Interaction-driven Floquet engineering of topological superconductivity in Rashba nanowires

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We analyze, analytically and numerically, a periodically driven Rashba nanowire proximity coupled to an *s*-wave superconductor using bosonization and renormalization group analysis in the regime of strong electron-electron interactions. Because of the repulsive interactions, the superconducting gap is suppressed, whereas the Floquet Zeeman gap is enhanced, resulting in a higher effective value of *g* factor compared to the noninteracting case. The flow equations for different coupling constants, velocities, and Luttinger-liquid parameters explicitly establish that even for small initial values of the Floquet Zeeman gap compared to the superconducting proximity gap, the interactions drive the system into the topological phase and the interband interaction term helps to achieve larger regions of the topological phase in parameter space.

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I. INTRODUCTION

In the past decade, a variety of experimentally relevant setups have been proposed to realize topological superconductivity [1,2]. The exotic topological phases include topological insulators [3–8], Majorana fermions [9–27], and parafermions [28–39]. Topological superconductors hosting Majorana bound states provide a platform for topological qubits [40,41], which have potential applications in quantum computation if missing quantum gates are supplemented [42–44]. Quantum systems driven out of equilibrium by an external field have also received rapidly increasing interest in recent years, for example, in the field of time crystals [45–48]. Combining these two research concepts gives rise to new phases such as Floquet Majorana bound states [49–56], Floquet topological insulators [57–61], and higher order Floquet topological insulators [62–66].

In this work, we present a comprehensive study of a setup consisting of an interacting Rashba nanowire (NW) proximity coupled to an *s*-wave superconductor and driven by a time-dependent magnetic field; see Fig. 1. This setup exhibits Floquet Majorana bound states (FMBSs) at each end of the NW if the Floquet Zeeman gap is larger than the superconducting gap [53]. However, high-amplitude time-dependent magnetic fields are not only difficult to apply but they also have detrimental effects on the superconductor; thus, the starting point of the current work is to focus on a low-amplitude magnetic field such that the Floquet Zeeman gap is small compared to the superconducting gap. Moreover, in low-dimensional systems, electron-electron interactions are important as even for weak interaction strength, the system changes drastically and cannot be described as Fermi liquid anymore. A generic interacting many-body system heats up to infinite temperature at sufficiently long times as described by the eigenstate thermalization hypothesis [67]. However, as demonstrated in Ref. [68], for a periodically driven quantum many-body system, in a strong interaction regime, the prethermal Floquet state can be stabilized. Therefore, we consider prethermal region, which means that the time period for the Floquet term is short compared to the heating timescale and study the setup in the presence of electron-electron interaction [12]. Using bosonization and renormalization group (RG) analysis [69–73], we show that even if the Floquet Zeeman gap is small compared to the superconducting gap, one can still obtain topological phases as the interaction renormalizes both superconducting and Floquet Zeeman terms. Repulsive electron-electron interactions renormalize the Floquet Zeeman gap, which is of Peierls type [74], and make it larger, whereas superconductivity gets suppressed such that the superconducting gap shrinks [12,73]. Thus, interactions helps to resolve the requirement of strong magnetic fields to obtain FMBSs in the setup. In this sense, we obtain similar results known from static systems, where the topological phases of the noninteracting systems remain qualitatively the

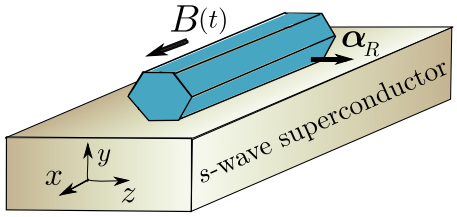

FIG. 1. Sketch of the setup consisting of a one-dimensional Rashba nanowire (blue cylinder) aligned along the *x* direction and brought into proximity to an *s*-wave superconductor (yellow slab). The uniform time-dependent magnetic field $\boldsymbol{B}(t)$ with frequency ω is applied along the NW axis and, thus, is applied perpendicular to the SOI vector $\boldsymbol{\alpha}_R$.

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same when interactions are turned on; in particular, Majorana bound states still exist but with parameters renormalized by interactions [12,75].

The outline of the paper is as follows. In Sec. II, we introduce the model for the double-NW system and the bosonization techniques to include electron-electron interactions. In Sec. III, we perform the RG analysis and calculate the RG flow equations. As a result, we obtain the topological phase diagram for interacting systems. Finally, in Sec. IV we conclude with a summary and outlook. The technical details are deferred to Appendixes A and B.

II. MODEL

We consider a one-band Rashba nanowire (NW) aligned along the x direction, which is proximity coupled to an s-wave superconductor. The spin-orbit interaction (SOI) vector with strength α_R defines the quantization axis along the z direction. We apply an external time-dependent periodic magnetic field $\boldsymbol{B}(t) = B\cos(\omega t)\hat{x}$, with magnitude B and frequency ω, along the axis of the NW. This choice ensures that $\boldsymbol{B}(t)$ is perpendicular to the Rashba SOI vector. The Hamiltonian contains three terms, namely, the kinetic energy term and SOI term as $H_{\rm kin}$, the superconducting pairing term $H_{\rm sc}$, and the time-dependent Zeeman term $H_Z(t)$:

$$\begin{aligned} H(t) &= H_{\rm kin} + H_{\rm sc} + H_Z(t), \\ H_{\rm kin} &= \sum_{\sigma}\int dx\,\Psi^{\dagger}_{\sigma}(x)\left[\frac{\hbar^2\hat{k}^2}{2m_0} - \mu\right]\Psi_{\sigma}(x) \\ &\quad - \alpha_R\sum_{\sigma\sigma'}\int dx\,\Psi^{\dagger}_{\sigma}(x)(\sigma_z)_{\sigma\sigma'}\hat{k}\,\Psi_{\sigma'}(x), \qquad (1) \\ H_{\rm sc} &= \frac{\Delta_{\rm sc}}{2}\sum_{\sigma\sigma'}\int dx\,\Psi_{\sigma}(x)(i\sigma_y)_{\sigma\sigma'}\Psi_{\sigma'}(x) + \text{H.c.}, \\ H_Z(t) &= 2t_F\cos(\omega t)\sum_{\sigma\sigma'}\int dx\,\Psi^{\dagger}_{\sigma}(x)\,(\sigma_x)_{\sigma\sigma'}\,\Psi_{\sigma'}(x), \end{aligned}$$

where $\Psi_\sigma(x)$ is the annihilation operator acting on an electron with spin $\sigma = \pm 1$ at position x, while the Pauli matrices $\sigma_{x,y,z}$ act on the spin space and $\hat{k} = -i\,\partial_x$ is the momentum operator. The amplitude of the time-dependent Floquet Zeeman term is given by $t_F = g\,\mu_B\,B/2$, where g and μ_B are the g factor and Bohr magneton, respectively. The proximity-induced superconducting pairing gap is of the size $\Delta_{\rm sc}$. The chemical potential μ is calculated from the SOI energy, $E_{\rm so} = \hbar^2\,k_{\rm so}^2/2m_0$, where $k_{\rm so} = m_0\,\alpha_R/\hbar^2$ is the SOI wave vector and m_0 is the mass.

We work in the Floquet formalism [49–52,76] to convert a time-dependent problem to a static one. The Hamiltonian is periodic in time, $H(t) = H(t+T)$, with period $T = 2\pi/\omega$. The frequency ω is chosen such that the Floquet Zeeman terms become resonant. As discussed in Ref. [53], this setup does not require the tuning of chemical potential to the SOI energy; instead the chemical potential μ has to be below the SOI energy such that the smallest Fermi wave vectors in both effective bands coincide (see Fig. 2). The eigenstates of Floquet operator, $H_F = H(t) - i\hbar\partial_t$, are given by periodic functions $e^{in\omega t}$, where the integer n labels different Floquet bands separated by an energy $\hbar\omega$. This periodicity allows

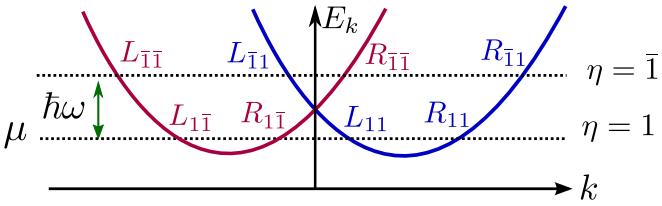

FIG. 2. Energy spectrum of Rashba NW consisting of spin up ($\sigma = 1$, blue) and spin down ($\sigma = \bar{1}$, red) branches. The chemical potential μ is set below the SOI energy and the driving frequency ω is chosen such that the smallest Fermi wave vector in the two Floquet bands, labeled by $\eta = \pm 1$, coincide. Close to the Fermi surfaces, the slowly varying right (left) fermionic field is denoted by $R_{\eta\sigma}$ ($L_{\eta\sigma}$).

us to write $H(t) = \sum_n H_n e^{in\omega t}$, where $H_n = \int_0^T e^{-in\omega t}H(t)/T$. The Floquet Hamiltonian H_F acquires a block diagonal form, written as

$$H_F = \begin{pmatrix} \cdots & H_0 + \hbar\omega & H_1 & 0 & \cdots \\ \cdots & H_{-1} & H_0 & H_1 & \cdots \\ \cdots & 0 & H_{-1} & H_0 - \hbar\omega & \cdots \end{pmatrix}, \qquad (2)$$

where the static terms in $H(t)$, i.e., $H_0 = H_{\rm kin} + H_{\rm sc}$, act on each Floquet band, while the oscillatory magnetic field couples different Floquet bands that are separated by an energy $\hbar\omega$. We note that we work in the quasiequilibrium limit such that we have a well-defined quasi-Fermi energy.

For simplicity, we consider only the single-photon absorption processes and thus focus only on the two lowest Floquet bands denoted by $\eta = 1$ and $\bar{1}$. This perturbative approach is justified in the weak driving regime in which the amplitude of driving is much smaller than the distance between subbands, $t_F \ll \hbar\omega$. Otherwise, one needs to include more Floquet bands and go beyond the rotating-wave approximation [55,77,78], which is beyond the scope of the present work. As a result, the effective Hamiltonian in the basis $\chi_{\eta\sigma} = (\Psi_{11}, \Psi_{1\bar{1}}, \Psi^{\dagger}_{11}, \Psi^{\dagger}_{1\bar{1}}, \Psi_{\bar{1}1}, \Psi_{\bar{1}\bar{1}}, \Psi^{\dagger}_{\bar{1}1}, \Psi^{\dagger}_{\bar{1}\bar{1}})$ is written as

$$\begin{aligned} H'_F = \int dx\,\chi^{\dagger}_{\eta\sigma}\Big[&\left(\frac{\hbar^2\hat{k}^2}{2m_0} - \mu - \alpha_R\sigma_z\hat{k}\right)\tau_z + \Delta_{\rm sc}\tau_y\sigma_y \\ &+ t_F\tau_z\sigma_x\eta_x + \frac{\hbar\omega}{2}(1-\eta_z)\tau_z\Big]\chi_{\eta\sigma}. \qquad (3) \end{aligned}$$

Here, the Pauli matrices $\tau_{x,y,z}$ and $\eta_{x,y,z}$ act on particle-hole and Floquet band spaces, respectively. We first linearize the spectrum close to the Fermi momenta. The Fermi points $k_{F,\eta\sigma r}$ of the Floquet band η with the spin σ have the form $k_{F,1\sigma\pm} = \sigma\,k_{\rm so} \pm k_{\rm so}\sqrt{1+\mu/E_{\rm so}}$ and $k_{F,\bar{1}\sigma\pm} = \sigma\,k_{\rm so} \pm k_{\rm so}\sqrt{1+(\mu+\hbar\omega)/E_{\rm so}}$. The resonance condition is satisfied, if $k_{F,1\bar{1}+} = k_{F,\bar{1}1-}$ and $k_{F,\bar{1}\bar{1}+} = k_{F,11-}$. We rewrite the fermionic fields in terms of slowly varying right- and left-moving fields by $R(x)$ and $L(x)$ as

$$\Psi_{11} = R_{11}e^{ik_{F,11+}} + L_{11}e^{ik_{F,11-}}, \qquad (4)$$

$$\Psi_{1\bar{1}} = R_{1\bar{1}}e^{ik_{F,1\bar{1}+}} + L_{1\bar{1}}e^{ik_{F,1\bar{1}-}}, \qquad (5)$$

$$\Psi_{\bar{1}1} = R_{\bar{1}1}e^{ik_{F,\bar{1}1+}} + L_{\bar{1}1}e^{ik_{F,\bar{1}1-}}, \qquad (6)$$

$$\Psi_{\bar{1}\bar{1}} = R_{\bar{1}\bar{1}}e^{ik_{F,\bar{1}\bar{1}+}} + L_{\bar{1}\bar{1}}e^{ik_{F,\bar{1}\bar{1}-}}. \qquad (7)$$

Further, we linearize the sum of kinetic energy and SOI terms, which takes the following form:

$$H'_{\rm kin} = -i\,\hbar\,v_F \sum_{\eta\sigma} \int dx\,[R^\dagger_{\eta\sigma}(x)\partial_x R_{\eta\sigma}(x) - L^\dagger_{\eta\sigma}(x)\partial_x L_{\eta\sigma}(x)], \tag{8}$$

where v_F is the Fermi velocity in the NW. The linearized form of the s-wave pairing term has the following form:

$$H'_{\rm sc} = \Delta_{\rm sc} \sum_{\eta} \int dx \left[R^\dagger_{\eta\bar{1}} L^\dagger_{\eta 1} - R^\dagger_{\eta 1} L^\dagger_{\eta\bar{1}} + \text{H.c.}\right]. \tag{9}$$

Finally, in terms of fermionic right and left movers, the Floquet Zeeman term that couples the lower and upper Floquet bands is given by

$$H'_Z = t_F \sum_{\eta} \int dx \left[R^\dagger_{\eta\bar{1}} L_{\bar{\eta}1} + \text{H.c.}\right]. \tag{10}$$

