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The Impact of Heterogeneous Signals on Stock Price Predictability in a Strategic Trade Model *

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Abstract

Generalizing the idea that price momentum can be explained by different levels of uncertainty inherent in the information structure, we implement signal-specific differences in uncertainty in a Kyle type model of strategic trading. We derive the equilibrium in a single-auction setting as well as a two-trading-period model. We show that the two-period equilibrium supports price patterns like momentum and reversal/under- and over-reaction without relying on any additional behavioral assumptions. Furthermore, the two-period setting can be extended to a multiple-trading-period equilibrium model with very similar equilibrium conditions to the original sequential auction equilibrium proposed by Kyle (1985), while preserving the price pattern of the two-period model.

JEL Classification: D43, D82, G12, G14.

Keywords: Market Structure; Asset Pricing; Market Efficiency; Asymmetric Information, Equilibrium.

*Any errors are solely my responsibility.

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1 Introduction

As brought up by Jegadeesh and Titman (1993), price momentum and reversal are the two most pervasive pricing anomalies existing on financial markets. They are difficult to explain and extremely inconsistent with information efficient markets as well as rational agents. According to rational explanations, the existence of these anomalies should be almost impossible; otherwise, they should immediately be exploited and traded away. Besides Holden and Subrahmanyam (2002) and Cespa and Vives (2012), who explain momentum with an increase in information precision and Andrei and Cujean (2017), who provide a rational model by employing word-of-mouth communication in a Grossman and Stiglitz (1980) setting, most of the prevailing models trying to explain these phenomenoa are settled in the behavioral literature. The basic idea in this field of research is to explain momentum and reversal by the irrational behavior of agents, like Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (2007). These explanations attribute to individuals various behavioral biases and assume that agents would persistently act irrationally.

In this paper we shift focus from the agent to the information signal and propose the idea that momentum and reversal are not caused by possible irrational behaviour of agents, but rather due to the information structure on the market. We impose that not all information is equal but differs regarding its uncertainty. Using this line of thought, implementing a heterogeneous signal structure results in the occurrence of signal-dependent under- and over-reaction, compared to a model with homogeneous information. This establishes momentum and reversal patterns without the necessity to assume the irrationality of agents.

In the literature on asset pricing under asymmetric information, there are two competing equilibrium concepts, namely Rational Expectations Equilibrium (REE) and game theoretic Bayesian Nash Equilibrium (BNE). We translate the above stated idea of heterogeneous information by implementing the information structure of signal-specific differences in uncertainty in a Bayesian equilibrium model of strategic trading. We show that in such a model, momentum and reversal—two patterns seemingly inconsistent with rational behavior—can be explained without recourse to the irrationality of traders. This provides some evidence that the rational existence of momentum and reversal is nested in the posed idea concerning the presence of heterogeneous information with different levels of uncertainty.

The analysis in this paper is based on a Kyle (1985) type of model. The original setting is modified in two dimensions: first, the informed trader only observes a noisy signal about the true liquidation value of the risky security and not the original liquidation value; and second, a heterogeneous information structure is introduced. There exist two kinds of different signals, one of them having a higher degree of uncertainty than the other. The difference in uncertainty is modelled with the help of a higher variance in the noise term of the signal.

We show that given these two modifications, the basic mechanics of the model still work, although very interesting new insights into the behavior of agents emerge. Furthermore, equilibrium prices support patterns of momentum and reversal.

The remainder of the paper is structured as follows. First, the modified structure of the model is introduced by discussing a single-auction equilibrium to show the main implications of the modifications. In a second step, we establish a two-period model and prove that equilibrium prices in this setting on average support price patterns like momentum and reversal. The two-period model is then further extended to a sequential auction equilibrium with N periods, which features very similar equilibrium conditions as those in Kyle (1985), the original paper. The main finding is that besides other implications, a heterogeneous signal structure supports price momentum and price reversal in a simple game theoretic model with a BNE.[1]

