger et al., 2013).

Cognitive Flexibility

To measure cognitive flexibility, a verbal fluency task was used which is a typical operationalization (van der Elst et al., 2011; for a summary, cf. Diamond, 2013). Children and adolescents were asked to name as many words as possible in a given category. Younger children (7 to- 9 years) were presented with the categories "animals" and "food". Older participants were additionally presented with the categories "words beginning with E" and "words beginning with alternatingly S or L". The number of correct words within a time limit of 90 seconds for each category was summed up and used as cognitive flexibility score. This subtest showed satisfying reliabilities (Cronbach's αs: .71; Schmitt, 1996).

Emotion regulation

Emotion regulation was measured by assessing participants' knowledge of how to cope with three negative emotions (anger, anxiety, and sadness). The subtest is based on the questionnaire of emotion-regulation strategies in children and adolescents (Fragebogen zur Erhebung der Emotionsregulation bei Kindern und Jugendlichen, FEEL-KJ; Grob & Smolenski, 2009). This original instrument shows high reliability and validity in measuring emotion-regulation strategies (Cracco et al, 2015; Greuel et al, 2018). Furthermore, it was found that children and adolescents with depression and social anxiety disorder reported fewer adaptive strategies as compared to a control group (Lange & Tröster, 2014, 2018), which suggests good differential validity.

Participants heard various scenarios about a child or adolescent who experiences anger, anxiety, or sadness. Simultaneously, they were presented with a photo showing a child or adolescent with a facial expression indicating the corresponding emotion (see Figure 1). Each scenario closed with an open-ended question (e.g., "Imagine that this boy is angry because another child broke his toy. What would you recommend this boy so that he will no longer be angry?"). If participants named various strategies, they were asked to choose their best-fitting strategy. Answers were categorized into three different types of strategies: adaptive strategies, other strategies, and maladaptive strategies based on their functionality to cope with these negative emotions (Compas et al., 1988). An emotion-regulation strategy was considered to be adaptive when allowing an efficient coping in which emotions were proactively regulated by the respective person herself and social relationships were maintained (e.g., by elevating his or her own mood, re-evaluating or addressing the problem). Other strategies were defined as strategies in which the emotions were not regulated by the person itself but by seeking social support (e.g., by teachers, parents or friends) or by cutting off contact with people who were involved in evoking these negative emotions. Maladaptive emotion-regulation strategies were defined as strategies resulting in damaging oneself and/or other people (e.g., aggressive behavior, being self-deprecating, giving up, perseverating). These categories were validated by correlational analyses with the FEEL-KJ questionnaire, showing no relation of other strategies to external criteria such as well-being, whereas adaptive strategies were positively and maladaptive strategies were negatively related to well-being (Grob & Smolenski, 2009). Similar to other open-ended formats of test batteries (e.g., verbal comprehension of the WISC-IV), a pool of adequate answers was given and the experimenter coded given answers as adaptive, maladaptive or other strategies. Children received two points for adaptive strategies, one point for other strategies, and zero points for maladaptive strategies. Consequently, higher scores indicate more knowledge about adaptive emotion regulation. The content and number of scenarios depended on the age of participants (cf. Figure 1a and Figure 1b). Eight- to 10-year-olds' emotion-regulation strategies were assessed using six scenarios (two items for each of the three negative emotions anger, anxiety, or sadness); 11- to 15-year-olds' strategies were tested using nine scenarios (three items for each of the three negative emotions anger, anxiety, or sadness). The subtest showed a satisfying reliability of .78 across age (Grob & Hagmann- von Arx, 2018). Scores for each trial were summed up which served as an emotion-regulation score. This variable was then age-standardized (M = 10, SD = 3) to reduce age-specific variance and to allow for comparability between younger and older children.