The spectrum of the linearized Hamiltonianconsists of states with energy dispersion $E_{1\pm\pm} = \pm\sqrt{(\hbar v_F k)^2 + (t_F \pm \Delta_{\rm sc})^2}$ and with energy dispersion $E_{2\pm} = \pm\sqrt{(\hbar v_F k)^2 + (\Delta_{\rm sc})^2}$. If the Floquet Zeeman gap exceeds the superconducting pairing gap [79], $t_F > \Delta_{\rm sc} > 0$, the system hosts two zero-energy bound states protected by an effective time-reversal symmetry $\mathcal{T} = -i\eta_z\sigma_y\mathcal{K}$, where $\mathcal{K}$ is the complex conjugation operator [53]. To include the electron-electron interactions, we consider only the low-lying excitations close to the Fermi level. As the particle-hole excitation are bosonic in nature, we bosonize the Hamiltonian by expressing the left- and right-moving fermions in terms of the charge $(\phi_{\eta c}, \theta_{\eta c})$ and spin $(\phi_{\eta s}, \theta_{\eta s})$ bosonic fields as follows:

$$\begin{aligned} R_{\eta\sigma} &= \frac{1}{\sqrt{2\pi\alpha}} e^{-\frac{i}{\sqrt{2}}[\phi_{\eta c} - \theta_{\eta c} + \sigma(\phi_{\eta s} - \theta_{\eta s})]}, \\ L_{\eta\sigma} &= \frac{1}{\sqrt{2\pi\alpha}} e^{\frac{i}{\sqrt{2}}[\phi_{\eta c} + \theta_{\eta c} + \sigma(\phi_{\eta s} + \theta_{\eta s})]}, \end{aligned} \tag{11}$$

where α is the short-distance cutoff of the theory which we assume to be equal to the lattice constant. The bosonic fields satisfy the commutation relation $[\phi_{\eta\beta}(x), \theta_{\eta'\beta}(x')] = i\,\pi\,\delta_{\eta\eta'}\,\text{sgn}(x' - x)$. The field $\phi_{\eta\beta}$ and $\theta_{\eta\beta}$ relate to the $\beta = c$ (charge) and $\beta = s$ (spin) density and current in the η band, respectively. We note that with this choice of commutators between bosonic fields one can neglect Klein factors; see Refs. [71,72]. Thus, we rewrite the linearized Hamiltonian in bosonized fields without Klein factors and obtain

$$\begin{aligned} H'_{\rm kin} = \sum_{\eta\beta} \int \frac{dx}{2\pi} \Bigg[u_{\eta\beta} \left[\frac{(\partial_x \phi_{\eta\beta})^2}{K_{\eta\beta}} + K_{\eta\beta} (\partial_x \theta_{\eta\beta})^2 \right] \\ + 2\,\tilde{u} (\partial_x \phi_{1c})(\partial_x \phi_{\bar{1}c}) \Bigg], \end{aligned} \tag{12}$$

where the indices of the bosonic fields $\phi_{\eta\beta}$ and $\theta_{\eta\beta}$ refer to $\beta = c\,(s)$ for the charge (spin) sectors of the η-Floquet band. We set $\hbar = 1$ here as well as in the following calculation. Further, $u_{\eta\beta}$ and $K_{\eta\beta}$ correspond to the velocity and Luttinger liquid (LL) parameter, respectively. The cross term, characterized by the velocity $\tilde{u}$, describes the repulsive interaction between the Floquet bands. To simplify the problem, we fix $K_{1\beta} = K_{\bar{1}\beta} = K_\beta$ and $u_{1\beta} = u_{\bar{1}\beta} = u_\beta$. For an ideal LL, $u_\beta/v_F = 1/K_\beta$ and $\tilde{u}/v_F = 1/K_c^2 - 1$, where v_F is the Fermi velocity [80,81]. Notably, we consider only the charge density interaction between the Floquet bands, as we are interested in the parameter regime in which interactions are spin-rotational symmetric with $K_s \sim 1$. The superconducting pairing term and the Floquet Zeeman term are rewritten as

$$H'_{\rm sc} = \sum_{\eta} \int dx\,\frac{2\,\Delta_{\rm sc}}{\pi\,\alpha} \cos(\sqrt{2}\,\theta_{\eta c}) \cos(\sqrt{2}\,\phi_{\eta s}), \tag{13}$$

$$H'_Z = \sum_{\eta} \int dx\,\frac{t_F}{\pi\,\alpha} \cos(\tilde{\varphi}_\eta/\sqrt{2}), \tag{14}$$

where $\tilde{\varphi}_\eta = \phi_{\eta c} - \theta_{\eta c} - \phi_{\eta s} + \theta_{\eta s} + \phi_{\bar{\eta}c} + \theta_{\bar{\eta}c} + \phi_{\bar{\eta}s} + \theta_{\bar{\eta}s}$ and α is the renormalized lattice constant of the NW, which grows under RG. As a result, the total effective Hamiltonian is given by $H' = H'_{\rm kin} + H'_{\rm sc} + H'_Z$. In addition, there is a backscattering term involving spin-up and spin-down electrons for each Floquet band separately of the form

$$\begin{aligned} H_g &= \sum_{\eta} g_\eta \int dx\, R^\dagger_{\eta 1} L_{\eta 1} L^\dagger_{\eta\bar{1}} R_{\eta\bar{1}} + \text{H.c.} \\ &= \sum_{\eta} \frac{g_\eta}{2\pi^2\alpha^2} \int dx \cos(2\sqrt{2}\phi_{\eta s}), \end{aligned} \tag{15}$$

where g_η is the coupling strength. However, in the regime of $K_s \geqslant 1$, this term is either marginal or irrelevant [73]. Thus, we drop this term in the following calculation.

III. RG EQUATIONS AND ANALYSIS

Next, we derive the RG equations for different coupling constants, velocities, and LL parameters in H'. Most important, we are interested in finding out whether it is possible to reach the topological phase even if the initial (nonrenormalized) value of t_F is smaller than $\Delta_{\rm sc}$ and, thus, one expects the system to be in the trivial phase in the absence of interactions. To compare different competing terms, we work in dimensionless units and define $\tilde{\Delta}_{\rm sc} = \alpha\,\Delta_{\rm sc}/u_c$ and $\tilde{t}_F = \alpha\,t_F/u_c$. Further, performing an RG analysis, our goal is to determine when $\tilde{t}_F$ dominates over $\tilde{\Delta}_{\rm sc}$. In RG language, this means finding the parameter regime when the system reaches the strong coupling limit, i.e., $\tilde{t}_F \sim 1$ and $\tilde{t}_F > \tilde{\Delta}_{\rm sc}$.

To derive the RG equations, first we calculate different correlation functions between $\phi_{\eta\beta}$ and $\theta_{\eta\beta}$ for the kinetic part $H'_{\rm kin}$ using Green's functions (see Appendix A). Subsequently, utilizing the operator product expansion (OPE) method [69], we compute the RG equations up to the second order in Appendix B. However, to solve the problem analytically, we consider that the LL parameters and velocities do not flow and write the first-order RG equations as

$$\begin{aligned} d\tilde{\Delta}_{\rm sc}/dl &= [2 - (\lambda_+ + \lambda_- + K_s)/2]\,\tilde{\Delta}_{\rm sc}, \\ d\tilde{t}_F/dl &= [2 - (2\,\nu_+ + 2\,\lambda_- + \nu_s)/4]\,\tilde{t}_F. \end{aligned} \tag{16}$$

Here, we define $\lambda_\pm = (u_c \pm \tilde{u}\,K_c)/(2\,K_c\,u_\pm)$, $u_\pm^2 = u_c^2 \pm u_c\,\tilde{u}\,K_c$, $\nu_s = K_s + 1/K_s$, and $\nu_+ = K_c\,u_c/2\,u_+$. The dimensionless RG flow parameter is $l = \ln(\alpha/\alpha_0)$, where α_0 is

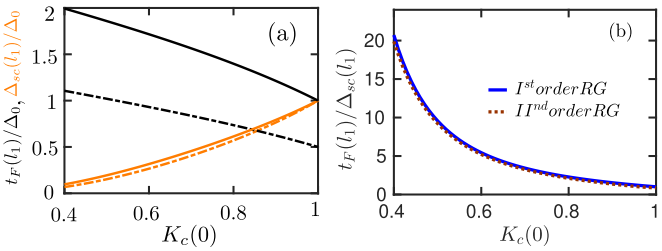

FIG. 3. (a) The strength of the physical Floquet Zeeman term $t_F(l_1)/\Delta_0$ (black) and superconducting pairing term $\Delta_{sc}(l_1)/\Delta_0$ (orange) as a function of $K_c(0)$ for two initial values of $t_F(0)/\Delta_{sc}(0) = 0.5$ (dashed) and 1 (solid) for $\Delta_0 = 0.01\, u_c(0)/a_0$ [see Eq. (17)]. The system is in the topological phase when $t_F(l_1) > \Delta_{sc}(l_1)$. As the ratio $t_F(0)/\Delta_{sc}(0)$ decreases, we need stronger interactions to reach the topological phase. (b) To compare the results of first-order RG [blue curve, see Eq. (17)] and second-order RG [red curve, see Eq. (B40)] flows, we plot the ratio $t_F(l_1)/\Delta_{sc}(l_1)$ as function of initial value of K_c for $t_F(0)/\Delta_{sc}(0) = 1$. The second-order RG gives only small corrections to the first-order RG results. The correction is most relevant close to the point $K_c = 1$, where analytical results predict that the system is topological [$t_F(l_1) > \Delta_{sc}(l_1)$]; however, due to the second-order correction, the system is in the trivial phase. Other parameters used in the second-order RG numerics are $K_s(0) = 1$, $u_c(0)/v_F = 1/K_c(0)$, $u_s(0)/v_F = 1$, and $\tilde{u}(0)/v_F = 1/K_c^2(0) - 1$.

the bare value of the lattice constant. Notably, the velocities and LL parameters do not flow under first-order RG. We are interested in the gapped regime where $\tilde{\Delta}_{sc}$ and $\tilde{t}_F$ are RG relevant (terms growing as a function of l). Moreover, the superconducting pairing term $\tilde{\Delta}_{sc}$ ($\tilde{t}_F$) is RG relevant for $\lambda_+ + \lambda_- + K_s < 4$ ($2\nu_+ + 2\lambda_- + \nu_s < 8$). To estimate the relevant parameter regime for K_c, we consider the limiting case $\tilde{u} = 0$ and $K_s = 1$, which results in $K_c > 1/3$ ($K_c + 1/K_c < 6$) for $\tilde{\Delta}_{sc}$ ($\tilde{t}_F$) to be RG relevant, thus providing us the lower bound. Therefore, in what follows, we focus on the repulsive interaction regime with $1/3 < K_c < 1$.

We solve the first-order RG [see Eq. (16)] for $\tilde{\Delta}_{sc}$ and $\tilde{t}_F$ considering that the latter reaches the strong coupling limit, i.e., $t_F(l_1) = 1$, where l_1 is the dimensionless RG flow parameter. Therefore, one obtains $l_1 = 4 \ln \tilde{t}_F(0)/(2\,\nu_+ + 2\,\lambda_- + \nu_s - 8)$ and $\tilde{\Delta}_{sc}(l_1) = \tilde{\Delta}_{sc}(0)\,[\tilde{t}_F(0)]^{\gamma}$, where $\gamma = (4 - \lambda_+ - \lambda_- - K_s)/(\nu_+ + \lambda_- + \nu_s/2 - 4)$. For $K_s = 1$, the values of the physical superconducting pairing gap and Floquet Zeeman gap are given by

$$\Delta_{sc}(l_1) = \frac{u_c(0)\,\tilde{\Delta}_{sc}(0)}{\alpha_0}\,[\tilde{t}_F(0)]^{\frac{\lambda_+ + \lambda_- - 1}{3 - \nu_+ - \lambda_-}},$$

$$t_F(l_1) = \frac{u_c(0)}{\alpha_0}\,[\tilde{t}_F(0)]^{\frac{2}{3 - \nu_+ - \lambda_-}},$$

$$\nu_+ + \lambda_- = \left(K_c(0)\Big/\sqrt{2 - K_c^2(0)} + 1\right)/2,$$

$$\lambda_+ + \lambda_- = \left(\sqrt{2 - K_c^2(0)}\Big/K_c(0) + 1\right)/2. \qquad (17)$$

In the presence of strong repulsive interaction, the Floquet Zeeman gap tends to exceed the superconducting pairing gap even if this was not the case for the initial values; see Fig. 3(a). Thus, interaction drives the system into the topological phase by satisfying the criterion $t_F > \Delta_{sc}$. Generally, there is a

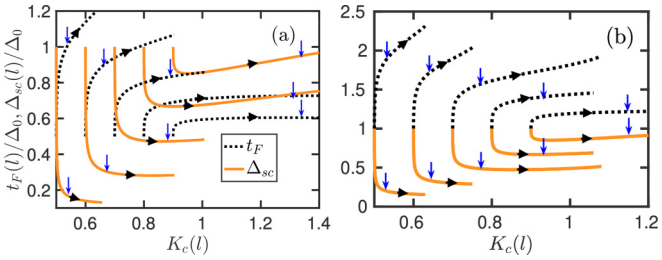

FIG. 4. RG flow of the physical gaps Δ_{sc}/Δ_0 (orange solid) and t_F/Δ_0 (black dotted) as a function of LL parameter K_c after solving the second-order RG equations for (a) $t_F(0)/\Delta_{sc}(0) = 0.5$ and (b) $t_F(0)/\Delta_{sc}(0) = 1$. The black arrows show the RG flow direction and the vertical blue arrows denote the point when $\tilde{t}_F = 1$. For the values $K_c(0) > 0.8$ [$K_c(0) > 1$], the system is in the trivial phase for panel (a) [(b)]. The LL parameters stay close to the initial value before reaching the blue vertical arrow, which justifies the assumption of first order RG. The smaller the ratio $t_F(0)/\Delta_{sc}(0) < 1$, the stronger interactions are required to reach the topological phase. Other initial conditions are chosen as $K_c(0) = 0.5, 0.6, 0.7, 0.8, 0.9$ from left to right, $K_s(0) = 1$, $u_c(0)/v_F = 1/K_c(0)$, $u_s(0)/v_F = 1$, and $\tilde{u}(0)/v_F = 1/K_c^2(0) - 1$.

crossover between t_F and Δ_{sc} depending upon the ratio of their initial values. When $t_F(0)/\Delta_{sc}(0) = 1$, the crossover happens at $K_c = 1$ and thus for $K_c < 1$ ($K_c > 1$), the system is in topological (trivial) phase. As the ratio $t_F(0)/\Delta_{sc}(0)$ decreases, the crossover point for K_c, which can be calculated by putting $\Delta_{sc}(l_1) = t_F(l_1)$ in Eq. (17), shifts to smaller values, indicating that one requires stronger repulsive interaction in the NW to reach the topological phase.