2 The static model

There exist three different groups of risk-neutral players. An investor who is informed, since he observes a signal concerning the true fundamental value of a risky asset (henceforth referred to as the informed investor or insider). Many different liquidity traders who do not maximize their utility but simply trade for reasons outside of the model ("noise traders"). Their demand might e.g. stem from idiosyncratic information that is not of common interest, such as the need to hedge against endowment shocks or private investment opportunities in incomplete market settings (see Brunnermeier, 2005, p. 421). A risk-neutral market maker who knows the signal structure of the informed, as well as the aggregate order flow of the informed investor and noise traders. However, he does not know the exact signal and the exact distribution it comes from. The basic process is as follows. The informed investor observes a signal about the true value of the risky asset. Given this information and taking into account his influence on price, he acts like an information monopolist and places a market order. The noise traders— trading for reasons outside of the model—submit their individual demand. Given this information, the market maker sets a price, after observing aggregate order flow. When doing so, he considers the information structure on which the informed trader bases his orders. The whole model has the flavor of a BNE model as all players take the strategies of all the other players as given and the market maker updates his beliefs using Bayes' rule. How-

[1]The stated results are established under the restriction that the informed agent is only allowed to play linear strategies. Relaxing this assumption should not alter the general mechanics of the model, however it extremely increases the mathematical complexity and precludes closed form solutions.

ever, we concentrate our analysis on the case in which the informed agent is restricted to playing linear strategies, as they are the only ones allowing for a closed-form solution.

2.1 Derivation of the single-auction equilibrium

In a first step and to gain a better understanding, we study the major implications of the modified setting by analyzing a static model of one-shot trading.

2.1.1 Environment

In this setting, a single risk-neutral informed trader and a number of uninformed liquidity traders submit market orders to a risk-neutral market maker. The market maker observes the aggregated order flow and clears the market at a single price.[2] The informed trader does not know the actual demand of the noise traders when submitting his order, but only its distribution. The ex-post liquidation value of the risky asset is denoted by θ, which is distributed normally with mean $\hat{\theta} > 0$ and variance σ_θ^2. The quantity demanded by noise traders denoted by u is normally distributed with mean zero and variance σ_u^2.[3] There are two possible noisy signals, S_i with $i \in \{H, L\}$ about the true value of θ, $S_H = \theta + \epsilon_H$ and $S_L = \theta + \epsilon_L$, which differ in their noise term ϵ. Both noise terms are normally distributed with mean zero, but have different variances $\sigma_{\epsilon_H}^2$ and $\sigma_{\epsilon_L}^2$. Their relationship is restricted by the inequality $\sigma_{\epsilon_H}^2 \geq \sigma_{\epsilon_L}^2$. This implies that S_L is a more valuable signal than S_H. It has higher precision and hence incorporates less uncertainty. The informed trader always observes only one signal, either S_H or S_L, with probability p and $1-p$, respectively. However, he knows whether the signal that he observes is S_H or S_L. The demand of the insider is labeled x and aggregate demand X, which comprises the quantities requested by informed and noise traders $X = x + u$, with u being the demand of the noise traders. The price is referred to as P.

The time line of events is almost identical to the original setting in (Kyle, 1985). First, θ is realized, the informed trader observes the signal S_i about the true value of θ and chooses his order size x. Additionally, the uninformed traders' demand u is realized. Second, the market maker observes aggregate order flow $x + u$ and sets a single price P to clear the market. Finally, uncertainty resolves and the asset pay off is realized. Figure 1 gives a time line of the events.

[2] As is common in this literature, the market maker is assumed to set semi-strong informationally-efficient prices; thus, his expected profit is zero. We do not explicitly model the underlying Bertrand competition with potential rival market makers in this paper.

[3] The fact that noise traders' demand is assumed to be normally distributed makes it unnecessary to specify off-equilibrium beliefs. Due to the normality assumption, the support of aggregate order flow ranges from $-\infty$ to ∞ and any order flow can potentially arise in equilibrium.

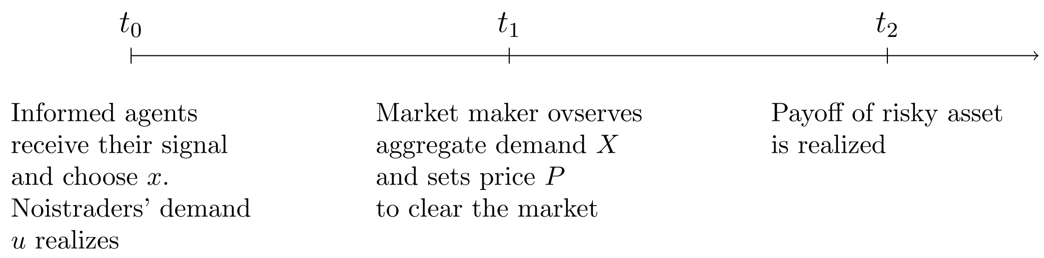

Figure 1: The time line shows the sequence of events in the model. At $t = 0$, the informed agents receive their private signal S_i and posit their market order x. Further noise traders' demand u is realized. In the next step at $t = 1$, financial markets open, the market maker observes aggregate demand and sets a price P to clear the market. Uncertainty is resolved at $t = 3$ and the pay off of the risky asset θ materializes

