

1 Behavioural attainability of evolution-  
2 arily stable strategies in repeated in-  
3 teractions

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10

## 11 **Abstract**

12 Theory for the evolution of social interactions based on continuous  
13 strategies often assumes for simplicity that expressed behaviours are  
14 independent from previous encounters. In reality, however, such  
15 dependencies are likely to be widespread and often strong, generating  
16 complex behavioural dynamics. To model this process and illustrate  
17 potential consequences for the evolution of behavioural interactions, we  
18 consider the behavioural dynamics of the interaction between caring  
19 parents and their demanding offspring, a prime example for long series of  
20 interdependent and highly dynamic interactions. These dynamics can be  
21 modelled using functions describing mechanisms for how parents and their  
22 offspring respond to each other in the interaction. We establish the general  
23 conditions under which the behavioural dynamics converge towards a  
24 proximate equilibrium and refer to such converging interactions as  
25 behaviourally stable strategies (BSSs). We further demonstrate that there  
26 is scope for behavioural instability under realistic conditions; that is,  
27 whenever parents and/or offspring 'overreact' beyond some threshold. By  
28 applying the derived condition for behavioural stability to evolutionary  
29 models of parent-offspring conflict resolution, we show by numerical  
30 simulations that evolutionarily stable strategies (ESSs) of current models  
31 are not necessarily behaviourally stable. Because behavioural instability  
32 implies that expressed levels of behaviours deviate from the ESS,  
33 behavioural stability is required for strict evolutionary stability in repeated  
34 behavioural interactions.

35 Key words: behavioural dynamics; ESS; game theory; interacting  
36 phenotypes; parent-offspring conflict; social interaction

37 Conspicuous offspring behaviours and displays to demand resources from  
38 their parents can be observed in most animal species with parental care  
39 (Clutton-Brock 1991). According to parent-offspring conflict (POC) theory,  
40 offspring are usually selected to demand more resources from their parents  
41 than the parents should provide (Trivers 1974). This phenomenon can be  
42 evolutionarily explained in terms of the outcome of the POC over resource  
43 distribution, where offspring begging and parental provisioning strategies  
44 reflect an evolved resolution of this conflict (Trivers 1974; Parker 1985;  
45 Godfray 1995; Kilner & Johnstone 1997; Mock & Parker 1997; Royle et al.  
46 2002). There are two main types of game-theoretic approaches to modelling  
47 conflict resolution. The 'scramble competition' model assumes that  
48 offspring control parental food allocation (Parker & Macnair 1979; Parker  
49 1985; Parker et al. 2002). The 'honest signalling' model assumes, that  
50 parents control food allocation (Godfray 1991; Godfray & Johnstone 2000).  
51 Both types of models can explain the evolution of condition-dependent,  
52 conspicuous and costly offspring begging and parental response.

53 These models generally assume for simplicity a single interaction, or  
54 equivalently multiple independent interactions, between parent and  
55 offspring (Maynard-Smith 1982; Parker 1985; Houston et al. 1988; Godfray  
56 1999; McNamara et al. 1999; Godfray & Johnstone 2000; Royle et al. 2002;  
57 but see Johnstone 1996 for a two-step exception) and the evolving  
58 strategies are taken to be adequately represented by the behavioural  
59 outcome of a one-off interaction. However, in reality the interactions  
60 between parents and offspring are behaviourally very dynamic and  
61 typically involve repeated encounters that are interdependent (McNamara  
62 et al. 1999; Godfray & Johnstone 2000). The issue of stability has been  
63 thoroughly explored in conflict resolution models from the perspective of

64 evolutionary stability (e.g. Takada & Kigami 1991; Rodríguez-Gironés  
65 et al. 1998; McNamara et al. 1999). But it is not known whether, and  
66 under what conditions, the behavioural dynamics of repeated  
67 parent-offspring interactions allow the behaviours actually to be expressed  
68 according to the idealized strategies in the evolutionary models.

69 Here, we model the repeated parent-offspring interaction using a  
70 behavioural reaction norm approach (Smiseth et al. 2008) in an expanded  
71 negotiation model framework (Moore et al. 1997; McNamara et al. 1999;  
72 Taylor & Day 2004; Johnstone & Hinde 2006), where a demand function  
73 defines how offspring adjust their begging to variation in parental  
74 provisioning, and a supply function defines how parents adjust their  
75 provisioning to variation in offspring begging (Fig. 1; Hussell 1988). These  
76 functions define how parents and offspring react to each other, and the  
77 resulting dynamics can be considered as a negotiation process. Under this  
78 perspective, evolutionary strategies can be represented by the slopes  
79 and/or shapes of these functions, mediate the behavioural dynamics of the  
80 interaction (Smiseth et al. 2008). In his graphical model, Hussell (1988)  
81 focussed on the expected behavioural equilibrium over multiple  
82 interdependent parent-offspring interactions (see also Kölliker 2003). This  
83 equilibrium represents the behavioural strategies considered in the game  
84 theoretic conflict resolution (Godfray 1991; Mock & Parker 1997; Parker  
85 et al. 2002) and quantitative genetic coadaptation models (Wolf & Brodie  
86 1998; Kölliker et al. 2005). However, given a pair of supply and demand  
87 functions, the behavioural dynamics of the repeated parent-offspring  
88 interactions may, or may not, lead to this equilibrium. Thereby, the shapes  
89 of the supply and demand functions affect the stability of the behavioural  
90 equilibrium (Samuelson 1976). Understanding the conditions under which