Verbal reasoning

Two tasks were used to measure verbal reasoning. In the *find-opposite task,* participants were asked to name the opposite of words (e.g., cold). In the *find-a-category task*, participants had to name the category of a set of three pictures (e.g., showing different vegetables). There was no time limit in each of these tasks and each task consisted of 34 items. Difficulty increased from one item to the next. Each task was stopped when three subsequent items were answered incorrectly. The number of correct answers were summed up in both tasks and this score was age-standardized. Both tasks were found to be highly reliable (Cronbach's αs: .87 - .90).

Socioeconomic status

SES was operationalized using the educational level of children's and adolescents' mother. Information was given via a parent questionnaire which is in line with other studies (cf. Hoff, 2013). Maternal educational levels ranged on a five-point Likert scale (1 = obligatory school education; 5 = university degree).

Data analytic procedure

Analyses were conducted using IBM SPSS Statistics 23.0. To investigate whether WM, inhibition, cognitive flexibility, and emotion regulation are independently associated with mathematical achievement, we calculated a linear regression. To examine whether relations between each EF component and mathematical achievement varied across different levels of children's emotion regulation in form of a moderation effect, we conducted three models, including one EF component as a predictor variable (e.g., WM), an interaction term with emotion regulation and the respective EF component (e.g., emotion regulation*WM), and the other respective EFs as control variables (e.g., inhibition and cognitive flexibility). The variables sex, SES, and verbal reasoning were included in all models as control variables. Age was not entered as an additional control variable given that we used age-standardized values. Significant interactions were followed up by simple slopes analyses which enabled to visualize interactions (cf. Aiken et al., 1991). Computations were made with PROCESS which is a modeling tool for SPSS for moderation analyses (Hayes, 2018). Descriptive statistics of each single task can be seen in Table 1.

Results

WM, inhibition, cognitive flexibility, emotion regulation, and mathematical achievement were small-to-moderately correlated with each other (see Table 2). Furthermore, all EFs were small-to-moderately correlated with sex, SES, and verbal reasoning. Similar relations with control variables were found for emotion regulation (see Table 2).

The regression models showed that all EFs and emotion regulation were specifically and distinctly related with mathematical achievement even after accounting for sex, SES, and verbal reasoning (see Table 3, 4, and 5). These relations indicate that children and adolescents who showed higher scores on WM, inhibition, cognitive flexibility and emotion regulation scored also higher in the mathematical achievement test. Each model (including control variables) was able to explain approximately 40% of the variance in children's and adolescents' mathematical achievement. With respect to a potential compensation effect between the EFs and emotion regulation, it was found that the interaction term WM*emotion regulation was significant ($\beta = -.07$, $p = .001$), indicating that the effect of WM on mathematical achievement was moderated by emotion regulation. Importantly, the interaction terms inhibition*emotion regulation and cognitive flexibility*emotion regulation were not significant (inhibition: $\beta = -.04$, $p = .07$; cognitive flexibility: $\beta = -.02$, $p = .29$).

In order to illustrate the interaction effect, we conducted simple slope analyses (Aiken et al., 1991). We followed the common procedure and illustrated the relation between WM and mathematical achievement one standard deviation above and below the mean value of emotion regulation in the current sample. To categorize children into those with low and high WM abilities, we categorized children into those with one standard deviation above and below the mean value of WM. As can be seen in Figure 2, those children with lower WM capacities showed higher mathematical skills when having more knowledge on how to regulate emotions.

Discussion

The present study investigated relations among emotion regulation, EFs, and mathematical achievement in a large, cross-sectional sample of children and adolescents aged 7 to 15 years. After accounting for the variables sex, SES, age, and verbal reasoning, our findings indicated that knowledge about emotion regulation and all EFs were independently associated with mathematical achievement across age. Furthermore, it was found that participants who scored low on WM could compensate their low WM skills when having higher knowledge of how to adequately cope with emotions. Importantly, we did not find similar compensating effects for other EFs such as inhibition or cognitive flexibility. All variables explained approximately 40% of the variance in our sample's mathematical achievement.