The expected profits $E[\pi|S_i]$ of the informed trader are given by $E[\pi|S_i] = (E[\theta|S_i] - P)x'$. In equilibrium, two conditions have to hold.

1. Profit maximization: For any trading strategy x' and for any signal S

$$E[\pi(x, P)|S_i] \geq E[\pi(x', P)|S_i].$$

2. Market efficiency: The price set by the market maker satisfies

$$P(X, P) = E[\theta|x + u].$$

Informed demand x is a function of the type of the signal S_i, the price P as well as the parameters of the distribution of uninformed demand u. It can be written as $x(S_i, P, u)$. The informed agent chooses his demand taking into account his impact on price as well as the pricing rule of the risk-neutral market maker. The signal structure of the model can be summarized as follows.

Signal structure

The terminal value of the asset is given by $\theta \sim \mathcal{N}(\hat{\theta}, \sigma_\theta^2)$. As the informed trader observes one of two possible noisy signals S_H or S_L of the true value of the risky asset with probability p and $1 - p$, respectively. The signals follow a mixture distribution, with a probability density function f defined as

$$f(S) = pf_{S_H}(S) + (1 - p)f_{S_L}(S) \tag{1}$$

The components of this mixture distribution are distributed as follows

$$\begin{aligned} S_L &= \theta + \epsilon_L \qquad \epsilon_L \sim \mathcal{N}(0, \sigma_{\epsilon_L}^2) \qquad \longrightarrow \qquad S_L \sim \mathcal{N}(\hat{\theta}, \sigma_\theta^2 + \sigma_{\epsilon_L}^2) \\ S_H &= \theta + \epsilon_H \qquad \epsilon_H \sim \mathcal{N}(0, \sigma_{\epsilon_H}^2) \qquad \longrightarrow \qquad S_H \sim \mathcal{N}(\hat{\theta}, \sigma_\theta^2 + \sigma_{\epsilon_H}^2) \end{aligned} \tag{2}$$

The informed trader knows which of the two signals S_H or S_L he observes. θ and S_L as well as θ and S_H are distributed bivariate normal $\mathcal{N} \sim (\boldsymbol{\mu_{S_L}}, \boldsymbol{\Sigma_{S_L}})$ and $\mathcal{N} \sim (\boldsymbol{\mu_{S_H}}, \boldsymbol{\Sigma_{S_H}})$, with

$$\boldsymbol{\mu_{S_L}} = \boldsymbol{\mu_{S_H}} = \begin{pmatrix} \hat{\theta} \\ \hat{\theta} \end{pmatrix}, \quad \boldsymbol{\Sigma_{S_L}} = \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_{\epsilon_L}^2 \end{pmatrix}, \quad \boldsymbol{\Sigma_{S_H}} = \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_{\epsilon_H}^2 \end{pmatrix}.$$

Given the above distributions and applying the projection theorem for jointly normal distributed variables, we can calculate the values of $E[\theta|S_H],\ Var[\theta|S_H],\ E[\theta|S_L]$ $and\ Var[\theta|S_L]$.

$$E[\theta|S_L] = \hat{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_L}^2}(S_L - \hat{\theta}) \quad and \quad Var[\theta|S_L] = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_{\epsilon_L}^2} \tag{3}$$

$$E[\theta|S_H] = \hat{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_H}^2}(S_H - \hat{\theta}) \quad and \quad Var[\theta|S_H] = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2 + \sigma_{\epsilon_H}^2} \tag{4}$$