91 the behavioural equilibrium is actually reached in the interaction is critical,  
92 because only a stable behavioural equilibrium can adequately represent  
93 strategies in current evolutionary resolution models (see Samuelson 1976  
94 for an economical context).

95 We provide a formal model and results from numerical analyses exploring  
96 the stability of behavioural equilibria and behavioural strategies in  
97 parent-offspring interactions for different shapes of supply and demand  
98 functions. We also show for a range of realistic conditions that current  
99 conflict resolution models predict evolutionarily stable strategies (ESSs)  
100 that are behaviourally not stable.

## 101 **The Model**

### 102 *Behavioural equilibria*

103 The offspring behavioural reaction norm for demand (Taylor & Day 2004;  
104 Smiseth et al. 2008) is defined by the demand function  $D = f(S)$ ,  
105 describing the dependence of offspring begging on parental provisioning.  
106 The parental behavioural reaction norm for supply (Taylor & Day 2004;  
107 Smiseth et al. 2008) is defined by the supply function  $S = g(D)$ , describing  
108 the dependence of parental provisioning on offspring begging. The  
109 interdependence of the two recursive functions can be used to model the  
110 behavioural dynamics over repeated interactions of parents and offspring  
111 (Fig. 1).

112 To display and formalize this feedback and find the behavioural  
113 equilibrium, the arguments of the two functions need to be expressed in the

114 same currency. This can graphically be achieved by inverting the supply  
115 function and superimposing it on the demand function (Hussell 1988). The  
116 intersection point of the two functions represents the behavioural  
117 equilibrium. Formally, the behavioural equilibrium is derived by inverting  
118 the supply function, setting it equal to the demand function:

$$f(S_{eq}) = g^{-1}(S_{eq}).$$

119 Then solving for  $S_{eq}$ , where  $S_{eq}$  is the equilibrium value for the supply and  
120 setting  $S_{eq}$  in the demand function yields the equilibrium value for demand  
121  $D_{eq}$  (Kölliker 2003).

### 122 *Stability of behavioural equilibria*

123 To address the behavioural stability of the equilibria, the dynamics of  
124 parent and offspring behaviours over repeated interactions need to be  
125 explored explicitly. Only when repeated parent-offspring interactions  
126 converge towards the equilibrium, the equilibrium and the strategies are  
127 behaviourally stable (Fig. 1a). At such an equilibrium we refer to the pairs  
128 of strategies as behaviourally stable strategies (BSSs), represented by the  
129 slopes and/or shapes of the supply and demand functions. When repeated  
130 parent-offspring interactions diverge away from the equilibrium, the  
131 equilibrium and the strategies are behaviourally not stable (Fig. 1b). We  
132 used the standard mathematical techniques based on discrete-time  
133 dynamics to address the stability of behavioural equilibria (Otto & Day  
134 2007, pp. 163 - 169). In our model, discrete time steps are from one  
135 specific interaction (offspring demands, parent provides) to the next and

136 the process is started with an offspring begging event.

137 Our model in principle explores a behavioural negotiation process between  
138 parents and offspring (e.g. McNamara et al. 1999; Taylor & Day 2004;  
139 Johnstone & Hinde 2006) in which parent and offspring interact according  
140 to their behavioural reaction norms. In contrast to previous models (e.g.  
141 McNamara et al. 1999; Taylor & Day 2004; Johnstone & Hinde 2006), we  
142 focus on the behavioural process and allow for asymmetric functions, as we  
143 have parental supply and offspring demand in our model, which are  
144 represented by response mechanisms for different kinds of behaviour.

#### 145 *Numerical analyses*

146 To further explore the behavioural dynamics of parent-offspring  
147 interactions explicitly, we also used numerical simulations for the changes of  
148 demand (begging) level and supply (provisioning) level over a series of 2000  
149 time steps for a given set of supply and demand functions (Fig. 1). For the  
150 first time step the initial demand level  $D_0$  was given and for subsequent  
151 steps the value of the supply function was used as argument of the demand  
152 function and vice versa. The initial demand level was randomly chosen  
153 within 10% around the equilibrium  $D_{eq}$ . This choice increased the  
154 likelihood of reaching the equilibrium with 2000 interactions even when the  
155 process of convergence was slow. But any initial demand level could have  
156 been chosen. There was no effect of initial condition on the outcome  
157 (stability/instability) of the interaction (see Results). Values for supply  
158 and demand were represented on a standardized scale from 0 (minimum) to  
159 1 (maximum). The functions we used for supply and demand were strictly  
160 monotonic (see below). A maximum level of demand was set at the point of

161 no supply, resulting in intercepts of 1 for the demand function and 0 for the  
162 supply function. The interactions between parents and offspring were  
163 assumed to be error free. That is, the dynamics of the behavioural  
164 interaction strictly followed the supply and demand functions without any  
165 deviation. For simplicity we assumed a single offspring interacting with a  
166 single parent (Hussell 1988; Godfray 1991; Kölliker et al. 2005).