Our findings are in line with previous research indicating that children and adolescents with more adaptive regulation strategies perform better in mathematics (e.g., Graziano et al., 2007; Gumora & Asenio, 2002; Oberle et. al., 2014). However, for the first time, in the present study, we investigated these relations in a large sample of children and adolescents. This relation holds true after accounting for SES, sex, verbal reasoning, and EFs. Overall, we can conclude that emotion regulation seems to be an important factor for mathematical achievement in children and adolescents.

Our second aim of the study was to investigate whether emotion regulation can compensate WM capacities in the mathematical domain. We were able to show that emotion regulation compensated low WM capacities in children's and adolescent's mathematical achievement. Interestingly, emotion regulation did not compensate low capacities in inhibition or cognitive flexibility, although a trend of this effect can be observed for inhibition[2]. Our results lend support for the cognitive load theory in a mathematical context (Sweller, 1994; Paas et al., 2004) which claims that adaptive emotion regulation can compensate WM by keeping demands of the executive processes to a low level, so that WM resources can in turn be devoted to mathematical operations. Our results show that emotion regulation does not compensate performance in any cognitive skill, highlighting the specific interactive nature between emotion regulation and WM. Furthermore, we have found that high emotion regulation could only buffer children's and adolescents' *low* WM capacities, which qualifies previous research on mathematics anxiety (Ashcraft & Moore, 2009; Ramirez et al., 2013, 2016).

Whereas our study provides an important step into clarifying who and why children profit from emotion regulation in educational settings, many questions remain unresolved. Future research should especially focus on the operationalization of variables and on the mechanisms underlying the beneficial role of emotion regulation in mathematical achievement in typically and atypically developing children and adolescents. On a variable level, it seems necessary to use more unified operationalizations of emotion regulation and mathematical achievement. Such an approach would help to increase comparability among studies. Experimental designs and intervention studies are clearly needed to shed light into causal relations. Investigating the mechanisms of the associations that we have found would be especially important for training studies. For example, it was found that improving WM skills resulted

[2] Even though the interaction term between inhibitory skills and emotion regulation was non-significant (p = .07), we conducted simple slopes analyses in order to explore the pattern of this trend. Similar to the interaction effect between WM and emotion regulation, it was found that children with low inhibitory skills showed higher mathematical skills when reporting more adaptive emotion-regulation strategies.

in higher emotion-regulation abilities (Schweizer et al., 2013; Xiu et al., 2016). Another intervention study showed that a training of emotion regulation led to higher mathematical achievement in dyscalculic children (Narimani et al., 2013). Given that our findings indicated interactions between emotion regulation and WM on children's mathematical achievement, it seems that intervention studies could profit from integrating WM and emotion regulation in a single training. This assumption could be investigated by an intervention study with a wait list control group, testing whether single trainings on emotion regulation, working memory, or EF lead to higher mathematical achievement than a combined training, in which both skills emotion regulation and working memory are trained simultaneously.

In the current study, we consider it an advantage that our sample covers a wide age span from childhood to adolescence with all participants being tested with the same standardized instrument. This test battery allowed assessing all variables in a conceptually similar way. With respect to the mathematical assessment, items were created in accordance to the curriculum, which provides high ecological validity given that this measure covered several mathematical topics and domains that children and adolescents are confronted with at school.

However, there are some limitations of our study that should be kept in mind when interpreting the present findings. First, we used a cross-sectional design which precluded interpreting any causal effects of emotion regulation and WM on children's mathematical understanding. Second, emotion regulation was measured using imaginary scenarios which is comparable to previous operationalizations (e.g., Lange & Tröster, 2014; Otterpohl & Wild, 2015). Even though these scenarios focused on different emotions, it was not possible to distinguish between specific emotion-regulation skills (e.g., anxiety) due to the small number of items per emotion and the lack of age-standardized values for each emotion. Moreover, no measures were included in order to assess mathematics anxiety or students' stress level while doing mathematics. Therefore, it was difficult to estimate the actual emotional burden of individual participants. Furthermore, the assessments (including the mathematical achievement test) took place in an individual face-to-face setting with a trained research assistant which may have contributed to a rather positive atmosphere in the test situation. Finally, the mathematical achievement test was based on the school curriculum, covering various topics students are confronted with at school. We focused our results on the combined score but were not able to make conclusions with respect to specific domains of mathematics (e.g., word problems, geometry etc.).