2.1.2 Optimization

According to the market efficiency condition and due to risk neutrality, the pricing rule of the market maker in equilibrium is a function of aggregate demand and is given by $E[\theta|X]$. The informed agent maximizes his expected profit, $E[\pi|S_i]$, which comprises the difference between the expected pay off of the asset given the signal and the price of the asset times the quantity of his stock holdings. The maximization problem of the informed agent writes

$$\max_x \ E[\pi|S_i] = E[(\theta - P_1)x|S_i]. \tag{5}$$

The risk-neutral market maker follows a linear pricing rule of the form $P_1 = P_0 + \lambda X$ while implying that the informed investor's demand is linear in θ and has the form $x = \beta(E[\theta|S_i] - P_0)$. λ can be interpreted as the responsiveness of the market maker to changes in aggregate supply and hence market depth. β defines the trading aggressiveness of the informed investor. If β is high, the informed investor reacts more strongly upon his private information. P_0 denotes the unconditional expectation of the risky asset's pay off, $P_0 = E[\theta] = \hat{\theta}$. The market maker can only observe the aggregate demand of informed and noise traders, $X = x + u$, and is unable to distinguish between x and u. He sets the price conditional on his information set $\mathcal{F}^{\mathcal{U}}$ which comprises aggregate demand X, $\mathcal{F}^{\mathcal{U}} = \{X\}$. Given this pricing rule, the optimization problem 5 of the informed trader can be rewritten as

$$\max_x \; E[(\theta - P_0 - \lambda(x+u))x|S_i],$$

resulting in the FOC

$$E[\theta|S_i] - P_0 - 2\lambda x = 0,$$

with x being

$$x = \frac{1}{2\lambda}(E[\theta|S_i] - P_0). \tag{6}$$

The SOC implies $\lambda > 0$, which we have imposed. Given $x = \beta(E[\theta|S_i] - P_0)$, equation 6 yields $\beta = \frac{1}{2\lambda}$. It becomes visible that x inherits its distribution from S_H and S_L. Using the results 3 and 4 and specifying

$$F_L = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_L}^2} \quad and \quad F_H = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_H}^2} \tag{7}$$

and hence,

$$Var\left[E[\theta|S_H]\right] = F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) \quad and \quad Var\left[E[\theta|S_L]\right] = F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2).$$

Informed demand, x, is defined as

$$\begin{aligned} x_H &= \beta(E[\theta|S_H] - \hat{\theta}) &&= \beta F_H(S_H - \hat{\theta}), \\ x_L &= \beta(E[\theta|S_L] - \hat{\theta}) &&= \beta F_L(S_L - \hat{\theta}), \end{aligned}$$

and distributed

$$x_H \sim \mathcal{N}(0, \beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)) \quad and \quad x_L \sim \mathcal{N}(0, \beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)).$$

Aggregate demand, X, is defined and distributed as

$$\begin{aligned} &X_H = x_H + u = \beta F_H(S_H - \hat{\theta}) + u \quad and \quad X_L = x_L + u = \beta F_L(S_L - \hat{\theta}) + u, \\ &X_H \sim \mathcal{N}(0, \beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2) \quad and \quad X_L \sim \mathcal{N}(0, \beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2). \end{aligned}$$

Given the distributions of X_H and X_L, X and θ are distributed jointly normal.

$$\begin{aligned}
\begin{matrix}\theta \\ X_H\end{matrix} &\sim \mathcal{N}\left[\begin{pmatrix}\hat{\theta} \\ 0\end{pmatrix}, \begin{pmatrix}\sigma_\theta^2 & \beta F_H \sigma_\theta^2 \\ \beta F_H \sigma_\theta^2 & \beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2\end{pmatrix}\right] \\
\begin{matrix}\theta \\ X_L\end{matrix} &\sim \mathcal{N}\left[\begin{pmatrix}\hat{\theta} \\ 0\end{pmatrix}, \begin{pmatrix}\sigma_\theta^2 & \beta F_L \sigma_\theta^2 \\ \beta F_L \sigma_\theta^2 & \beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2\end{pmatrix}\right]
\end{aligned}$$

Based on the joint distribution of X and θ, the market maker sets the price, P, conditional on the amount of aggregate demand X that he observes, $P = E[\theta|X]$. The market maker cannot distinguish between the two demands X_H and X_L. He simply observes X, which is either X_H or X_L with probability p and $1-p$. Therefore, X is distributed as a Gaussian mixture. Due to the inherited mixture structure of X, the conditional expectation is a weighted average

$$\begin{aligned}
P_1 &= E[\theta|X] = \omega_L E[\theta|X_L] + \omega_H E[\theta|X_H] \\
&= \hat{\theta} + \left(\omega_L \frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + \omega_H \frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}\right) X,
\end{aligned}$$

with the weights ω_L and ω_H being defined as

$$\begin{aligned}
\omega_L &= \frac{(1-p) f_{X_L}(X)}{(1-p) f_{X_L}(X) + p f_{X_H}(X)}, \\
\omega_H &= \frac{p f_{X_H}(X)}{(1-p) f_{X_L}(X) + p f_{X_H}(X)},
\end{aligned} \tag{8}$$

and $\omega_L = (1 - \omega_H)$.