### 167 *Specific function types*

168 We numerically investigated the impact of two different types of functions  
169 on behavioural stability. Linear functions are the simplest and usual way to  
170 represent the parent-offspring interaction. They are standard in  
171 quantitative genetic maternal effect (Kirkpatrick & Lande 1989) and  
172 coadaptation models (Wolf & Brodie 1998; Kölliker 2003; Kölliker et al.  
173 2005), and reflect the local gradients in game-theoretic conflict resolution  
174 models (Godfray 1991; Parker et al. 2002). In addition, to simulate the  
175 consequences of slightly more complex responses on the behavioural  
176 dynamics, we used power functions (Smiseth et al. 2008), for which the  
177 slopes are not constant but change with the level of supply and demand.  
178 This may have important consequences for the behavioural dynamics and  
179 stability of the parent-offspring interaction.

180 In the linear case, the supply function was defined as  $S = g(D) = aD + y$ ,  
181 with slope  $a$  and intercept  $y = 0$  (Fig. 2a). The demand function was  
182 defined as  $D = f(S) = bS + x$ , with slope  $b$  and intercept  $x = 1$  (Fig. 2b).  
183 The intercepts reflect the previously mentioned assumption of maximal  
184 begging in the absence of provisioning. The slopes  $a$  of the supply function  
185 could vary between zero and infinity ( $0 < a < \infty$ ) and the slopes  $b$  for the



186 demand function between minus infinity to zero ( $-\infty < b < 0$ ).

187 In the case of power functions, the supply function was defined as

188  $S = g(D) = 1 - (1 - D)^k$  (Fig. 2c) and the demand function was defined as

189  $D = f(S) = (1 - S)^l$  (Fig. 2d). The parameters  $k$  and  $l$  define the

190 corresponding strength and direction of the curvature and could both range

191 from zero to infinity ( $0 < k < \infty$ ,  $0 < l < \infty$ ).

### 192 *Numerical application to ESS models*

193 The link between behavioural dynamics and evolutionary stability was

194 explored by applying our BSS condition to an already existing ESS model

195 in a numerical sensitivity analysis. We chose the scramble competition

196 resolution model, for which the role of supply and demand functions has

197 been made explicit (Parker 1985; Mock & Parker 1997; Parker et al. 2002).

198 But our analysis in principle also applies to honest signalling models

199 (Godfray 1991; Godfray & Johnstone 2000), although this is based on a

200 different (i.e. additive rather than multiplicative) fitness model. Scramble

201 competition models assume that parents have a fixed quantity of resources

202 available for reproduction. A unit of investment in a given offspring

203 enhances that offspring's survival chances, but at the expense of other

204 offspring the parent can produce. Offspring survival chances follow a curve

205 of diminishing returns with respect to the parental resources obtained

206 (Smith & Fretwell 1974; Parker 1985). Further, for evolutionary stability,

207 offspring begging has to be costly. For simplicity, we assume that offspring

208 survival decreases linearly with increased begging (Parker et al. 2002). To

209 allow direct comparison with the evolutionary model, the dimensionless

210 level of supply in our behavioural model can be interpreted in units of

211 parental investment.

212 The scramble competition ESS is defined via the local gradients of supply  
213 and demand functions at the ESS (Parker 1985; Mock & Parker 1997; see  
214 Appendix), which are parameters in the model and assumed to be  
215 nonevolving. We carried out a broad numerical sensitivity analysis, varying  
216 the gradients (i.e. the slopes) of the supply and demand functions and the  
217 parameters  $p$  and  $q$  of the associated cost and benefit functions of begging  
218 and provisioning, respectively (see Parker et al. 2002 and Appendix for  
219 details of the functions), to test for the behavioural stability of the ESS for  
220 specific sets of supply and demand functions (with known slopes and  
221 intercepts) and, hence, whether they are BSSs or not.