Until to date, research often focuses mainly on cognitive skills and their relations to mathematical achievement. Indeed, in the present study, WM was (again) identified as a strong predictor for mathematical achievement (for related findings, see an overview from Bull & Lee, 2014). At the same time, decades of research have shown that emotional and cognitive abilities show bidirectional relations and predict later academic achievement (e.g., Liew, 2012). In accordance to these findings, several researchers claim that emotional and cognitive abilities should be integrated when investigating children's academic success (for reviews, see Blair, 2002; Koole, 2009). The present study represents an important step in this direction and widens our understanding of cognitive and affective domain-general skills and their relation to mathematical achievement. Overall, our results support an integrative view on mathematical achievement by taking cognition and emotion across children's and adolescents' development into account.

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Table 1.

Descriptive Statistics of the Tasks Using Raw Scores (n = 992)

Task	*M*	*SD*	Min	Max	Score limit
Working Memory	3.97	1.43	0	8	10
Inhibition	34.18	17.29	11	222	--
Cognitive Flexibility	60.30	22.87	15	132	--
Emotion Regulation (Total)	12.67	3.65	0	18	18
Emotion Regulation (Children)	9.79	1.98	1	12	12
Emotion Regulation (Adolescents)	15.32	2.74	0	18	18
Verbal Reasoning					
Find-Opposite Task	17.21	4.10	3	30	34
Find-Category Task	19.55	5.25	2	33	34
Mathematical Achievement	34.24	10.33	5	59	64

Table 2.

Correlations Among Working Memory, Inhibition, Cognitive Flexibility, Emotion Regulation, Mathematical Achievement, and the Control Variables Sex, SES, and Verbal Reasoning (n = 992)

Variable	1	2	3	4	5	6	7
1 Sex	_						
2 SES	-.016	_					
3 Verbal Reasoning	.023	.287**	_				
4 Working Memory	.082	.239**	.413**	_			
5 Inhibition	.116**	.131**	.273**	.307**	_		
6 Cognitive Flexibility	.180**	.213**	.544**	.381**	.371**	--	
7 Emotion Regulation	.106**	.083**	.255**	.176**	.192**	.238**	
8 Math. Achievement	-.042	.267**	.564**	.431**	316**	.453**	.242**

Note: SES = socioeconomic status*;* Sex: 0 = male, 1 = female.
**p* < .05, ***p* < .001. Age-standardized values of all variables, apart from sex and SES, were used for the correlations.

Table 3.

Relations Among Working Memory, Emotion Regulation, and Mathematical Achievement (n = 992): A Regression Analysis

Variables	β	*T*	Sign. *T*	R^2	*F (df)*	Sign. *F*
				.42	114 (8, 983)	< .001
Sex	-.11	-4,41	< .001			
SES	.07	2.61	.009			
Verbal Reasoning	.34	10.06	< .001			
Inhibition	.10	3.58	< .001			
Cognitive Flexibility	.13	4.05	< .001			
WM	.18	5.94	< .001			
Emotion Regulation	.06	2.20	.028			
WM*Emotion Regulation	-.07	-3.29	.001			

Note: Working memory = WM; SES = socioeconomic status*;* Sex: 0 = male, 1 = female. Age-standardized values were used for the analysis. Mathematical achievement served as outcome variable.

Table 4.