To check that the flow of LL parameters and velocities do not affect the analytical results obtained in the first-order, we consider the RG equations up to the second order (see Appendix B). As these are involved coupled differential equations, we solve them numerically. Generally, working with the second-order RG equations leads only to small corrections to the first-order result; see Fig. 3(b). However, close to $K_c = 1$, the corrections are more relevant, rendering the system trivial. Examining the RG flow of the physical gaps in the second-order theory [see Fig. (4)], we observe that, up to the strong coupling limit point, K_c stays close to the initial value and hardly flows. This justifies our assumption of focusing only on the first-order RG equations. Also the effect of the ratio $t_F(0)/\Delta_{sc}(0)$ on the physical gaps in both RG orders matches exactly [see Figs. 3(a) and 4]. Generally, the repulsive interaction suppresses the superconducting gap [12,73]. In contrast to that, the Floquet Zeeman gap is enhanced [74], which results in a higher value of the effective g factor compared to the noninteracting case. One can understand the enhancement in a simple way: The Floquet Zeeeman term has a form similar to a spin-flip backscattering term. As discussed in Ref. [71], the backscattering amplitude increases as interactions get stronger, thus resulting in larger gaps in comparison to the noninteracting case. This allows one to satisfy the topological criterion $t_F(l_1) > \Delta_{sc}(l_1)$ even if the noninteracting bare value of $t_F(l_0)$ is smaller than $\Delta_{sc}(l_0)$.

Finally, we also compute the qualitative phase diagram as a function of the ratio $\Delta_{sc}(0)/t_F(0)$ and the initial LL parameter value $K_c(0)$ (see Fig. 5). For a noninteracting system ($K_c = 1$),

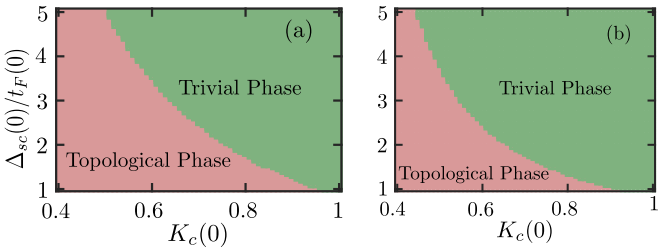

FIG. 5. (a) Qualitative phase diagram as a function of $\Delta_{sc}(0)/t_F(0)$ and $K_c(0)$ in the presence of the interband interaction cross term, $\tilde{u}(0)/v_F = 1/K_c^2(0) - 1$, obtained numerically by solving second-order RG equation [see Eq. (B40)]. (b) The same in the absence of $\tilde{u}(0)$. If the topological criterion $t_F(l_1) > \Delta_{sc}(l_1)$ is satisfied, the system is in the topological phase (red area), but otherwise in the trivial phase (green area). The area corresponding to the topological phase is substantially increased in the presence of $\tilde{u}$. Other initial conditions: $K_s(0) = 1$, $u_c(0)/v_F = 1/K_c(0)$, and $u_s(0)/v_F = 1$.

if $\Delta_{sc}(0)/t_F(0) \geqslant 1$, system is always in the trivial phase. However, when $K_c < 1$, the topological phase emerges due to the presence of interactions. As the ratio $\Delta_{sc}(0)/t_F(0)$ increases, we require lower values of K_c, i.e., stronger interaction strength to reach the topological phase. We also would like to emphasize the role played by the interband interaction cross term $\tilde{u}$. If $\tilde{u}$ is absent [see Fig. 5(b)], the phase boundary between the topological and trivial phase is shifted to lower values of $K_c(0)$ compared to the case of finite $\tilde{u}$ [see Fig. 5(a)]. Thus, the interband interaction term results in larger parameter space corresponding to the topological phase.

In experimental setups, depending on the coupling between the NW and the s-wave superconductor, the proximity-induced superconducting pairing gap Δ_{sc} is of the order of 0.1 meV [13]. For a g factor of 10, it would require us to apply a magnetic field of the strength of around 0.3 T. As interactions effectively renormalize the g factor by a factor of 10 (see Fig. 3), one can instead work with magnetic fields of the strength as small as 0.03 T. The oscillation frequency ω is of the order of E_{so}, which implies it should be in the GHz range. In addition, we note that nonequilibrium heating, an issue generally pertinent to Floquet systems, is important here as well. However, this difficulty can be overcome by involving, e.g., adiabatic buildup of states or engineered baths [57,82–84].

IV. CONCLUSIONS

We studied the effects of electron-electron interactions on a driven Rashba NW with proximity gap and analyzed the interplay between the Floquet Zeeman term and superconducting pairing term using bosonization techniques and RG analysis. The repulsive Coloumb interaction drives the system into the topological phase even if the initial (bare) value of the Floquet Zeeman gap is smaller than the superconducting proximity gap. Under RG flow, the physical Floquet Zeeman gap is enhanced whereas the proximity gap is suppressed, pushing the system into the topological phase.

Important features of the proposed setup are that it does not require the tuning of the chemical potential close to the spin-orbit energy and that it exhibits topological superconductivity, and thus Floquet Majorana bound states, even for weak strengths of the driving magnetic field due to the presence of electron-electron interactions.

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APPENDIX A: GREEN'S FUNCTIONS OF UNPERTURBED HAMILTONIAN H_{kin} IN THE PRESENCE OF INTERBAND CROSS TERM

In this Appendix, we derive first the Matsubara Green's functions and later different correlation functions of $\phi - \theta$ fields in the presence of the cross term for the unperturbed Hamiltonian in H'_{kin} defined in Eq. (4) of the main text. The quadratic part of the Matsubara action is written as $S = S_c + S_s$. Here we split the action into two parts S_c and S_s, corresponding to the charge and spin sectors, respectively, with the following form:

$$S_c = \sum_{\eta=1,\bar{1}} \frac{1}{\pi} \int dx\, d\tau \left[i\partial_x\theta_{\eta c}\, \partial_\tau\phi_{\eta c} - u_{\eta c}\frac{(\partial_x\phi_{\eta c})^2}{2K_{\eta c}} - \frac{u_{\eta c}K_{\eta c}}{2}(\partial_x\theta_{\eta c})^2 \right] - \frac{\tilde{u}}{\pi}\int dx\, d\tau\ (\partial_x\phi_{1c})(\partial_x\phi_{\bar{1}c}),$$

$$S_s = \sum_{\eta=1,\bar{1}} \frac{1}{\pi} \int dx\, d\tau \left[i\partial_x\theta_{\eta s}\, \partial_\tau\phi_{\eta s} - u_{\eta c}\frac{(\partial_x\phi_{\eta s})^2}{2K_{\eta s}} - \frac{u_{\eta s}K_{\eta s}}{2}(\partial_x\theta_{\eta s})^2 \right]. \tag{A1}$$

The correlation functions for the spin sector are unchanged as the cross term between different Floquet bands appears in the charge sector only and has the following form:

$$\langle[\phi_{\eta s}(x,\tau) - \phi_{\eta s}(0,0)]^2\rangle_0 = K_{\eta s}\ln\frac{\sqrt{x^2 + u_{\eta s}^2\tau^2}}{\alpha}, \qquad \langle[\phi_{\eta s}(0,0)]^2\rangle_0 \sim -\frac{K_{\eta s}}{2}\ln\alpha,$$

$$\langle[\theta_{\eta s}(x,\tau) - \theta_{\eta s}(0,0)]^2\rangle_0 = \frac{1}{K_{\eta s}}\ln\frac{\sqrt{x^2 + u_{\eta s}^2\tau^2}}{\alpha}, \qquad \langle[\theta_{\eta s}(0,0)]^2\rangle_0 \sim -\frac{1}{2K_{\eta s}}\ln\alpha, \tag{A2}$$

where the expectation value $\langle\cdots\rangle_0$ is taken with respect to the LL action S_s defined in Eq. (A1). For the charge sector, we calculate the action in Fourier space which then becomes

$$S_c = \frac{1}{2}\int \frac{dq\,d\omega}{4\pi^2}\,\Phi_c^\dagger(q,\omega)\check{G}_c^{-1}(q,\omega)\Phi_c(q,\omega), \tag{A3}$$

where $\Phi_c(q,\omega) = [\phi_{1c}(q,\omega),\ \theta_{1c}(q,\omega),\ \phi_{\bar{1}c}(q,\omega),\ \theta_{\bar{1}c}(q,\omega)]$. Here, we use the definition $\phi_{\eta c}(q,\omega) = \int dx\,d\tau\,\phi_{\eta c}(x,\tau)\,e^{i(qx-\omega\tau)}$. The action S_c [see Eq. (A1)] determines the inverse Green's function as

$$\check{G}_c^{-1}(q,\omega) = \frac{1}{\pi}\begin{pmatrix} q^2u_{1c}K_{1c}^{-1} & iq\omega & q^2\tilde{u} & 0 \\ iq\omega & q^2u_{1c}K_{1c} & 0 & 0 \\ q^2\tilde{u} & 0 & q^2u_{\bar{1}c}K_{\bar{1}c}^{-1} & iq\omega \\ 0 & 0 & iq\omega & q^2u_{\bar{1}c}K_{\bar{1}c} \end{pmatrix}. \tag{A4}$$

We calculate the inverse of Eq. (A4) to obtain the matrix of Green's functions for the charge sector, which yields

$$\check{G}_c(q,\omega) = \begin{pmatrix} G_{\phi_{1c}\phi_{1c}} & G_{\phi_{1c}\theta_{1c}} & G_{\phi_{1c}\phi_{\bar{1}c}} & G_{\phi_{1c}\theta_{\bar{1}c}} \\ G_{\theta_{1c}\phi_{1c}} & G_{\theta_{1c}\theta_{1c}} & G_{\theta_{1c}\phi_{\bar{1}c}} & G_{\theta_{1c}\theta_{\bar{1}c}} \\ G_{\phi_{\bar{1}c}\phi_{1c}} & G_{\phi_{\bar{1}c}\theta_{1c}} & G_{\phi_{\bar{1}c}\phi_{\bar{1}c}} & G_{\phi_{\bar{1}c}\theta_{\bar{1}c}} \\ G_{\theta_{\bar{1}c}\phi_{1c}} & G_{\theta_{\bar{1}c}\theta_{1c}} & G_{\theta_{\bar{1}c}\phi_{\bar{1}c}} & G_{\theta_{\bar{1}c}\theta_{\bar{1}c}} \end{pmatrix}. \tag{A5}$$

For the calculations to follow below, we need to compute the correlations $\langle\phi_{\eta c}\phi_{\eta c}\rangle_0$ and $\langle\theta_{\eta c}\theta_{\eta c}\rangle_0$ in each of the η-NW and $\langle\phi_{\eta c}\phi_{\bar{\eta}c}\rangle_0$ and $\langle\theta_{\eta c}\theta_{\bar{\eta}c}\rangle_0$ in between $\eta-\bar{\eta}$ NWs. We write the corresponding Green's functions as

$$\begin{aligned}
G_{\theta_{\eta c}\theta_{\eta c}}(\omega,q) &= -\frac{\pi\left(K_{\eta c}K_{\bar{\eta}c}q^2\tilde{u}^2u_{\bar{\eta}c} - q^2u_{\eta c}u_{\bar{\eta}c}^2 - u_{\eta c}\,\omega^2\right)}{\left[\left(\omega^2+u_{\eta c}^2\,q^2\right)\left(\omega^2+u_{\bar{\eta}c}^2\,q^2\right)\right]K_{\eta c} - K_{\eta c}^2K_{\bar{\eta}c}\,q^4\,\tilde{u}^2\,u_{\eta c}\,u_{\bar{\eta}c}}, \\
G_{\theta_{\eta c}\theta_{\bar{\eta}c}}(\omega,q) &= \frac{\pi\,\tilde{u}\,\omega^2}{\left(\omega^2+u_{\eta c}^2\,q^2\right)\left(\omega^2+u_{\bar{\eta}c}^2\,q^2\right) - K_{\eta c}\,K_{\bar{\eta}c}\,q^4\tilde{u}^2\,u_{\eta c}\,u_{\bar{\eta}c}}, \\
G_{\phi_{\eta c}\phi_{\eta c}}(\omega,q) &= \frac{\pi\,K_{\eta c}\,u_{\eta c}\left(\omega^2+q^2\,u_{\bar{\eta}c}^2\right)}{\left(\omega^2+u_{\eta c}^2\,q^2\right)\left(\omega^2+u_{\bar{\eta}c}^2\,q^2\right) - K_{\eta c}\,K_{\bar{\eta}c}\,q^4\,\tilde{u}^2\,u_{\eta c}\,u_{\bar{\eta}c}}, \\
G_{\phi_{\eta c}\phi_{\bar{\eta}c}}(\omega,q) &= -\frac{\pi\,K_{\eta c}\,K_{\bar{\eta}c}\,q^2\,u_{\eta c}\,u_{\bar{\eta}c}\tilde{u}}{\left(\omega^2+u_{\eta c}^2\,q^2\right)\left(\omega^2+u_{\bar{\eta}c}^2\,q^2\right) - K_{\eta c}\,K_{\bar{\eta}c}\,q^4\,\tilde{u}^2\,u_{\eta c}\,u_{\bar{\eta}c}}.
\end{aligned} \tag{A6}$$