222 To explore numerically the parameter space of the scramble competition  
223 ESSs for converging behavioural dynamics we performed the following  
224 steps: 1) We generated a behavioural equilibrium grid for the supply and  
225 demand function pairs (1'000'000 behavioural equilibria; 1'000 demand  
226 levels and 1'000 supply levels, evenly distributed) in the parameter range of  
227 our model. 2) We calculated the ESSs according to the scramble  
228 competition model (Parker & Macnair 1979; Parker 1985; Parker et al.  
229 2002) in terms of a demand and supply function pairs (according to Eqs  
230 A-2 and A-3). 3) For all of these function pairs we checked whether the  
231 intercept of the demand function was close to 1 and that for the supply  
232 function close to 0 to fulfil the assumptions we made to derive the BSS  
233 conditions. Intercepts were deemed close enough when they deviated by less  
234 than 0.005. 4) The equilibria of those function pairs which satisfied these  
235 criteria were matched to the behavioural equilibrium grid generated before  
236 (see step 1). We took the numerical values for the behavioural equilibrium

237 and the evolutionary equilibrium to be equal when they were within a  
238 margin of  $\pm 0.001$ , which corresponds to the resolution of the behavioural  
239 equilibrium grid. 5) In cases where we found more than one function pair  
240 that numerically satisfied our matching conditions, we chose the one with  
241 the smallest mismatch to be the 'true' one, as we expected only one ESS  
242 per parameter combination  $p$  and  $q$ . Choosing another pair did not alter  
243 the final result (i.e. whether an ESS was behaviourally stable or not). 6)  
244 For the slope combinations where behavioural and evolutionary equilibrium  
245 matched, we checked whether or not the equilibrium was also behaviourally  
246 stable. For this we applied the stability conditions of our formal model.

247 We ran our model for 90 different fitness parameter value combinations  $p$   
248 and  $q$  in the cost and benefit functions of the scramble competition model  
249 (see Parker et al. 2002 and Appendix for details of the functions).  
250 Variation in these fitness parameters may have an impact on the outcome  
251 of the behavioural stability of the POC resolution, because different slopes  
252 for demand and supply functions are required to reach the ESS.

253 We used R version 2.4.0 (2006-10-03) and Mathematica 6.0.1.0 for the  
254 analyses and simulations (R Development Core Team 2006; Wolfram  
255 Research, Inc. 2007).

## 256 **Results**

### 257 *Stability of behavioural equilibria and numerical analyses*

258 We calculated the stability index  $\lambda$  following Otto & Day (2007) for  
259 discrete-time dynamics systems.  $\lambda$  allows us to determine if a dynamic

260 system that is close to an equilibrium converges towards, or diverges away  
261 from, this equilibrium from one time-step to the next. In our case, it is the  
262 derivative of the function describing the begging level at the next time  
263 step, which is a combination of the supply and demand function. In our  
264 model this is

$$\lambda = f'(S_n),$$

265 where  $f(S_n) = D_{n+1}$  is the begging level after one interaction when  
266 starting at  $D_n$ , with

$$D_{n+1} = f(S_n) = f(g(D_n)).$$

267  $S_n$  is the supply level and  $f(S_n)$  the demand function.  $f(g(D_n))$  is again  
268 the demand function, expressed as a function of the demand level one  
269 interaction before. The resulting value for  $\lambda$  is

$$\lambda = f'(S_n) = f'(g(D_n))g'(D_n) = f'(S_n)g'(D_n).$$

270 (Note that  $g(D_n) = S_n$ ).

271 For linear demand and supply functions we get

$$f(S_n) = D_{n+1} = b(aD_n + y) + x,$$

272 where  $x$  and  $y$  are the intercepts of the demand and supply function,  
273 respectively. And for  $\lambda$  we get

$$\lambda = ab.$$

274 A value of  $\lambda$  between  $-1$  and  $1$  indicates a stable dynamic equilibrium  
275 (Otto & Day 2007). So the general stability condition for the behavioural  
276 interaction between a parent and its offspring is

$$\lambda = |ab| < 1. \tag{1}$$

277 For the case of linear supply and demand functions, this condition is for  
278 local and global stability; it is not only valid at/near the equilibrium but  
279 for any initial begging level  $D_n$ , because the functions involved are linear  
280 and the slopes  $a$  and  $b$  apply over the whole range. This stability condition  
281 can also be proven by using the convergence criteria for infinite repeated  
282 interactions between parent and offspring (see Appendix).

### 283 *Numerical simulations*

284 Our numerical simulations deal only with a subset of all possible functions,  
285 namely a supply function with a positive slope  $a$  (parent increases supply  
286 level with increasing demand level) and a demand function with a negative  
287 slope  $b$  (offspring decreases demand level with increasing supply level) (Figs  
288 2a and b). This assumption is also made in scramble competition models  
289 (Parker et al. 2002). The stability condition under these assumptions and  
290 for linear supply and demand functions is

$$-1 < ab < 0.$$

291 This solution is a partial solution of the conditions for general stability  
 292 derived earlier (Eq. 1), confirming the result of our formal model. The  
 293 product of the two parameters  $a$  and  $b$  defines the behavioural stability of  
 294 linear parent-offspring interactions. When  $ab$  has a value between  $-1$  and  $0$   
 295 ( $-1 < ab < 0$ ), then the repeated interactions of parent and offspring  
 296 converge towards the equilibrium and reflect a BSS. In all these cases the  
 297 two functions intersect at a demand level between  $0.5$  and  $1$   
 298 ( $0.5 < D_{eq} < 1$ , Fig. 3a), under the assumptions of our model (i.e. slopes of  
 299 the functions are of opposite sign and the intercepts are  $0$  and  $1$ ,  
 300 respectively). The threshold level of  $0.5$  can be derived analytically (see  
 301 Appendix, A-1). That half of the function pairs that intersect above a  
 302 demand level of  $0.5$  represent a BSS. The half of the function pairs that  
 303 intersect below a demand level of  $0.5$  represent behaviourally not stable  
 304 strategies, resulting in divergence of the behavioural interaction.