Relations Among Inhibition, Emotion Regulation, and Mathematical Achievement (n = 992): A Regression Analysis

Variables	β	*T*	Sign. *T*	R^2	*F (df)*	Sign. *F*
				.42	106 (8, 983)	< .001
Sex	-.11	-4,41	< .001			
SES	.07	2.61	.009			
Verbal Reasoning	.34	10.06	< .001			
Cognitive Flexibility	.14	4.21	< .001			
WM	.18	6.05	< .001			
Inhibition	.10	3.52	< .001			
Emotion Regulation	.06	2.28	.023			
Inhibition*Emotion Regulation	-.04	-1.83	.068			

Note: Working memory = WM; SES = socioeconomic status*;* Sex: 0 = male, 1 = female. Age-standardized values were used for the analysis. Mathematical achievement served as outcome variable.

Table 5.

Relations Among Cognitive Flexibility, Emotion Regulation, and Mathematical Achievement (n = 992): A Regression Analysis

Variables	β	*T*	Sign. *T*	R^2	*F (df)*	Sign. *F*
				.41	103 (8, 983)	< .001
Sex	-.11	-4,38	< .001			
SES	.07	2.71	.007			
Verbal Reasoning	.34	9.91	< .001			
WM	.18	6.04	< .001			
Inhibition	.10	3.70	< .001			
Cognitive Flexibility	.14	.14	< .001			
Emotion Regulation	.07	2.41	.016			
Cognitive Flexibility*Emotion Regulation	-.02	-1.06	.29			

Note: Working memory = WM; SES = socioeconomic status; Sex: 0 = male, 1 = female. Age-standardized values were used for the analysis. Mathematical achievement served as outcome variable.

Figure Captions

Figure 1. A selection of two scenarios task (©Hogrefe AG, Bern: With kind permission of Hogrefe AG) used to measure emotion regulation for anger in children (A) and for anxiety in adolescents (B). Participants were asked how to deal with the situation by open-ended questions. Participant's answers were categorized into adaptive strategies (two points), other strategies (one point), or maladaptive strategies (zero points). Higher points indicate higher knowledge of how to proactively regulate emotions.

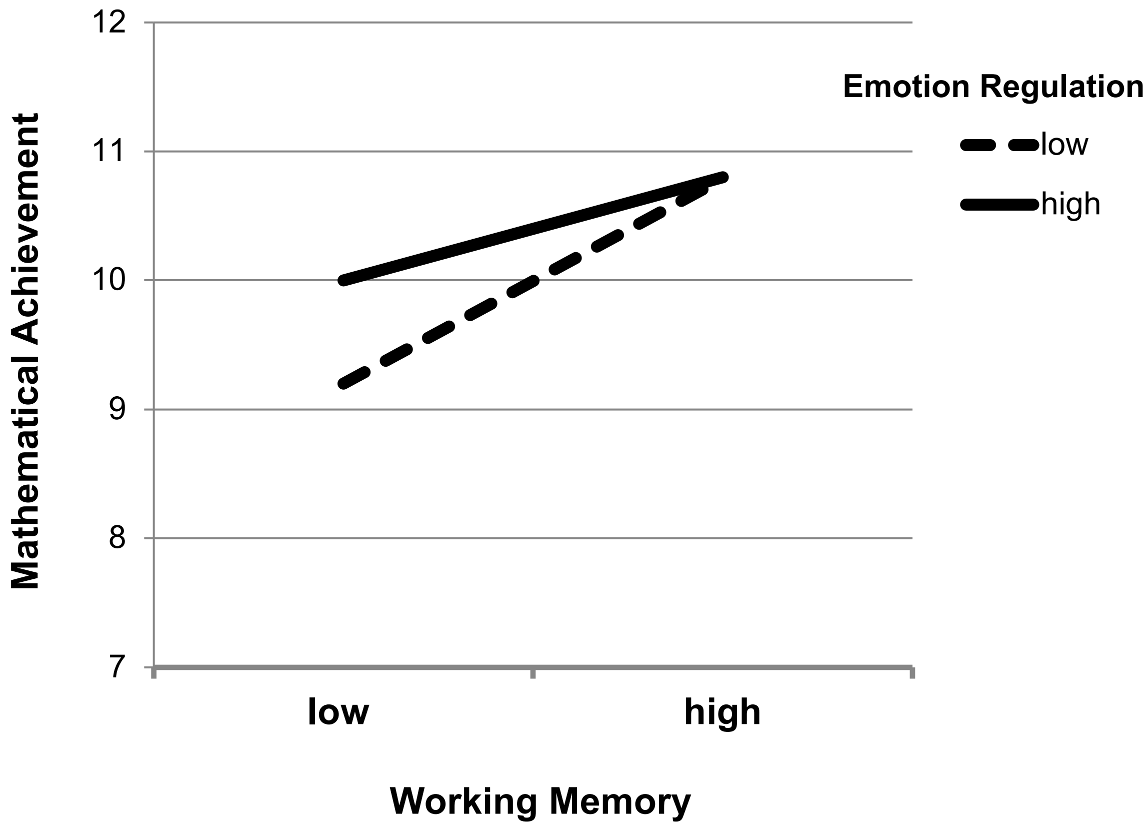
Figure 2. Participants' age-standardized mathematical achievement scores (M = 10, SD = 3) as a dependent variable of individual differences of WM. Separate graphs are shown for participants with one standard deviation above or below average emotion regulation and with one standard deviation above or below average WM.