These expressions can be simplified in case of a band symmetry, $u_{\bar{1}c} = u_{1c} = u_c$, and $K_{1c} = K_{\bar{1}c} = K_c$, as follows:

$$\begin{aligned}
G_{\theta_{\eta c}\theta_{\eta c}}(\omega,q) &= \frac{\pi}{2K_c}\left(\frac{u_c+\tilde{u}K_c}{\omega^2+u_+^2q^2} + \frac{u_c-\tilde{u}K_c}{\omega^2+u_-^2q^2}\right), \quad G_{\theta_{\eta c}\theta_{\bar{\eta}c}}(\omega,q) = \frac{\pi}{2K_c}\left(\frac{\tilde{u}\,K_c+u_c}{\omega^2+u_+^2q^2} + \frac{\tilde{u}\,K_c-u_c}{\omega^2+u_-^2q^2}\right), \\
G_{\phi_{\eta c}\phi_{\eta c}}(\omega,q) &= \frac{K_c}{2}\left(\frac{\pi\,u_c}{\omega^2+q^2\,u_+^2} + \frac{\pi\,u_c}{\omega^2+q^2u_-^2}\right), \quad G_{\phi_{\eta c}\phi_{\bar{\eta}c}}(\omega,q) = \frac{\pi K_c\,u_c}{2}\left(\frac{1}{\omega^2+q^2\,u_+^2} - \frac{1}{\omega^2+q^2\,u_-^2}\right),
\end{aligned} \tag{A7}$$

where we define $u_\pm^2 = u_c^2 \pm K_cu_c\,\tilde{u}$. If the cross term is absent, $\tilde{u}=0$, we get $u_\pm = u_c$ and recover the usual form for the correlation functions:

$$\begin{aligned}
G_{\theta_{\eta c}\theta_{\eta c}}(\omega,q) &= \frac{1}{K_c}\frac{\pi u_c}{\omega^2+u_c^2q^2}, \quad G_{\theta_{\eta c}\theta_{\bar{\eta}c}}(\omega,q) = 0, \\
G_{\phi_{\eta c}\phi_{\eta c}}(\omega,q) &= \frac{K_c}{2}\frac{\pi u_c}{\omega^2+u_c^2q^2}, \quad G_{\phi_{\eta c}\phi_{\bar{\eta}c}}(\omega,q) = 0.
\end{aligned} \tag{A8}$$

In the time-coordinate representation, $\pi u/(\omega^2+u^2q^2)$ leads to $F_1(x,u\tau) = \ln(\sqrt{x^2+u^2\tau^2}\,/\alpha)$ [71]. Thus, for the nonzero cross term, $\tilde{u}\neq 0$, we get by analogy

$$\begin{aligned}
G_{\theta_{\eta c}\theta_{\eta c}} &\equiv \frac{1}{2K_c}\frac{u_c+\tilde{u}K_c}{u_+}F_1(x,u_+\tau) + \frac{1}{2K_c}\frac{u_c-\tilde{u}K_c}{u_-}F_1(x,u_-\tau), \\
G_{\theta_{\eta c}\theta_{\bar{\eta}c}} &\equiv \frac{1}{2K_c}\frac{\tilde{u}K_c+u_c}{u_+}F_1(x,u_+\tau) - \frac{1}{2K_c}\frac{u_c-\tilde{u}K_c}{u_-}F_1(x,u_-\tau),
\end{aligned}$$

$$G_{\phi_{\eta c}\phi_{\eta c}} \equiv \frac{K_c\,u_c}{2u_+}F_1(x,u_+\tau)+\frac{K_c\,u_c}{2u_-}F_1(x,u_-\tau),$$
$$G_{\phi_{\eta c}\phi_{\bar{\eta} c}} \equiv \frac{K_c\,u_c}{2\,u_+}F_1(x,u_+\tau)-\frac{K_c\,u_c}{2\,u_-}F_1(x,u_-\tau). \tag{A9}$$

Therefore, in the presence of the interband cross term, the modified form of the correlation functions is given by

$$\langle[\theta_{\eta c}(x,\tau)-\theta_{\eta c}(0,0)]^2\rangle_0 = \lambda_+\ln\frac{\sqrt{x^2+u_+^2\tau^2}}{\alpha}+\lambda_-\ln\frac{\sqrt{x^2+u_-^2\tau^2}}{\alpha},$$
$$\langle[\theta_{\eta c}(x,\tau)-\theta_{\bar{\eta} c}(0,0)]^2\rangle_0 = \lambda_+\ln\frac{\sqrt{x^2+u_+^2\tau^2}}{\alpha}-\lambda_-\ln\frac{\sqrt{x^2+u_-^2\tau^2}}{\alpha},$$
$$\langle[\phi_{\eta c}(x,\tau)-\phi_{\eta c}(0,0)]^2\rangle_0 = \frac{K_c\,u_c}{2\,u_+}\ln\frac{\sqrt{x^2+u_+^2\tau^2}}{\alpha}+\frac{K_c\,u_c}{2\,u_-}\ln\frac{\sqrt{x^2+u_-^2\tau^2}}{\alpha},$$
$$\langle[\phi_{\eta c}(x,\tau)-\phi_{\bar{\eta} c}(0,0)]^2\rangle_0 = \frac{K_c\,u_c}{2\,u_+}\ln\frac{\sqrt{x^2+u_+^2\tau^2}}{\alpha}-\frac{K_c\,u_c}{2\,u_-}\ln\frac{\sqrt{x^2+u_-^2\tau^2}}{\alpha}, \tag{A10}$$

where we define $\lambda_\pm = \frac{1}{2K_c}\frac{u_c\pm\tilde{u}K_c}{u_\pm}$. We will utilize these correlation functions in the next section to obtain the RG flow equations for different system parameters.

APPENDIX B: RG EQUATIONS FOR COUPLING CONSTANTS, VELOCITIES, AND LL PARAMETERS

1. RG equations for $H'_{\rm sc}$

Below we show the calculation of RG equations for two terms, namely $H'_{\rm sc}$ and H'_Z. We define complex coordinates $(z_{\kappa\nu},\bar{z}_{\kappa\nu})$ as $z_{\kappa\nu}=-i\,x_\kappa+u_\nu\tau_\kappa$ and $\bar{z}_{\kappa\nu}=i\,x_\kappa+u_\nu\tau_\kappa$, where $\nu\in\{c,s,+,-\}$ is the index for different velocities and κ is the index for position-time coordinates. We also define the center-of-mass coordinate for charge-spin sector as $(z_{1\nu},z_{2\nu})\Rightarrow(\chi_\nu,\zeta_\nu)$, where $\chi_\nu=(z_{1\nu}-z_{2\nu})$ and $\zeta_\nu=(z_{1\nu}+z_{2\nu})/2$, and $(\bar{z}_{1\nu},\bar{z}_{2\nu})\Rightarrow(\bar{\chi}_\nu,\bar{\zeta}_\nu)$, where $\bar{\chi}_\nu=(\bar{z}_{1\nu}-\bar{z}_{2\nu})$ and $\bar{\zeta}_\nu=(\bar{z}_{1\nu}+\bar{z}_{2\nu})/2$.

First, we focus on the superconducting term $H'_{\rm sc}=\Delta_{\rm sc}\sum_\eta(\cos[\sqrt{2}\,(\theta_{\eta c}+\phi_{\eta s})]+\cos[\sqrt{2}\,(\theta_{\eta c}-\phi_{\eta s})])/\pi\,\alpha$ and obtain the RG flow equations for a dimensionless coupling constant $\tilde{\Delta}_{\rm sc}=\Delta_{\rm sc}\,\alpha/u_c$. Also, below we show the calculation just for the first cosine term; however, we obtain the same corrections from the second cosine term. We make use of the operator product expansion from Refs. [69,73] and expanded the partition function in powers of the cosine term characterized by a real constant λ up to the second order:

$$Z_a = Z_{0a}\Bigg\langle 1-\sum_\eta\frac{\tilde{\Delta}_{\rm sc}u_c}{\pi\,\alpha^2}\int dx\,d\tau\;\cos(\lambda[\theta_{\eta c}(x,\tau)+\phi_{\eta s}(x,\tau)])+\frac{(\tilde{\Delta}_{\rm sc})^2\,u_c^2}{2\,\pi^2\,\alpha^4}\int dx_1\,dx_2\,d\tau_1\,d\tau_2\;\cos(\lambda\,[\theta_{\eta c}(x_1,\tau_1)$$
$$+\,\phi_{\eta s}(x_1,\tau_1)])\cos(\lambda\,[\theta_{\eta c}(x_2,\tau_2)+\phi_{\eta s}(x_2,\tau_2)])+\cdots\Bigg\rangle_0. \tag{B1}$$

The second term has the following form in complex coordinates

$$\cos(\lambda\,[\theta_{\eta c}(z_{1c},\bar{z}_{1c})+\phi_{\eta s}(z_{1s},\bar{z}_{1s})])\,\cos(\lambda\,[\theta_{\eta c}(z_{2c},\bar{z}_{2c})+\phi_{\eta s}(z_{2s},\bar{z}_{2s})])$$
$$\simeq(e^{i\,\lambda[\theta_{\eta c}(z_{1c},\bar{z}_{1c})+\phi_{\eta s}(z_{1s},\bar{z}_{1s})]}e^{-i\,\lambda[\theta_{\eta c}(z_{2c},\bar{z}_{2c})+\phi_{\eta s}(z_{2s},\bar{z}_{2s})]}+e^{-i\,\lambda[\theta_{\eta c}(z_{1c},\bar{z}_{1c})+\phi_{\eta s}(z_{1s},\bar{z}_{1s})]}e^{i\,\lambda[\theta_{\eta c}(z_{2c},\bar{z}_{2c})+\phi_{\eta s}(z_{2s},\bar{z}_{2s})]})/4$$
$$\simeq\frac{1}{2(|\chi_+|/\alpha)^{\lambda^2\lambda_+/2}(|\chi_-|/\alpha)^{\lambda^2\lambda_-/2}(|\chi_s|/\alpha)^{\lambda^2K_s/2}}\Bigg(1-\lambda^2|\chi_c|^2\,[J_{\theta_{\eta c}}\bar{J}_{\theta_{\eta c}}]_{(\zeta_c,\bar{\zeta}_c)}-\lambda^2|\chi_s|^2\,[J_{\phi_{\eta s}}\bar{J}_{\phi_{\eta s}}]_{(\zeta_s,\bar{\zeta}_s)}$$
$$+\frac{\lambda^2}{2}\big[(\chi_c)^2\big[J^2_{\theta_{\eta c}}\big]_{(\zeta_c,\bar{\zeta}_c)}+(\bar{\chi}_c)^2\big[\bar{J}^2_{\theta_{\eta c}}\big]_{(\zeta_c,\bar{\zeta}_c)}+(\chi_s)^2\big[J^2_{\phi_{\eta s}}\big]_{(\zeta_s,\bar{\zeta}_s)}+(\bar{\chi}_s)^2\big[\bar{J}^2_{\phi_{\eta s}}\big]_{(\zeta_s,\bar{\zeta}_s)}\big]+\cdots\Bigg), \tag{B2}$$

where $J_{\phi_{\eta\beta}}=i\partial_{\zeta_\beta}\phi_{\eta\beta}$, $\bar{J}_{\phi_{\eta\beta}}=-i\partial_{\zeta_\beta}\phi_{\eta\beta}$, $J_{\theta_{\eta\beta}}=i\partial_{\zeta_\beta}\theta_{\eta\beta}$ and $\bar{J}_{\theta_{\eta\beta}}=-i\partial_{\zeta_\beta}\theta_{\eta\beta}$ with $\beta=c/s$. For the case of identical Floquet bands, we omit the index η such that $K_{\eta\beta}=K_\beta$ and $u_{\eta\beta}=u_\beta$. We also use the relation between two bosonic operators $e^Ae^B=:e^{A+B}:e^{\langle AB+\frac{A^2+B^2}{2}\rangle_0}$. The expectation value is taken with respect to the Hamiltonian $H'_{\rm kin}$. Further, we utilize the correlation functions

described in Appendix A by rewriting them in terms of complex coordinate as

$$\begin{aligned}
\langle[\theta_{\eta c}(z_{1c},\bar{z}_{1c})-\theta_{\eta c}(z_{2c},\bar{z}_{2c})]^2\rangle_0 &= \lambda_+\ln\frac{|\chi_+|}{\alpha}+\lambda_-\ln\frac{|\chi_-|}{\alpha},\\
\langle[\theta_{\eta c}(z_{1c},\bar{z}_{1c})-\theta_{\bar{\eta} c}(z_{2c},\bar{z}_{2c})]^2\rangle_0 &= \lambda_+\ln\frac{|\chi_+|}{\alpha}-\lambda_-\ln\frac{|\chi_-|}{\alpha},\\
\langle[\phi_{\eta c}(z_{1c},\bar{z}_{1c})-\phi_{\eta c}(z_{2c},\bar{z}_{2c})]^2\rangle_0 &= \frac{K_c\,u_c}{2\,u_+}\ln\frac{|\chi_+|}{\alpha}+\frac{K_c\,u_c}{2\,u_-}\ln\frac{|\chi_-|}{\alpha},\\
\langle[\phi_{\eta c}(z_{1c},\bar{z}_{1c})-\phi_{\bar{\eta} c}(z_{2c},\bar{z}_{2c})]^2\rangle_0 &= \frac{K_c\,u_c}{2\,u_+}\ln\frac{|\chi_+|}{\alpha}-\frac{K_c\,u_c}{2\,u_-}\ln\frac{|\chi_-|}{\alpha},\\
\langle[\phi_{\eta s}(z_{1s},\bar{z}_{1s})-\phi_{\eta s}(z_{2s},\bar{z}_{2s})]^2\rangle_0 &= K_s\ln\frac{|\chi_s|}{\alpha},\quad \langle[\phi_{\eta\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})]^2\rangle_0\sim\langle\phi_{\eta\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})\phi_{\bar{\eta}\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})\rangle_0\propto-\frac{K_\beta}{2}\ln\alpha,\\
\langle[\theta_{\eta s}(z_{1s},\bar{z}_{1s})-\theta_{\eta s}(z_{2s},\bar{z}_{2s})]^2\rangle_0 &= \frac{1}{K_s}\ln\frac{|\chi_s|}{\alpha},\quad \langle[\theta_{\eta\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})]^2\rangle_0\sim\langle\theta_{\eta\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})\theta_{\bar{\eta}\beta}(z_{\kappa\beta},\bar{z}_{\kappa\beta})\rangle_0\propto-\frac{1}{2K_\beta}\ln\alpha\,.
\end{aligned}\tag{B3}$$