305 For the power functions (Figs 2c and d), the local stability conditions  
 306 follow the general rule

$$-1 < f'(S_{eq})g'(D_{eq}) < 1. \quad (2)$$

307  $f'(S_{eq})$  is the derivative (local gradient) of the demand function at the  
 308 equilibrium and  $g'(D_{eq})$  is the derivative of the supply function at the  
 309 equilibrium.

310 According to the results of our simulation, this stability condition seems  
 311 not only valid at the equilibrium but over the whole value range of possible  
 312 initial demand levels (data not shown). So we can state the general  
 313 stability conditions for the power functions within the value range of our

314 numerical simulation as follows:

$$-1 < f'(S)g'(D) < 0.$$

315 This is equivalent to the stability condition for linear functions, but  
316 generalized to the case of non-linear power functions. The behavioural  
317 stability of the parent-offspring interaction depends only on the derivatives  
318 (the slopes) of the describing functions of demand and supply. Again,  
319 under the assumptions of our model, all supply and demand function pairs  
320 that intersect at a level of demand between 0.5 and 1 ( $0.5 < D_{eq} < 1$ ) have  
321 a behaviourally stable equilibrium (Fig. 3b), whereas those that intersect  
322 between 0 and 0.5 ( $0 < D_{eq} < 0.5$ ) do not.

### 323 *Numerical application to ESS models*

324 For 17 out of the 90 simulations the ESS was outside the considered  
325 parameter space or the intercepts deviated too much from 1 (for the  
326 demand function) or 0 (for the supply function). For 73 simulations we  
327 found ESSs inside the considered parameter space where the intercepts of  
328 the functions matched to 1 and 0, respectively. For 16 cases the difference  
329 between ESS and BSS was larger than our matching criteria (i.e. the  
330 difference in either demand level or supply level was larger than 0.001 (our  
331 grid resolution) and the function pair could hence not be clearly assigned  
332 to one grid point). Out of the remaining 57 cases where the intercepts  
333 fulfilled our matching criteria, 33 turned out to be behaviourally stable  
334 (BSS) and 24 were behaviourally not stable (Table 1).

335 ESSs were behaviourally stable in the lower range of explored begging costs

336 ( $q$  ranging from -0.1 to -0.35), representing higher numerical values for ESS  
337 levels of demand. For higher begging cost parameter values, and the  
338 correspondingly lower values for ESS levels of demand, the ESSs were  
339 behaviourally not stable. The parameter  $p$ , determining the benefit of  
340 parental supply for offspring survival, was not associated with behavioural  
341 stability (Table 1).for the begging cost parameter predicted ESSs that are  
342 also BSSs, and higher values for the begging cost parameter lead to  
343 predicted ESSs that are behaviourally not attainable (Table 1).

## 344 **Discussion**

345 Behavioural interactions based on continuous strategies generate complex  
346 dependencies and behavioural dynamics over time, raising the question of  
347 behavioural stability. To define a condition for behavioural stability in  
348 repeated interactions we proposed the novel concept of the behaviourally  
349 stable strategy (BSS): a BSS is characterized by behavioural dynamics of  
350 repeated interactions that converge towards the behavioural equilibrium.  
351 While the BSS is a proximate condition for the outcome of behavioural  
352 interactions, it has repercussions on evolutionary stability in a stricter  
353 sense. Non converging behavioural dynamics imply deviation from the  
354 expressed behavioural levels that would represent the ESS. Thus, by  
355 definition, behaviourally unstable strategies lead to deviation from the  
356 single-interaction or negotiation ESS (see below) and to corresponding  
357 fitness penalties. To illustrate this argument further, we have shown by  
358 simulations that there are realistic conditions under which ESSs for  
359 parent-offspring conflict resolution are behaviourally not stable. While this  
360 analysis is based on scramble competition resolution models (Parker et al.



361 2002), in principle the same basic conclusion apply to honest signalling  
362 models (Godfray 1991) because neither type of model incorporates the  
363 dynamics of repeated interactions.

364 Negotiation models (McNamara et al. 1999; Taylor & Day 2004; Johnstone  
365 & Hinde 2006) and quantitative genetic models of interacting phenotypes  
366 (Moore et al. 1997; Kölliker 2003) both assume  $|\lambda|$  to be smaller than 1 and  
367 thereby ensure behavioural stability in the predicted evolutionary  
368 outcomes. Our model provides the biological rationale for this critical  
369 assumption in negotiation and interacting phenotype models.