Figure 1.

A) **B)**

Selected scenario for dealing with anger in children	Selected scenario for dealing with anxiety in adolescents
« Imagine that this boy is angry because another child broke his toy. What do you recommend this boy so that he will not be angry anymore? »	« Imagine that you are afraid because you will go to a new school soon and you do not know whether you will find friends. What could you do to become less afraid? »

Figure 2.



Supplementary Table 1.

Age and Grade Distribution of the Sample (n = 992)

Grade	n_{total}	Age_{Range}	Age_{Mean}	Age_{SD}
2	158	7;00 - 9;85	8.15	.54
3	150	7;73 - 10;72	9.12	.54
4	130	8;17 – 12;07	10.11	.64
5	125	9;92 – 13;37	11.16	.55
6	124	10;89 – 14;14	12.16	.61
7	105	11;85 – 15;55	13.31	.76
8	110	12;71 – 15;99	14.32	.68
9	90	13;59 – 15;98	15.04	.53

Note: Students are asked to either skip a grade or to repeat a year at the schools tested when outperforming or not meeting the academic goals of the year which explains the varying age range in each grade.

APPENDIX D: ERKLÄRUNG ZUR WISSENSCHAFTLICHEN LAUTERKEIT

Ich erkläre hiermit, dass die vorliegende Arbeit ohne die Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel selbstständig verfasst habe. Zu Hilfe genommene Quellen sind als solche gekennzeichnet. Die veröffentlichten oder zur Veröffentlichung in Zeitschriften eingereichten Manuskripte wurden in Zusammenarbeit mit den Koautoren erstellt und von keinem der Beteiligten an anderer Stelle publiziert, zur Publikation eingereicht, oder einer anderen Prüfungsbehörde als Qualifikationsarbeit vorgelegt. Es handelt sich dabei um folgende Manuskripte:

- **Kahl, T.,** Grob, A., Segerer, R., & Möhring, W. (2019). Executive Functions and Visual-Spatial Skills Predict Mathematical Achievement: Asymmetrical Associations Across Age. *Psychological Research*, 1-11. http://doi:10.1007/s00426-019-01249-4
- **Kahl, T,** Grob, A., Möhring, W. (2020). Does emotion regulation compensate deficits in various executive functions in children’s and adolescent’s mathematical achievement? *Manuscript in revision.*
- **Kahl, T.,** Segerer, R., Grob, A., & Möhring, W. (2020). Bidirectional associations among executive functions and visual-spatial skills and mathematical achievement in primary school students: Results from a longitudinal study. *Manuscript submitted for publication.*

Basel, 29. 01. 2021

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