We also define $\partial_{\zeta_\beta}=-\frac{1}{2}(\frac{\partial_\mathcal{T}}{u_\beta}-i\,\partial_X)$ and $\partial_{\bar{\zeta}_\beta}=-\frac{1}{2}(\frac{\partial_\mathcal{T}}{u_\beta}+i\,\partial_X)$ as well as

$$\begin{aligned}
J_{\phi_{\eta\beta}}\bar{J}_{\phi_{\eta\beta}} &= \partial_{\zeta_\beta}\phi_{\eta\beta}\,\partial_{\bar{\zeta}_\beta}\phi_{\eta\beta}=\frac{(\partial_X\phi_{\eta\beta})^2+(\partial_\mathcal{T}\phi_{\eta\beta})^2/u_\beta^2}{4},\\
J^2_{\phi_{\eta\beta}} &= -(\partial_{\zeta_\beta}\phi_{\eta\beta})^2=\frac{(\partial_X\phi_{\eta\beta})^2-(\partial_\mathcal{T}\phi_{\eta\beta})^2/u_\beta^2+2\,i\,(\partial_X\phi_{\eta\beta})(\partial_\mathcal{T}\phi_{\eta\beta})/u_\beta}{4},\\
\bar{J}^2_{\phi_{\eta\beta}} &= -(\partial_{\bar{\zeta}_\beta}\phi_{\eta\beta})^2=\frac{(\partial_X\phi_{\eta\beta})^2-(\partial_\mathcal{T}\phi_{\eta\beta})^2/u_\beta^2-2\,i\,(\partial_X\phi_{\eta\beta})(\partial_\mathcal{T}\phi_{\eta\beta})/u_\beta}{4}.
\end{aligned}\tag{B4}$$

We can write the same relations for the θ field. Next, for $\lambda=\sqrt{2}$, the scaling dimension of the second term in Eq. (B1) is $(\lambda_++\lambda_-+K_s)/2$. Thus, the RG equation is given by $d\tilde{\Delta}_{\rm sc}/dl=[2-(\lambda_++\lambda_-+K_s)/2]\,\tilde{\Delta}_{\rm sc}$. We can go back to $(x,\tau,X,\mathcal{T})$ center-of-mass notation and utilize the fact that for the odd terms the integral over $dx\,d\tau$ gives zero and hence calculate the terms contributing to the kinetic energy in charge sector from Eq. (B2) as follows:

$$-\lambda^2|\chi_c|^2\,[J_{\theta_{\eta c}}\bar{J}_{\theta_{\eta c}}]_{(\zeta_c,\bar{\zeta}_c)}+\frac{\lambda^2}{2}(\chi_c)^2\big[J^2_{\theta_{\eta c}}\big]_{(\zeta_c,\bar{\zeta}_c)}+(\bar{\chi}_c)^2\big[\bar{J}^2_{\theta_{\eta c}}\big]_{(\zeta_c,\bar{\zeta}_c)}=-\frac{\lambda^2}{2}[x^2\,(\partial_X\theta_{\eta c})^2+\tau^2(\partial_\mathcal{T}\theta_{\eta c})^2].\tag{B5}$$

The second-order term in the Z_a for the charge sector has the form

$$\begin{aligned}
I_c=-\sum_\eta\frac{(\tilde{\Delta}_{\rm sc})^2\,u_c^2\,\lambda^2}{8\,\pi^2\,\alpha^4}\int dX\,d\mathcal{T}\Bigg[(\partial_X\theta_{\eta c})^2\int\frac{x^2\,dx\,d\tau}{(|\chi_+|/\alpha)^{\lambda^2\lambda_+/2}(|\chi_-|/\alpha)^{\lambda^2\lambda_-/2}(|\chi_s|/\alpha)^{\lambda^2K_s/2}}\\
+\,(\partial_\mathcal{T}\theta_{\eta c})^2\int\frac{\tau^2\,dx\,d\tau}{(|\chi_+|/\alpha)^{\lambda^2\lambda_+/2}(|\chi_-|/\alpha)^{\lambda^2\lambda_-/2}(|\chi_s|/\alpha)^{\lambda^2K_s/2}}\Bigg].
\end{aligned}\tag{B6}$$

We change to polar coordinates such that $(x,u_c\tau)$ is $[r\cos\theta',r\sin\theta']$ such that $\int dx\,(u_c\,d\tau)=\int r\,dr\,d\theta'$ and calculate the integral over r for the range $[\alpha,\alpha+d\alpha]$.

$$I_c=-\sum_\eta\frac{(\tilde{\Delta}_{\rm sc})^2\,u_c\,\lambda^2}{2}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left[(\partial_X\theta_{\eta c})^2h_x^\Delta+\frac{(\partial_\mathcal{T}\theta_{\eta c})^2}{u_c^2}h_\tau^\Delta\right],\tag{B7}$$

where we define

$$\begin{aligned}
2\,\pi\,h_x^\Delta &= \int\frac{\cos^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\lambda^2\lambda_+/4}\left(\cos^2\theta'+\frac{u_-^2}{u_c^2}\sin^2(\theta')\right)^{\lambda^2\lambda_-/4}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\lambda^2K_s/4}},\\
2\,\pi\,h_\tau^\Delta &= \int\frac{\sin^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\lambda^2\lambda_+/4}\left(\cos^2\theta'+\frac{u_-^2}{u_c^2}\sin^2\theta'\right)^{\lambda^2\lambda_-/4}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\lambda^2K_s/4}}.
\end{aligned}\tag{B8}$$

From the equation of motion, we get $\partial_{\mathcal{T}}\theta_{\eta c} = -i[\frac{u_c}{K_c}\partial_X\phi_{\eta c} + \tilde{u}\,\partial_X\phi_{\bar{\eta}c}]$. Therefore, for $\lambda = \sqrt{2}$, Eq. (B7) takes following form:

$$I_c = -(\tilde{\Delta}_{\rm sc})^2\,u_c\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\Bigg[(\partial_X\theta_{1c})^2 h_x^{\Delta} + (\partial_X\theta_{\bar{1}c})^2 h_x^{\Delta} - \frac{4\,\tilde{u}\,(\partial_X\phi_{1c})\,(\partial_X\phi_{\bar{1}c})}{u_c\,K_c}h_\tau^{\Delta} - \frac{\left(u_c^2/K_c^2+\tilde{u}^2\right)(\partial_X\phi_{1c})^2 + \left(u_c^2/K_c^2+\tilde{u}^2\right)(\partial_X\phi_{\bar{1}c})^2}{u_c^2}h_\tau^{\Delta}\Bigg]. \tag{B9}$$

Compared with Eq. (12) in the main text, we obtain the RG equation for the charge sector as

$$\frac{d}{dl}\left(\frac{u_c}{K_c}\right) = -(\tilde{\Delta}_{\rm sc})^2\left(\frac{u_c}{K_c^2}+\frac{\tilde{u}^2}{u_c}\right)h_\tau^{\Delta},\quad \frac{d}{dl}(u_c\,K_c) = (\tilde{\Delta}_{\rm sc})^2\,u_c\,h_x^{\Delta},\quad \frac{d\tilde{u}}{dl} = \frac{-2\,\tilde{u}}{K_c}(\tilde{\Delta}_{\rm sc})^2\,h_\tau^{\Delta}, \tag{B10}$$

$$\frac{du_c}{dl} = \frac{(\tilde{\Delta}_{\rm sc})^2}{2}\left[\frac{u_c}{K_c}\left(h_x^{\Delta}-h_\tau^{\Delta}\right) - \frac{h_\tau^{\Delta}\,K_c\,\tilde{u}^2}{u_c}\right],\quad \frac{dK_c}{dl} = \frac{(\tilde{\Delta}_{\rm sc})^2}{2}\left[\left(h_x^{\Delta}+h_\tau^{\Delta}\right) + \frac{h_\tau^{\Delta}\,K_c^2\,\tilde{u}^2}{u_c^2}\right]. \tag{B11}$$

For the spin sector (corresponding to the field $\phi_{\eta s}$), we get an expression similar to that in Eq. (B7); however, $\theta_{\eta c}$ gets replaced by $\phi_{\eta s}$,

$$I_s = -\frac{(\tilde{\Delta}_{\rm sc})^2\,u_c\,\lambda^2}{2}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left[(\partial_X\phi_{\eta s})^2 h_x^{\Delta} + \frac{(\partial_{\mathcal{T}}\phi_{\eta s})^2}{u_c^2}h_\tau^{\Delta}\right]. \tag{B12}$$

We use $\partial_{\mathcal{T}}\phi_{\eta s} = -i\,u_s\,K_s\,\partial_X\theta_{\eta s}$ and obtain

$$I_s = -\frac{(\tilde{\Delta}_{\rm sc})^2\,u_c\,\lambda^2}{2}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left[(\partial_X\phi_{\eta s})^2 h_x^{\Delta} - \frac{u_s^2\,K_s^2(\partial_X\theta_{\eta s})^2}{u_c^2}h_\tau^{\Delta}\right]. \tag{B13}$$

As a result, the RG equations become

$$\frac{d}{dl}\left(\frac{u_s}{K_s}\right) = (\tilde{\Delta}_{\rm sc})^2\,u_c\,h_x^{\Delta},\quad \frac{d}{dl}(u_s\,K_s) = -\frac{u_s^2\,K_s^2(\tilde{\Delta}_{\rm sc})^2\,h_\tau^{\Delta}}{u_c},$$
$$\frac{du_s}{dl} = \frac{(\tilde{\Delta}_{\rm sc})^2\,u_c\,K_s}{2}\left(h_x^{\Delta} - \frac{u_s^2}{u_c^2}h_\tau^{\Delta}\right),\quad \frac{dK_s}{dl} = -\frac{(\tilde{\Delta}_{\rm sc})^2\,K_s^2}{2}\left(\frac{u_c}{u_s}h_x^{\Delta} + \frac{u_s}{u_c}h_\tau^{\Delta}\right). \tag{B14}$$

We also note that the RG equations for $\cos[\sqrt{2}\,(\theta_{\eta c}-\phi_{\eta s})]/\pi\,\alpha$ have exactly the same form as written in Eqs. (B11) and (B14). Thus, in the final RG equations, we double the corrections and obtain the final contribution from $\tilde{\Delta}_{\rm sc}$ as

$$\frac{d\tilde{u}}{dl} = \frac{-4\,\tilde{u}}{K_c}(\tilde{\Delta}_{\rm sc})^2\,h_\tau^{\Delta},\quad \frac{du_c}{dl} = (\tilde{\Delta}_{\rm sc})^2\left[\frac{u_c}{K_c}\left(h_x^{\Delta}-h_\tau^{\Delta}\right) - \frac{h_\tau^{\Delta}\,K_c\,\tilde{u}^2}{u_c}\right],\quad \frac{dK_c}{dl} = (\tilde{\Delta}_{\rm sc})^2\left[\left(h_x^{\Delta}+h_\tau^{\Delta}\right) + \frac{h_\tau^{\Delta}\,K_c^2\,\tilde{u}^2}{u_c^2}\right],$$
$$\frac{du_s}{dl} = (\tilde{\Delta}_{\rm sc})^2\,u_c\,K_s\left(h_x^{\Delta} - \frac{u_s^2}{u_c^2}h_\tau^{\Delta}\right),\quad \frac{dK_s}{dl} = -(\tilde{\Delta}_{\rm sc})^2\,K_s^2\left(\frac{u_c}{u_s}h_x^{\Delta} + \frac{u_s}{u_c}h_\tau^{\Delta}\right). \tag{B15}$$

2. RG equations for H_Z'

In the following section, we calculate the RG equations for the Floquet Zeeman term H_Z' given by

$$H_Z' = \sum_\eta\frac{t_F}{\pi\,\alpha}\cos\left[\frac{1}{\sqrt{2}}\left(\phi_{\eta c}-\theta_{\eta c}-\phi_{\eta s}+\theta_{\eta s}+\phi_{\bar{\eta}c}+\theta_{\bar{\eta}c}+\phi_{\bar{\eta}s}+\theta_{\bar{\eta}s}\right)\right]. \tag{B16}$$