### 370 *Behavioural stability*

371 The behavioural stability of parent-offspring interactions at the equilibrium  
372 depends only on the derivatives (the change rates) of the supply and  
373 demand function. The absolute value of the product of the two derivatives  
374 has to be smaller than 1 (Eq. 2). Samuelson (1941, 1976) found analogous  
375 results for linear demand and supply functions in economics for the  
376 equilibrium prices of products. This result can be biologically interpreted  
377 such that the stability condition is likely to be violated when at least one  
378 interactant reacts too sensitively (i.e. it 'overreacts') to a behavioural  
379 change in the other interactant, leading to unsteady (oscillating) dynamics.

380

381 Under the assumptions of an intercept of 0 for the supply and of 1 for the  
382 demand function, the equilibrium is behaviourally stable for linear response  
383 functions as well as for power response functions, when the begging level at  
384 the equilibrium is larger than 0.5 (i.e. generally speaking the average

385 between the minimum and maximum level, see Appendix). We present the  
386 derivation for the linear case, although an equivalent solution can be shown  
387 for power functions (G. Nöldeke, personal communication; R. Dobler and  
388 M. Kölliker, unpublished results)

389 The increased complexity in the response functions from a linear to a  
390 power function had no impact on the outcome of the behavioural stability  
391 in our simulations. Nevertheless, we cannot reject the possibility that  
392 response functions with a more elaborate complexity (e.g. logistic  
393 functions, quadratic functions or higher order functions) may influence the  
394 behavioural stability, including the possibility of multiple alternative  
395 behavioural equilibria (i.e. more than one intersection point). In such more  
396 complex cases the stability may not only depend on the response functions  
397 and their derivatives but also on the starting conditions, adding another  
398 level of complexity to the model. For such models it would not only be  
399 interesting to find stability conditions but also to find possible conditions  
400 and circumstances that allow the interaction to change from one  
401 behavioural equilibrium to another. Under such more complex conditions,  
402 behaviourally stable equilibria may not always be attainable.

403 Communication errors and shifting response functions (e.g. due to changing  
404 environmental conditions or offspring age) may make the behavioural  
405 equilibrium shift over time, which would constantly reintroduce behavioural  
406 dynamics, likely rendering even stronger the expected selection pressure on  
407 behavioural reaction norms that allow fast behavioural convergence. In  
408 future models it would be interesting to incorporate perception errors, time  
409 lags and developmental/plastic function adjustments (Johnstone & Grafen  
410 1992; Johnstone 1994), and to study such plasticity in experimental work

411 (e.g. Hinde & Kilner 2007). Such inclusions would add realism and  
412 specificity to the model, but our major conclusion that the behavioural  
413 dynamics need to be addressed for an understanding of evolved strategies  
414 in repeated social interactions will most likely not be affected.

415 So far, empirical studies on behavioural dynamics mainly focused on the  
416 average effect which a change in behaviour (experimental or natural) of an  
417 interactant has on the behaviour of another interactant (e.g. Smith et al.  
418 1988; Kilner 1995; Ottosson et al. 1997; Kilner et al. 1999). Although these  
419 studies give valuable insight on the overall adjustment (change rate) and  
420 plasticity of behaviours, they do not address the underlying dynamics  
421 leading to the observed behavioural outcome. Experiments where a  
422 supposedly equilibrated system is deviated temporarily and the subsequent  
423 interaction-to-interaction dynamics analysed explicitly could provide the  
424 data required to determine to what degree a behavioural reaction norm  
425 (i.e. response rule) approach can actually be used to model the behavioural  
426 dynamics in repeated interactions (Roulin 2002; Hinde & Kilner 2007;  
427 Smiseth et al. 2008). Behaviourally stable strategies (BSSs) are expected to  
428 stabilise back to the initial equilibrium after the temporary disturbance.

#### 429 *Application of the BSS concept*

430 Applying our BSS model to scramble competition resolution models  
431 (Parker 1985; Mock & Parker 1997; Parker et al. 2002) confirms that  
432 predicted ESSs of single-interaction models are not necessarily  
433 behaviourally stable strategies. Some are either outside the parameter  
434 range or they represent a behaviourally unstable equilibrium. In both  
435 cases, what would be the ESS can behaviourally never be reached,

436 regenerating selection in the interactions. Under the specific conditions of  
437 the chosen intercepts, and from a perspective of behavioural stability,  
438 intermediate to high begging levels should be favoured over the course of  
439 selection, because only these can be the result of convergent behavioural  
440 dynamics. This may appear counterintuitive at first sight, as one would  
441 associate higher begging levels with increased costs, which should be  
442 evolutionarily unfavourable (Moreno-Rueda 2007). However, higher  
443 begging costs also imply lower evolved begging levels at the ESS (Parker  
444 1985; see Appendix) and when ESS begging levels exceed the level for  
445 behavioural stability, instability ensues (Table 1). Thus, there are  
446 behavioural stability constraints in parent-offspring interactions, which  
447 would favour evolutionary conflict resolutions with relatively high levels of  
448 begging and, hence, relatively low associated begging costs. In addition to  
449 the well-studied selection for optimal conflict resolution, we suggest  
450 selection on behavioural reaction norms that enhance the likelihood for  
451 behaviourally stable repeated interactions.