Further, introducing $\tilde{t}_F = t_F\,\alpha/u_c$ as a dimensionless coupling constant, $\tilde{\varphi}_\eta = \phi_{\eta c}-\theta_{\eta c}-\phi_{\eta s}+\theta_{\eta s}+\phi_{\bar{\eta}c}+\theta_{\bar{\eta}c}+\phi_{\bar{\eta}s}+\theta_{\bar{\eta}s}$ and using the definition given in Appendix B 1, we write the correlator $\langle[\tilde{\varphi}_\eta(x_1,t_1)-\tilde{\varphi}_\eta(x_2,t_2)]^2\rangle$ as

$$\begin{aligned}\langle[\tilde{\varphi}_\eta(x_1,t_1)-\tilde{\varphi}_\eta(x_2,t_2)]^2\rangle \sim\ & \langle[\phi_{\eta c}(z_{1c},\bar{z}_{1c})-\phi_{\eta c}(z_{2c},\bar{z}_{2c})]^2\rangle + \langle[\theta_{\eta c}(z_{1c},\bar{z}_{1c})-\theta_{\eta c}(z_{2c},\bar{z}_{2c})]^2\rangle\\
&+ \langle[\phi_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})-\phi_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})]^2\rangle + \langle[\theta_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})-\theta_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})]^2\rangle + \langle[\phi_{\eta s}(z_{1s},\bar{z}_{1s})-\phi_{\eta s}(z_{2s},\bar{z}_{2s})]^2\rangle\\
&+ \langle[\theta_{\eta s}(z_{1s},\bar{z}_{1s})-\theta_{\eta s}(z_{2s},\bar{z}_{2s})]^2\rangle + \langle[\phi_{\bar{\eta}s}(z_{1s},\bar{z}_{1s})-\phi_{\bar{\eta}s}(z_{2s},\bar{z}_{2s})]^2\rangle + \langle[\theta_{\bar{\eta}s}(z_{1s},\bar{z}_{1s})-\theta_{\bar{\eta}s}(z_{2s},\bar{z}_{2s})]^2\rangle\\
&- 2\,\langle\phi_{\eta c}(z_{1c},\bar{z}_{1c})\phi_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})\rangle - 2\,\langle\phi_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})\phi_{\eta c}(z_{2c},\bar{z}_{2c})\rangle + 2\,\langle\phi_{\eta c}(z_{1c},\bar{z}_{1c})\phi_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})\rangle + 2\,\langle\phi_{\eta c}(z_{2c},\bar{z}_{2c})\phi_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})\rangle\\
&+ 2\,\langle\theta_{\eta c}(z_{1c},\bar{z}_{1c})\theta_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})\rangle + 2\,\langle\theta_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})\theta_{\eta c}(z_{2c},\bar{z}_{2c})\rangle - 2\,\langle\theta_{\eta c}(z_{1c},\bar{z}_{1c})\theta_{\bar{\eta}c}(z_{1c},\bar{z}_{1c})\rangle - 2\,\langle\theta_{\eta c}(z_{2c},\bar{z}_{2c})\theta_{\bar{\eta}c}(z_{2c},\bar{z}_{2c})\rangle.\end{aligned} \tag{B17}$$

Here we neglect the cross terms between θ and ϕ fields. After utilizing the correlation functions written in Eq. (B3) and considering identical bands ($K_{\eta\beta} = K_{\bar{\eta}\beta} = K_\beta$ and $u_{\eta\beta} = u_{\bar{\eta}\beta} = u_\beta$) with $\beta = c, s$, we get the following form:

$$\begin{aligned}\langle[\tilde{\varphi}_\eta(x_1,t_1) - \tilde{\varphi}_\eta(x_2,t_2)]^2\rangle = 2\Bigg[&\frac{K_c\,u_c}{2\,u_+}\ln\frac{|\chi_+|}{\alpha} + \frac{K_c\,u_c}{2\,u_-}\ln\frac{|\chi_-|}{\alpha} + \lambda_+\ln\frac{|\chi_+|}{\alpha} + \lambda_-\ln\frac{|\chi_-|}{\alpha} + K_s\ln\frac{|\chi_s|}{\alpha} + \frac{1}{K_s}\ln\frac{|\chi_s|}{\alpha}\\ &+ \frac{K_c\,u_c}{2\,u_+}\ln\frac{|\chi_+|}{\alpha} - \frac{K_c\,u_c}{2\,u_-}\ln\frac{|\chi_-|}{\alpha} - \lambda_+\ln\frac{|\chi_+|}{\alpha} + \lambda_-\ln\frac{|\chi_-|}{\alpha}\Bigg]\\ &= 4\,\nu_+\ln\frac{|\chi_+|}{\alpha} + 4\,\lambda_-\ln\frac{|\chi_-|}{\alpha} + 2\,\nu_s\ln\frac{|\chi_s|}{\alpha}.\end{aligned} \tag{B18}$$

Here we define $\nu_+ = K_c u_c/(2\,u_+)$ and $\nu_s = K_s + 1/K_s$. We utilize the correlation function written in Eq. (B18) and obtain the most singular term in the OPEs for $\tilde{t}_F$ as

$$\begin{aligned}&\cos[\lambda\tilde{\varphi}_1(x_1,t_1)]\cos[\lambda\tilde{\varphi}_1(x_2,t_2)]\\ &= \frac{1}{2(|\chi_+|/\alpha)^{2\lambda^2\nu_+}(|\chi_-|/\alpha)^{2\lambda^2\lambda_-}(|\chi_s|/\alpha)^{\lambda^2\nu_s}}\times\Bigg(1 - \lambda^2|\chi_c|^2\,[J_{\phi_{1c}}\bar{J}_{\phi_{1c}} + J_{\theta_{1c}}\bar{J}_{\theta_{1c}} + J_{\phi_{\bar{1}c}}\bar{J}_{\phi_{\bar{1}c}} + J_{\theta_{\bar{1}c}}\bar{J}_{\theta_{\bar{1}c}}]_{(\zeta_c,\bar{\zeta}_c)}\\ &- \lambda^2|\chi_s|^2\,[J_{\phi_{1s}}\bar{J}_{\phi_{1s}} + J_{\theta_{1s}}\bar{J}_{\theta_{1s}} + J_{\phi_{\bar{1}s}}\bar{J}_{\phi_{\bar{1}s}} + J_{\theta_{\bar{1}s}}\bar{J}_{\theta_{\bar{1}s}}]_{(\zeta_s,\bar{\zeta}_s)} - \lambda^2|\chi_c|^2[J_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}} + J_{\phi_{\bar{1}c}}\bar{J}_{\phi_{1c}} - J_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}} - J_{\theta_{\bar{1}c}}\bar{J}_{\theta_{1c}}]_{(\zeta_c,\bar{\zeta}_c)}\\ &+ \frac{\lambda^2}{2}\Big[(\chi_c)^2\left[J^2_{\phi_{1c}} + J^2_{\theta_{1c}} + J^2_{\phi_{\bar{1}c}} + J^2_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + (\bar{\chi}_c)^2\left[\bar{J}^2_{\phi_{1c}} + \bar{J}^2_{\theta_{1c}} + \bar{J}^2_{\phi_{\bar{1}c}} + \bar{J}^2_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)}\Big]\\ &+ \lambda^2\Big[(\chi_c)^2\,[J_{\phi_{1c}}J_{\phi_{\bar{1}c}}]_{(\zeta_c,\bar{\zeta}_c)} + (\bar{\chi}_c)^2\,[\bar{J}_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}}]_{(\zeta_c,\bar{\zeta}_c)} - (\chi_c)^2\,[J_{\theta_{1c}}J_{\theta_{\bar{1}c}}]_{(\zeta_c,\bar{\zeta}_c)} - (\bar{\chi}_c)^2[\bar{J}_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}}]_{(\zeta_c,\bar{\zeta}_c)}\Big]\\ &+ \frac{\lambda^2}{2}\Big[(\bar{\chi}_s)^2\left[\bar{J}^2_{\phi_{1s}} + \bar{J}^2_{\theta_{1s}} + \bar{J}^2_{\phi_{\bar{1}s}} + \bar{J}^2_{\theta_{\bar{1}s}}\right]_{(\zeta_s,\bar{\zeta}_s)} + (\chi_s)^2\left[J^2_{\phi_{1s}} + J^2_{\theta_{1s}} + J^2_{\phi_{\bar{1}s}} + J^2_{\theta_{\bar{1}s}}\right]_{(\zeta_s,\bar{\zeta}_s)}\Big]\Bigg) + \cdots.\end{aligned} \tag{B19}$$

To obtain the RG flow equations, we expand the partition function up to second order in the cosine term,

$$\begin{aligned}Z = Z_0\Bigg\langle 1 - \sum_\eta\Bigg(&\frac{\tilde{t}_F\,u_c}{\pi\,\alpha^2}\int dx\,d\tau\,\cos\left[\frac{1}{\sqrt{2}}\tilde{\varphi}(x,\tau)\right]\\ &+ \frac{(\tilde{t}_F)^2\,u_c^2}{2\,\pi^2\,\alpha^4}\int dx_1\,d\tau_1\,dx_2\,d\tau_2\cos\left[\frac{1}{\sqrt{2}}\tilde{\varphi}(x_1,\tau_1)\right]\cos\left[\frac{1}{\sqrt{2}}\tilde{\varphi}(x_2,\tau_2)\right] + \cdots\Bigg)\Bigg\rangle.\end{aligned} \tag{B20}$$

Thus, from Eq. (B19), by scaling-dimension arguments, the RG flow of $\tilde{t}_F$ is given by

$$\frac{d\tilde{t}_F}{dl} = \left[2 - \frac{(2\,\nu_+ + 2\,\lambda_- + \nu_s)}{4}\right]\tilde{t}_F, \tag{B21}$$

where we take $\lambda = 1/\sqrt{2}$ and $dl = d\alpha/\alpha$. Below, we calculate the second-order correction from $\tilde{t}_F$ to the charge-spin LL parameters and velocities.

a. Terms contributing to the kinetic energy

We consider the third term in Eq. (B20) and collect all the terms contributing to the kinetic energy for the charge sector in the $\eta = 1$ NW from Eq. (B19) and find

$$\begin{aligned}I^{d\alpha} = \frac{(\tilde{t}_F)^2\,u_c^2}{2\,\pi^2\,\alpha^4}\sum_\eta\int dx\,d\tau\,dX\,d\mathcal{T}\,&\frac{1}{2(|\chi_+|/\alpha)^{2\lambda^2\nu_+}(|\chi_-|/\alpha)^{2\lambda^2\lambda_-}(|\chi_s|/\alpha)^{\lambda^2\nu_s}}\Bigg(-\lambda^2|\chi_c|^2\,[J_{\phi_{\eta c}}\bar{J}_{\phi_{\eta c}} + J_{\theta_{\eta c}}\bar{J}_{\theta_{\eta c}}]_{(\zeta_c,\bar{\zeta}_c)}\\ &+ \frac{\lambda^2\,(\chi_c)^2}{2}\left[J^2_{\phi_{\eta c}} + J^2_{\theta_{\eta c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + \frac{\lambda^2\,(\bar{\chi}_c)^2}{2}\left[\bar{J}^2_{\phi_{\eta c}} + \bar{J}^2_{\theta_{\eta c}}\right]_{(\zeta_c,\bar{\zeta}_c)}\Bigg),\end{aligned} \tag{B22}$$

where the terms containing $J_{\phi_{\eta c}/\theta_{\eta c}}$ and $\bar{J}_{\phi_{\eta c}/\theta_{\eta c}}$ take the form

$$
\begin{aligned}
&-\lambda^2|\chi_c|^2\left[J_{\phi_{\eta c}}\bar{J}_{\phi_{\eta c}}+J_{\theta_{\eta c}}\bar{J}_{\theta_{\eta c}}\right]_{(\zeta_c,\bar{\zeta}_c)}+\frac{\lambda^2}{2}(\chi_c)^2\left[J^2_{\phi_{\eta c}}+J^2_{\theta_{\eta c}}\right]_{(\zeta_c,\bar{\zeta}_c)}+\frac{\lambda^2}{2}(\bar{\chi}_c)^2\left[\bar{J}^2_{\phi_{\eta c}}+\bar{J}^2_{\theta_{\eta c}}\right]_{(\zeta_c,\bar{\zeta}_c)}\\
&=-\frac{\lambda^2}{4}\left(x^2+u_c^2\,\tau^2\right)\left[(\partial_X\phi_{\eta c})^2+\frac{(\partial_{\mathcal{T}}\phi_{\eta c})^2}{u_c^2}+(\partial_X\theta_{\eta c})^2+\frac{(\partial_{\mathcal{T}}\theta_{\eta c})^2}{u_c^2}\right]\\
&\quad+\frac{\lambda^2}{8}\left(-x^2+u_c^2\,\tau^2-2iu_cx\tau\right)\left[(\partial_X\phi_{\eta c})^2-\frac{(\partial_{\mathcal{T}}\phi_{\eta c})^2}{u_c^2}+\frac{2\,i\,(\partial_X\phi_{\eta c})(\partial_{\mathcal{T}}\phi_{\eta c})}{u_c}+(\partial_X\theta_{\eta c})^2-\frac{(\partial_{\mathcal{T}}\theta_{\eta c})^2}{u_c^2}+\frac{2\,i\,(\partial_X\theta_{\eta c})(\partial_{\mathcal{T}}\theta_{\eta c})}{u_c}\right]\\
&\quad+\frac{\lambda^2}{8}\left(-x^2+u_c^2\,\tau^2+2iu_cx\tau\right)\left[(\partial_X\phi_{\eta c})^2-\frac{(\partial_{\mathcal{T}}\phi_{\eta c})^2}{u_c^2}-\frac{2\,i\,(\partial_X\phi_{\eta c})(\partial_{\mathcal{T}}\phi_{\eta c})}{u_c}+(\partial_X\theta_{\eta c})^2-\frac{(\partial_{\mathcal{T}}\theta_{\eta c})^2}{u_c^2}-\frac{2\,i\,(\partial_X\theta_{\eta c})(\partial_{\mathcal{T}}\theta_{\eta c})}{u_c}\right]\\
&=-\frac{\lambda^2}{2}\left[x^2\,(\partial_X\phi_{\eta c})^2+\tau^2(\partial_{\mathcal{T}}\phi_{\eta c})^2+x^2\,(\partial_X\theta_{\eta c})^2+\tau^2(\partial_{\mathcal{T}}\theta_{\eta c})^2\right].
\end{aligned}
\tag{B23}
$$

We note that the odd terms in x and τ do not contribute to the RG equations. Next, we put $\lambda=1/\sqrt{2}$ in Eq. (B23) and make use of it in Eq. (B22) to get

$$
I^{d\alpha}=-\sum_{\eta}\frac{(\tilde{t}_F)^2\,u_c^2}{16\,\pi^2\,\alpha^4}\int dx\,dt\,dX\,d\mathcal{T}\,\frac{x^2\,(\partial_X\phi_{\eta c})^2+\tau^2(\partial_{\mathcal{T}}\phi_{\eta c})^2+x^2\,(\partial_X\theta_{\eta c})^2+\tau^2(\partial_{\mathcal{T}}\theta_{\eta c})^2}{(|\chi_+|/\alpha)^{\nu_+}(|\chi_-|/\alpha)^{\lambda_-}(|\chi_s|/\alpha)^{\nu_s/2}}.
\tag{B24}
$$

We again change to polar coordinates with $(x,u_c\tau)=(r\cos\theta',r\sin\theta')$ such that $I^{d\alpha}$ reads

$$
\begin{aligned}
I^{d\alpha}&=-\frac{(\tilde{t}_F)^2\,u_c}{16\,\pi^2\,\alpha^4}\sum_{\eta}\int dX\,d\mathcal{T}\int r\,dr\,d\theta\,\frac{r^2\,\cos^2\theta'[(\partial_X\phi_{\eta c})^2+(\partial_X\theta_{\eta c})^2]+\frac{r^2\,\sin^2(\theta')}{u_c^2}[(\partial_{\mathcal{T}}\phi_{\eta c})^2+(\partial_{\mathcal{T}}\theta_{\eta c})^2]}{(|\chi_+|/\alpha)^{\nu_+}(|\chi_-|/\alpha)^{\lambda_-}(|\chi_s|/\alpha)^{\nu_s/2}}\\
&=-\frac{(\tilde{t}_F)^2\,u_c}{4}\left(\frac{d\alpha}{\alpha}\right)\sum_{\eta}\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left([(\partial_X\phi_{\eta c})^2+(\partial_X\theta_{\eta c})^2]h_x^{t_F}+\frac{[(\partial_{\mathcal{T}}\phi_{\eta c})^2+(\partial_{\mathcal{T}}\theta_{\eta c})^2]\,h_t^{t_F}}{u_c^2}\right),
\end{aligned}
\tag{B25}
$$

where $h_x^{t_F}$ and $h_\tau^{t_F}$ are defined by

$$
\begin{aligned}
2\,\pi\,h_x^{t_F}&=\int_0^{2\pi}\frac{\cos^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\nu_+/2}\left(\cos^2\theta'+\frac{u_c^2}{u_c^2}\sin^2\theta'\right)^{\lambda_-/2}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\nu_s/4}},\\
2\,\pi\,h_\tau^{t_F}&=\int_0^{2\pi}\frac{\sin^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\nu_+/2}\left(\cos^2\theta'+\frac{u_c^2}{u_c^2}\sin^2\theta'\right)^{\lambda_-/2}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\nu_s/4}}.
\end{aligned}
\tag{B26}
$$

From the equation of motion, we obtain $\partial_{\mathcal{T}}\theta_{\eta c}=-i[\frac{u_c}{K_c}\partial_X\phi_{\eta c}+\tilde{u}\,\partial_X\phi_{\bar{\eta}c}]$, $\partial_{\mathcal{T}}\theta_{\eta s}=-i\,\frac{u_s}{K_s}\partial_X\phi_{\eta s}$ and $\partial_{\mathcal{T}}\phi_{\eta\nu}=-i\,u_\nu\,K_\nu\,\partial_X\theta_{\eta\nu}$. Therefore, Eq. (B25) reduces to

$$
\begin{aligned}
I^{d\alpha}&=-\frac{(\tilde{t}_F)^2\,u_c}{4}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left([(\partial_X\phi_{1c})^2+(\partial_X\phi_{\bar{1}c})^2+(\partial_X\theta_{1c})^2+(\partial_X\theta_{\bar{1}c})^2]h_x^{t_F}-K_c^2\,[(\partial_X\theta_{1c})^2+(\partial_X\theta_{\bar{1}c})^2]\,h_\tau^{t_F}\right.\\
&\quad\left.-\left[\frac{\left(u_c^2/K_c^2+\tilde{u}^2\right)(\partial_X\phi_{1c})^2+\left(u_c^2/K_c^2+\tilde{u}^2\right)(\partial_X\phi_{\bar{1}c})^2}{u_c^2}+\frac{4\,\tilde{u}\,(\partial_X\phi_{1c})\,(\partial_X\phi_{\bar{1}c})}{K_c\,u_c}\right]h_\tau^{t_F}\right)\\
&=-\frac{(\tilde{t}_F)^2\,u_c}{4}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\,\pi}\left(\left[h_x^{t_F}-\left(\frac{u_c^2}{K_c^2}+\tilde{u}^2\right)h_\tau^{t_F}\right](\partial_X\phi_{1c})^2+\left[h_x^{t_F}-\left(\frac{u_c^2}{K_c^2}+\tilde{u}^2\right)h_\tau^{t_F}\right](\partial_X\phi_{\bar{1}c})^2\right.\\
&\quad+\left[h_x^{t_F}-K_c^2\,h_\tau^{t_F}\right](\partial_X\theta_{1c})^2+\left[h_x^{t_F}-K_c^2\,h_\tau^{t_F}\right](\partial_X\theta_{\bar{1}c})^2-\left.\frac{4\,\tilde{u}\,(\partial_X\phi_{1c})\,(\partial_X\phi_{\bar{1}c})}{K_c\,u_c}\,h_\tau^{t_F}\right).
\end{aligned}
\tag{B27}
$$

As a result, the RG equations corresponding to ϕ_{1c}/θ_{1c} fields take the following form:

$$
\frac{d}{dl}\left(\frac{u_c}{K_c}\right)=\frac{(\tilde{t}_F)^2\,u_c\left[h_x^{t_F}-\left(\frac{u_c^2}{K_c^2}+\tilde{u}^2\right)h_\tau^{t_F}\right]}{4},\quad\frac{d}{dl}(u_c\,K_c)=\frac{(\tilde{t}_F)^2\,u_c\left[h_x^{t_F}-K_c^2h_\tau^{t_F}\right]}{4},\quad\frac{d\tilde{u}}{dl}=-\frac{\tilde{u}}{2\,K_c}(\tilde{t}_F)^2\,h_\tau^{t_F},
\tag{B28}
$$

$$
\frac{du_c}{dl}=\frac{(\tilde{t}_F)^2}{8}\left(h_x^{t_F}-h_\tau^{t_F}\right)\left(u_c\,K_c+\frac{u_c}{K_c}\right)-\frac{(\tilde{t}_F)^2\,\tilde{u}^2\,K_c\,h_\tau^{t_F}}{8\,u_c},
\tag{B29}
$$

$$
\frac{dK_c}{dl}=\frac{(\tilde{t}_F)^2}{8}\left(h_x^{t_F}+h_\tau^{t_F}\right)\left(1-K_c^2\right)+\frac{(\tilde{t}_F)^2\,\tilde{u}^2\,K_c^2\,h_\tau^{t_F}}{8\,u_c^2}.
\tag{B30}
$$

For the spin sector (corresponding to $\phi_{\eta s}$–$\theta_{\eta s}$), as follows from Eq. (B19), we obtain the form of I_s as

$$I_s = \frac{(\tilde{t}_F)^2\, u_c^2}{2\,\pi^2\,\alpha^4} \sum_\eta \int dx\, d\tau\, dX\, dT \frac{1}{2(|\chi_+|/\alpha)^{2\,\lambda^2\nu_+}(|\chi_-|/\alpha)^{2\,\lambda^2\lambda_-}(|\chi_s|/\alpha)^{\lambda^2\nu_s}} \times \left(-\lambda^2|\chi_s|^2\left[J_{\phi_{\eta s}}\bar{J}_{\phi_{\eta s}} + J_{\theta_{\eta s}}\bar{J}_{\theta_{\eta s}}\right]_{(\zeta_s,\bar{\zeta}_s)} + \frac{\lambda^2\,(\chi_s)^2}{2}\left[J^2_{\phi_{\eta s}} + J^2_{\theta_{\eta s}}\right]_{(\zeta_s,\bar{\zeta}_s)} + \frac{\lambda^2\,(\bar{\chi}_s)^2}{2}\left[\bar{J}^2_{\phi_{\eta s}} + \bar{J}^2_{\theta_{\eta s}}\right]_{(\zeta_s,\bar{\zeta}_s)}\right). \tag{B31}$$

We repeat the same steps as before for Eqs. (B23)–(B27) and after converting the integral to polar coordinates $(x, u_c\tau) = (r\cos\theta', r\sin\theta')$ and integrating r from α to $\alpha + d\alpha$, $I_s^{d\alpha}$ reads

$$\begin{aligned} I_s^{d\alpha} &= -\frac{(\tilde{t}_F)^2\, u_c\,\lambda^2}{2}\left(\frac{d\alpha}{\alpha}\right)\sum_\eta \int \frac{dX\,d\mathcal{T}}{2\,\pi}\left([(\partial_X\phi_{\eta s})^2 + (\partial_X\theta_{\eta s})^2]\,h_x^{t_F} + \frac{[(\partial_T\phi_{\eta s})^2 + (\partial_T\theta_{\eta s})^2]}{u_c^2}h_\tau^{t_F}\right) \\ &= -\frac{(\tilde{t}_F)^2\, u_c\,\lambda^2}{2}\left(\frac{d\alpha}{\alpha}\right)\sum_\eta \int \frac{dX\,d\mathcal{T}}{2\,\pi}\left\{\left[h_x^{t_F} - \frac{u_s^2}{u_c^2\,K_s^2}h_\tau^{t_F}\right](\partial_X\phi_{\eta s})^2 + \left[h_x^{t_F} - \frac{u_s^2\,K_s^2}{u_c^2}h_\tau^{t_F}\right](\partial_X\theta_{\eta s})^2\right\}. \end{aligned} \tag{B32}$$

Therefore, for $\lambda = 1/\sqrt{2}$, we obtain

$$\begin{aligned} \frac{d}{dl}\left(\frac{u_s}{K_s}\right) &= \frac{(\tilde{t}_F)^2\, u_c\left[h_x^{t_F} - \frac{u_s^2}{u_c^2 K_s^2}h_\tau^{t_F}\right]}{4}, \quad \frac{d}{dl}(u_s\,K_s) = \frac{(\tilde{t}_F)^2\, u_c\left[h_x^{t_F} - \frac{u_s^2 K_s^2}{u_c^2}h_\tau^{t_F}\right]}{4}, \\ \frac{du_s}{dl} &= \frac{(\tilde{t}_F)^2}{8}\left(h_x^{t_F} - \frac{u_s^2}{u_c^2}\,h_\tau^{t_F}\right)\left(u_c\,K_s + \frac{u_c}{K_s}\right), \quad \frac{dK_s}{dl} = \frac{(\tilde{t}_F)^2}{8}\left(\frac{u_c}{u_s}\,h_x^{t_F} + \frac{u_s}{u_c}\,h_\tau^{t_F}\right)\left(1 - K_s^2\right). \end{aligned} \tag{B33}$$

b. Terms contributing to the interband crossed terms between the NWs: $(\partial_x\phi_{\eta c}\,\partial_x\phi_{\bar{\eta}c})$ and $(\partial_x\theta_{\eta c}\,\partial_x\theta_{\bar{\eta}c})$

In this subsection, we calculate the terms generated under the RG flow and contributing to the interband cross term defined in Eq. (12). From the kinetic energy term calculations, we already get the contribution coming out from $\tilde{\Delta}_{sc}$ and $\tilde{t}_F$ and defined in Eqs. (B10) and (B28). Apart from these contributions, $\tilde{t}_F$ has direct terms as well as terms which contribute to the interband cross term. We collect this type of terms from Eq. (B19) and write the term similar to Eq. (B22) as

$$\begin{aligned} I_{cr}^{d\alpha} = \int dx\,dt\,dX\,d\mathcal{T}\frac{(\tilde{t}_F)^2\,u_c^2}{2\,\pi^2\,\alpha^4}\frac{1}{2(|\chi_+|/\alpha)^{2\,\lambda^2\nu_+}(|\chi_-|/\alpha)^{2\,\lambda^2\lambda_-}(|\chi_s|/\alpha)^{\lambda^2\,\nu_s}} \\ \times\left(-\lambda^2|\chi_c|^2\left[J_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}} + J_{\phi_{\bar{1}c}}\bar{J}_{\phi_{1c}} - J_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}} - J_{\theta_{\bar{1}c}}\bar{J}_{\theta_{1c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + \lambda^2(\chi_c)^2\left[J_{\phi_{1c}}J_{\phi_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)}\right. \\ \left. - \lambda^2(\chi_c)^2\left[J_{\theta_{1c}}J_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + \lambda^2(\bar{\chi}_c)^2\left[\bar{J}_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)} - \lambda^2(\bar{\chi}_c)^2\left[\bar{J}_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)}\right), \end{aligned} \tag{B34}$$