## 452 *Conclusions*

453 The importance of behavioural stability is not restricted to the functional  
454 context of parent-offspring interactions, but may include any context where  
455 fast-responding short-term interactions are involved (e.g. dominance  
456 interactions (Matsumura & Kobayashi 1998), biological markets (Noe &  
457 Hammerstein 1994, 1995), cell interactions (Hofmeyr & Cornish-Bowden  
458 2000), negotiation over care (McNamara et al. 1999; Taylor & Day 2004;  
459 Johnstone & Hinde 2006)).

460 There are usually many BSSs that are not an ESS. Many interactions

461 behaviourally converge but yield behavioural levels with suboptimal fitness  
462 consequences. This is not surprising, since behavioural stability alone tells  
463 us nothing about fitness. More revealing are the cases where a predicted  
464 ESS (in terms of a pair of slopes for the supply and demand reaction  
465 norms) is not a BSS. We could show for the scramble competition model  
466 (Parker 1985; Mock & Parker 1997; Parker et al. 2002) that behaviourally  
467 unstable ESSs are predicted when begging costs are of greater than some  
468 intermediate level. BSS and ESS are two conditions to evaluate the  
469 stability of repeated interactions that deal with the proximate and ultimate  
470 dimension of repeated interactions, respectively. Both need to be met for  
471 evolutionary stability in a stricter sense.

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477 **Appendix**

478 *Alternative proof of the behavioural stability conditions for*  
479 *linear behavioural reaction norms*

480 The change in begging over one time step is

$$D_{n+1} = b(aD_n + y) + x.$$

481 The change of begging level over  $n$  time steps is therefore

$$D_n = a^n b^n D_0 + a^{n-1} b^n y + a^{n-1} b^{n-1} x + a^{n-2} b^{n-1} y + a^{n-2} b^{n-2} x + \dots + abx + by + x.$$

482 Factoring out  $a^i b^i$  for  $0 \leq i \leq (n-1)$  leads to

$$D_n = a^n b^n D_0 + (x + yb) \sum_{i=0}^{i=n-1} a^i b^i.$$

483 The second term on the right hand side is a geometric series that for  
484  $n \rightarrow \infty$  only converges, when  $|ab| < 1$ . For this case ( $|ab| < 1$ ) the first  
485 term on the right hand side converges to 0. From this we can conclude that  
486 the repeated interaction only converges towards the equilibrium ( $D_{eq}$ )  
487 when  $|ab| < 1$ . Otherwise the interaction diverges.

488 The demand level at the equilibrium  $D_{eq}$  (for  $|ab| < 1$ ) is:

$$D_{eq} = \lim_{n \rightarrow \infty} a^n b^n D_0 + (x + yb) \sum_{i=0}^{i=n-1} a^i b^i = \frac{x + yb}{1 - ab}.$$

489 For the linear case in our simulation we have  $x = 1$  and  $y = 0$ , resulting in

$$D_{eq} = \frac{1}{1 - ab}. \quad (\text{A-1})$$

490 This proves that the behavioural equilibrium is only stable if and only if

491  $D_{eq} > 0.5$  (because  $|ab| < 1$ ).

492 More generally, the threshold level for stability/instability  $D_{eq}$  for any

493 intercepts  $x$  and  $y$  is

$$D_{eq} = \frac{x + y}{1 - ab} = \frac{x + y}{2}.$$

494 Because  $1 - ab < 2$ , the lowest value for  $D_{eq}$  is the mean of the two

495 intercepts  $x$  and  $y$ , representing the threshold demand level for BSS.

#### 496 *Details of the scramble competition model*

497 Following Mock & Parker (1997) and Parker et al. (2002), we used the

498 following functions for cost and benefit. For the costs of begging we used

499 the survival probability  $\kappa$ :

$$\kappa(D) = pD + 1,$$

500 with  $p$  as the parameter (slope) for the cost of begging ( $-1 < p < 0$ ) and

501 an intercept of 1. As the costs reduce the net benefit, the intercept of 1

502 indicates no costs when there is no begging. Values of  $p$  close to 0 indicate

503 a weak begging cost which becomes higher the more  $p$  diverges from 0.

504 With  $p = -1$ , maximal begging results in a survival probability of 0. For



505 the benefit ( $\mu$ ) of supply we used

$$\mu(S) = 1 - e^{-q(S-0.1)},$$

506 with the parameter  $q$  for the benefit of supply bigger than 0 ( $q > 0$ ).