To simplify the expression of $I_{cr}^{d\alpha}$, we calculate the explicit form of terms containing $J_{\phi_{\eta c}/\theta_{\eta c}}$ and $\bar{J}_{\phi_{\eta c}/\theta_{\eta c}}$ as

$$\begin{aligned} &-\lambda^2|\chi_c|^2\left[J_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}} + J_{\phi_{\bar{1}c}}\bar{J}_{\phi_{1c}} - J_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}} - J_{\theta_{\bar{1}c}}\bar{J}_{\theta_{1c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + \lambda^2(\chi_c)^2\left[J_{\phi_{1c}}J_{\phi_{\bar{1}c}} - J_{\theta_{1c}}J_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)} + \lambda^2(\bar{\chi}_c)^2\left[\bar{J}_{\phi_{1c}}\bar{J}_{\phi_{\bar{1}c}} - \bar{J}_{\theta_{1c}}\bar{J}_{\theta_{\bar{1}c}}\right]_{(\zeta_c,\bar{\zeta}_c)} \\ &= -\frac{\lambda^2}{2}\left(x^2 + u_c^2\,\tau^2\right)\left[(\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c}) + \frac{(\partial_\mathcal{T}\phi_{1c})(\partial_\mathcal{T}\phi_{\bar{1}c})}{u_c^2} - (\partial_X\theta_{1c})(\partial_X\theta_{\bar{1}c}) - \frac{(\partial_\mathcal{T}\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c})}{u_c^2}\right] \\ &\quad + \frac{\lambda^2}{4}\left(-x^2 + u_c^2\,\tau^2 - 2\,i\,u_c\,x\,\tau\right)\left\{(\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c}) - \frac{(\partial_\mathcal{T}\phi_{1c})(\partial_\mathcal{T}\phi_{\bar{1}c})}{u_c^2} + \frac{i\,[(\partial_X\phi_{1c})(\partial_\mathcal{T}\phi_{\bar{1}c}) + (\partial_\mathcal{T}\phi_{1c})(\partial_X\phi_{\bar{1}c})]}{u_c}\right. \\ &\quad \left. - (\partial_X\theta_{1c})(\partial_X\theta_{\bar{1}c}) + \frac{(\partial_\mathcal{T}\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c})}{u_c^2} - \frac{i\,[(\partial_X\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c}) + (\partial_\mathcal{T}\theta_{1c})(\partial_X\theta_{\bar{1}c})]}{u_c}\right\} \\ &\quad + \frac{\lambda^2}{4}\left(-x^2 + u_c^2\,\tau^2 + 2\,i\,u_c\,x\,\tau\right)\left\{(\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c}) - \frac{(\partial_\mathcal{T}\phi_{1c})(\partial_\mathcal{T}\phi_{\bar{1}c})}{u_c^2} - \frac{i\,[(\partial_X\phi_{1c})(\partial_\mathcal{T}\phi_{\bar{1}c}) + (\partial_\mathcal{T}\phi_{1c})(\partial_X\phi_{\bar{1}c})]}{u_c}\right. \\ &\quad \left. - (\partial_X\theta_{1c})(\partial_X\theta_{\bar{1}c}) + \frac{(\partial_\mathcal{T}\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c})}{u_c^2} + \frac{i\,[(\partial_X\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c}) + (\partial_\mathcal{T}\theta_{1c})(\partial_X\theta_{\bar{1}c})]}{u_c}\right\} \\ &\approx -\lambda^2\,x^2\,(\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c}) + \lambda^2\tau^2\,(\partial_\mathcal{T}\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c}) \\ &= -\lambda^2\left[x^2 + \frac{u_c^2\,\tau^2}{K_c^2} + \tilde{u}^2\,\tau^2\right](\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c}) - \frac{\lambda^2\tau^2\,\tilde{u}\,u_c}{K_c}[(\partial_X\phi_{1c})^2 + (\partial_X\phi_{\bar{1}c})^2]. \end{aligned} \tag{B35}$$

Here, we consider the term proportional to $(\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c})$ and $(\partial_\mathcal{T}\theta_{1c})(\partial_\mathcal{T}\theta_{\bar{1}c})$ and utilized the definition from the equation of motion: $\partial_\mathcal{T}\theta_{\eta c} = -i[\frac{u_c}{K_c}\partial_X\phi_{\eta c} + \tilde{u}\,\partial_X\phi_{\bar{\eta}c}]$. Therefore, for $\lambda = 1/\sqrt{2}$, we obtain

$$I_{cr}^{d\alpha} = -\frac{(\tilde{t}_F)^2\,u_c^2}{8\,\pi^2\,\alpha^4}\int dx\,d\tau\,dX\,d\mathcal{T}\,\frac{\left[x^2+\frac{u_c^2\,\tau^2}{K_c^2}+\tilde{u}^2\,\tau^2\right](\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c})+\frac{\tau^2\,\tilde{u}\,u_c}{K_c}[(\partial_X\phi_{1c})^2+(\partial_X\phi_{\bar{1}c})^2]}{(|\chi_+|/\alpha)^{\nu_+}(|\chi_-|/\alpha)^{\lambda_-}(|\chi_s|/\alpha)^{\nu_s/2}}. \tag{B36}$$

Further, converting the integral to polar coordinates and integrating from α to $\alpha + d\alpha$, we find

$$\begin{aligned}
I_{cr}^{d\alpha} &= -\frac{(\tilde{t}_F)^2\,u_c}{8\,\pi^2\,\alpha^4}\int r\,dr\,d\theta'\,dX\,d\mathcal{T}\,\frac{1}{(|\chi_+|/\alpha)^{\nu_+}(|\chi_-|/\alpha)^{\lambda_-}(|\chi_s|/\alpha)^{\nu_s/2}}\\
&\times\left\{\left[r^2\,\cos^2(\theta')+\frac{r^2\,\sin^2(\theta')}{K_c^2}+\frac{\tilde{u}^2\,r^2\,\sin^2(\theta')}{u_c^2}\right](\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c})+\frac{r^2\,\sin^2(\theta')\,\tilde{u}}{u_c\,K_c}[(\partial_X\phi_{1c})^2+(\partial_X\phi_{\bar{1}c})^2]\right\}\\
&= -\frac{(\tilde{t}_F)^2\,u_c}{2}\left(\frac{d\alpha}{\alpha}\right)\int\frac{dX\,d\mathcal{T}}{2\pi}\left\{\left[h_x^{t_F}+h_\tau^{t_F}\left(\frac{1}{K_c^2}+\frac{\tilde{u}^2}{u_c^2}\right)\right](\partial_X\phi_{1c})(\partial_X\phi_{\bar{1}c})+\frac{h_\tau^{t_F}\,\tilde{u}}{u_c\,K_c}[(\partial_X\phi_{1c})^2+(\partial_X\phi_{\bar{1}c})^2]\right\}.
\end{aligned} \tag{B37}$$

This generates the following contribution to the RG equations:

$$\frac{d\tilde{u}}{dl} = \frac{(\tilde{t}_F)^2 u_c}{4}\left[h_x^{t_F}+h_\tau^{t_F}\left(\frac{1}{K_c^2}+\frac{\tilde{u}^2}{u_c^2}\right)\right], \tag{B38}$$

$$\frac{d}{dl}\left(\frac{u_c}{K_c}\right) = \frac{(\tilde{t}_F)^2\tilde{u}\,h_\tau^{t_F}}{2\,K_c},\quad \frac{d}{dl}(u_c\,K_c) = 0,\quad \frac{du_c}{dl} = \frac{(\tilde{t}_F)^2\tilde{u}\,h_\tau^{t_F}}{4},\quad \frac{dK_c}{dl} = -\frac{(\tilde{t}_F)^2\,\tilde{u}\,K_c\,h_\tau^{t_F}}{4\,u_c}. \tag{B39}$$

To conclude, we collect all the terms from the Hamiltonians $H'_{\rm sc}$ and H'_Z, which are contributing to the dimensionless coupling constants, LL parameters, and velocities and obtain the final form of the RG equations:

$$\begin{aligned}
\frac{d}{dl}\tilde{\Delta}_{\rm sc} &= \left[2-\frac{\lambda_+ + \lambda_- + K_s}{2}\right]\tilde{\Delta}_{\rm sc},\quad \frac{d\tilde{t}_F}{dl} = \left[2-\frac{2\,\nu_+ + 2\,\lambda_- + \nu_s}{4}\right]\tilde{t}_F,\\
\frac{dK_c}{dl} &= \left[h_x^\Delta+\left(1+\frac{K_c^2\,\tilde{u}^2}{u_c^2}\right)h_t^\Delta\right](\tilde{\Delta}_{\rm sc})^2+\frac{\left(1-K_c^2\right)h_x^{t_F}+\left[\left(1-\frac{\tilde{u}}{u_c}K_c\right)^2-K_c^2\right]h_t^{t_F}}{8}\,(\tilde{t}_F)^2,\\
\frac{dK_s}{dl} &= -\frac{\left(\frac{u_c}{u_s}h_x^\Delta+\frac{u_s}{u_c}h_t^\Delta\right)}{2}\,K_s^2\,[(\tilde{\Delta}_{\rm sc})^2+(\tilde{\Delta}^{\rm ext})^2]+\frac{\left(\frac{u_c}{u_s}h_x^{t_F}+\frac{u_s}{u_c}h_t^{t_F}\right)\left(1-K_s^2\right)}{8}\,(\tilde{t}_F)^2,\\
\frac{du_c}{dl} &= \frac{\left[h_x^\Delta-\left(1+\frac{K_c^2\,\tilde{u}^2}{u_c^2}\right)h_t^\Delta\right]}{2\,K_c}\,u_c\,[(\tilde{\Delta}_{\rm sc})^2+(\tilde{\Delta}^{\rm ext})^2]+\frac{\left(1+K_c^2\right)h_x^{t_F}-\left[\left(1-\frac{\tilde{u}}{u_c}K_c\right)^2+K_c^2\right]h_t^{t_F}}{8\,K_c}\,u_c\,(\tilde{t}_F)^2,\\
\frac{du_s}{dl} &= \frac{\left(h_x^\Delta-\frac{u_s^2}{u_c^2}h_t^\Delta\right)}{2}\,u_c\,K_s\,[(\tilde{\Delta}_{\rm sc})^2+(\tilde{\Delta}^{\rm ext})^2]+\frac{\left(h_x^{t_F}-\frac{u_s^2}{u_c^2}h_t^{t_F}\right)\left(1+K_s^2\right)}{8\,K_s}\,u_c\,(\tilde{t}_F)^2,\\
\frac{d\tilde{u}}{dl} &= -\frac{2\,\tilde{u}\,h_t^\Delta}{K_c}[(\tilde{\Delta}_{\rm sc})^2+(\tilde{\Delta}^{\rm ext})^2]+\frac{\left[h_x^{t_F}+\left(\frac{1}{K_c}-\frac{\tilde{u}}{u_c}\right)^2h_t^{t_F}\right]}{4}\,u_c\,(\tilde{t}_F)^2,
\end{aligned} \tag{B40}$$

where we define

$$\begin{aligned}
h_x^\Delta &= \frac{1}{2\,\pi}\int_0^{2\,\pi}\frac{\cos^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\lambda_+/2}\left(\cos^2\theta'+\frac{u_-^2}{u_c^2}\sin^2(\theta')\right)^{\lambda_-/2}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{K_s/2}},\\
h_t^\Delta &= \frac{1}{2\,\pi}\int_0^{2\,\pi}\frac{\sin^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\lambda_+/2}\left(\cos^2\theta'+\frac{u_-^2}{u_c^2}\sin^2\theta'\right)^{\lambda_-/2}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{K_s/2}},\\
h_x^{t_F} &= \frac{1}{2\,\pi}\int_0^{2\,\pi}\frac{\cos^2\theta'\,d\theta'}{\left(\cos^2\theta'+\frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\nu_+/2}\left(\cos^2\theta'+\frac{u_-^2}{u_c^2}\sin^2(\theta')\right)^{\lambda_-/2}\left(\cos^2\theta'+\frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\nu_s/4}},
\end{aligned}$$

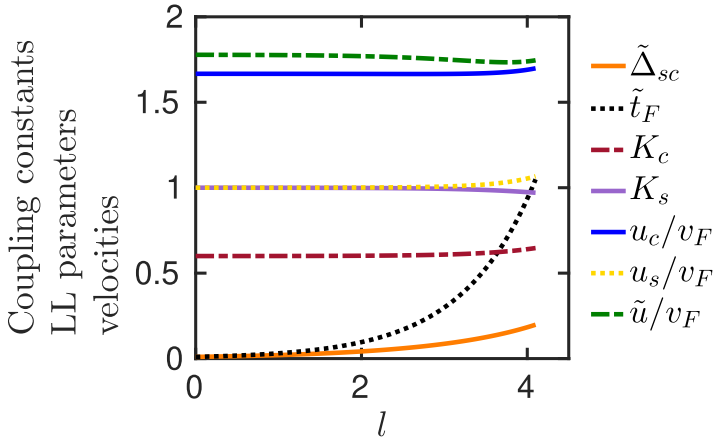

FIG. 6. The RG flow of different coupling constants, LL parameter, and velocities as function of the flow parameter l, with the initial conditions: $\tilde{\Delta}_{sc}(0) = \tilde{t}_F(0) = 0.01$, $K_c(0) = 0.6$, $K_s(0) = 1$, $u_v(0)/v_F = 1/K_v(0)$, and $\tilde{u}(0)/v_F = 1/K_c^2(0) - 1$, where v_F is the Fermi velocity. The dimensionless Floquet Zeeman term (black dotted) reaches the strong coupling limit much faster compared to the superconducting pairing gap $\tilde{\Delta}_{sc}$ (orange solid), which results in the topological phase for the given initial parameter values at $l = 0$.

$$h_t^{t_F} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\sin^2\theta'\, d\theta'}{\left(\cos^2\theta' + \frac{u_+^2}{u_c^2}\sin^2\theta'\right)^{\nu_+/2}\left(\cos^2\theta' + \frac{u_-^2}{u_c^2}\sin^2\theta'\right)^{\lambda_-/2}\left(\cos^2\theta' + \frac{u_s^2}{u_c^2}\sin^2\theta'\right)^{\nu_s/4}},$$

$$\lambda_\pm = \frac{1}{2K_c}\frac{u_c \pm \tilde{u}\,K_c}{u_\pm}, \quad \nu_+ = \frac{K_c\,u_c}{2\,u_+}, \quad \nu_s = \left(K_s + \frac{1}{K_s}\right) \quad u_\pm^2 = u_c^2 \pm u_c\,\tilde{u}\,K_c. \tag{B41}$$

The coupled RG equations [see Eq. (B40)] are solved numerically and the coupling constants, LL parameters, and velocities are plotted as a function of l in Fig. 6. The charge-spin velocities and LL parameter stay close to their initial values at $l = 0$, which again verifies the validity of the assumption used for the analytical calculations performed in the main text of the paper.

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