507 Applying these functions to the ESS conditions of the scramble

508 competition model (Parker 1985, Eqs 2 and 3; Mock & Parker 1997) with

$$\beta = \frac{0.5v}{v-1}$$

509 for the case of 'true monogamy' (see Parker 1985) and

$$\alpha + \beta = 1.$$

510  $\alpha$  and  $\beta$  are coefficients assigned to special levels of sib competition,

511 parental care and mating system.  $v$  is the product of the two gradients of

512 the supply and demand functions (Parker et al. 2002)

$$v = g'(D_{eq})f'(S_{eq}).$$

513 This leads to

$$D_{ESS} = \beta \frac{\kappa(D_{ESS})}{\kappa'(D_{ESS})} = \left( \frac{0.5ab}{ab-1} \right) \frac{pD_{ESS} + 1}{D_{ESS}} \quad (\text{A-2})$$

514 as the stability condition for demand and

$$S_{ESS} = \alpha \frac{\mu(S_{ESS})}{\mu'(S_{ESS})} = \left(1 - \frac{0.5ab}{ab-1}\right) \frac{1 - e^{-q(S_{ESS}-0.1)}}{e^{-q(S_{ESS}-0.1)k}} \quad (\text{A-3})$$

515 as the stability conditions for supply. This are the two strategies offspring  
 516 and parents, respectively, should play to solve the POC from an  
 517 evolutionary perspective.

518 Note:  $v$  (see Parker et al. 2002) is equivalent to  $\lambda$  in our condition for  
 519 behavioural stability.

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**Table 1:** Simulation results for behavioural and evolutionary stability for different parameter combinations  $p$  and  $q$  in the cost and benefit functions. 'NA' indicates parameter combinations with no evolutionary equilibrium in the parameter range of our simulation. 'YES' indicates parameter combinations with evolutionary equilibria which are also behavioural equilibria. 'yes' indicates parameter conditions with evolutionary equilibria that are also behavioural equilibria but where the difference of the two is larger than 0.001 (our grid resolution) in at least one dimension (supply or demand). 'no' indicates parameter combinations with evolutionary equilibria which are no behavioural equilibria. Begging cost increases from left to right. Provisioning benefit increases from top to bottom. The reason why we could not clearly assign some evolutionary equilibria to unique grid points in our simulation ( $p$  values of  $-0.15$  and  $-0.2$ ) is, that the sum of the deviations of the intercepts for the two functions was in these cases larger than the grid resolution, although taken separately each intercept fulfilled our matching criteria. Hence, our inability to assign these evolutionary equilibria is a result of our matching criteria for the intercepts rather than a methodological problem in the simulation of repeated interactions. Therefore, it is likely that these evolutionary equilibria represent the proper behavioural equilibria.

		Begging cost parameter $p$									
		-0.05	-0.1	-0.15	-0.2	-0.25	-0.3	-0.35	-0.4	-0.45	-0.5
Provisioning benefit parameter $q$	0.5	NA	YES	NA	NA	NA	NA	NA	NA	NA	NA
	1	NA	YES	yes	yes	YES	YES	YES	no	no	no
	1.5	NA	YES	yes	yes	YES	YES	YES	no	no	no
	2	NA	YES	yes	yes	YES	YES	YES	no	no	no
	2.5	NA	YES	yes	yes	YES	YES	YES	no	no	no
	3	NA	YES	yes	yes	YES	YES	YES	no	no	no
	3.5	NA	YES	yes	yes	YES	YES	YES	no	no	no
	4	NA	YES	yes	yes	YES	YES	YES	no	no	no
	4.5	NA	YES	yes	yes	YES	YES	YES	no	no	no

**Figure 1:** Dynamics of repeated interactions. By superimposing the demand function and the inverted supply function the repeated interdependent parent-offspring interaction can be graphically visualized. Starting at a demand level  $D_n$  (filled square) leads over repeated interactions to the supply level  $S_{n+4}$ . The equilibrium ( $S_{eq}, D_{eq}$ , solid diamond) represents the point of behavioural matching between parents and offspring over supply and demand level, respectively. a) A behaviourally stable parent-offspring interaction converges toward the equilibrium. b) An interaction that diverges away from the equilibrium is behaviourally not stable.

**Figure 2:** Illustration of the used supply and demand function types (behavioural reaction norms). a) Inverted linear supply functions (to superimpose with the demand function) for different slopes. b) Different slopes for linear demand functions. c) Several different inverted supply power functions (to superimpose with demand functions). d) Different demand power functions.

**Figure 3:** Supply (dashed) and demand (solid) functions which intersect in the grey shaded area represent behaviourally stable strategies (BSS). Function pairs with an intersection in the white area represent a behaviourally unstable strategy. a) Linear response functions. One supply function and two different demand functions. One combination leads to a BSS, the other is behaviourally not stable. The threshold level of 0.5 was derived analytically. b) Same as in a) but with power functions and threshold level according to the results of our numerical simulations. In a) and b) the inverted supply function is drawn to superimpose the two functions.