2.1.3 Equilibrium

Determining the coefficients in the linear BNE yields

$$\begin{aligned}
\beta &= \frac{1}{2\lambda}, \\
\lambda &= \omega_L \frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + \omega_H \frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}.
\end{aligned} \tag{9}$$

A special feature of the model is that λ is not a constant but rather a function of aggregate demand X, since according to 9, ω_i directly depends on the realization of S_i. However, in equilibrium the informed does not know the actual value of X, as he does not know noise traders' demand u. Hence, he does not know λ and he optimizes using the expected value $E[\lambda]$. Equation 9 becomes

$$\beta = \frac{1}{2E[\lambda]} \quad \textit{and} \quad E[\lambda] = (1-p)\frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + p\frac{\beta\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}. \quad (10)$$

In equilibrium, the coefficients $E[\lambda]$ and β are defined in terms of parameters of the distributions by solving for β and plugging in $E[\lambda] = \frac{1}{2\beta}$. After simplifying, β is implicitly characterized by

$$0 = (1-p)\frac{\beta^2\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + p\frac{\beta^2\sigma_\theta^4}{\beta^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} - \frac{1}{2}.$$

Solving the quadratic equation for β means solving a fourth-order polynomial, which has four solutions. According to the SOC, β has to be real and positive. Thus, three of the four solutions can be ruled out immediately, and the one surviving is

$$\beta = \frac{\sqrt{\sigma_u^2}}{\sqrt{2\sigma_\theta^2}}\sqrt{(2p-1)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2) + \sqrt{\left((\sigma_{\epsilon_L}^2 + \sigma_\theta^2) + (\sigma_{\epsilon_H}^2 + \sigma_\theta^2)\right)^2 - 4(p-p^2)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)^2}}. \quad (11)$$

For readability, define

$$B_p = \sqrt{\frac{1}{2}\left((2p-1)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2) + \sqrt{\left((\sigma_{\epsilon_L}^2 + \sigma_\theta^2) + (\sigma_{\epsilon_H}^2 + \sigma_\theta^2)\right)^2 - 4(p-p^2)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)^2}\right)}, \quad (12)$$

with $\sqrt{(\sigma_{\epsilon_L}^2 + \sigma_\theta^2)} < B_p < \sqrt{(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)}$ for $0 < p < 1$, $\sqrt{(\sigma_{\epsilon_L}^2 + \sigma_\theta^2)} = B_p$ for $p = 0$, $B_p = \sqrt{(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)}$ for $p = 1$, and $\frac{\partial B_p}{\partial p} > 0$.
equation 11 can be written as

$$\beta = \frac{\sqrt{\sigma_u^2}}{\sigma_\theta^2}B_p.$$

To gain a basic understanding of the underlying mechanics, we want to take a closer look at the special case $p = \frac{1}{2}$. This gives

$$\beta = \frac{\sqrt{\sigma_u^2}}{\sigma_\theta^2}\sqrt[4]{(\sigma_{\epsilon_L}^2 + \sigma_\theta^2)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)}. \quad (13)$$

The above expression is nothing but $\frac{\sqrt{\sigma_u^2}}{\sigma_\theta^2}$, which is the original equilibrium solution in (Kyle, 1985, p. 1319) without heterogeneous signals times the square root of a geometric average of the signal variances.

$E[\lambda]$ - which in the following will be denoted as $\hat{\lambda}$ - equals

$$\hat{\lambda} = \frac{\sigma_\theta^2}{2\sqrt{\sigma_u^2}} \frac{1}{B_p}. \tag{14}$$

A special feature of the model is that λ is not a constant in equilibrium. It crucially depends on ω_H, which itself is a function of aggregate demand X. In equilibrium, λ is hence dependent on the level of aggregate demand. The same is true for market depth, defined as $\frac{1}{\lambda}$. If market depth is high, meaning that λ is very low, then an increase in (aggregate) demand has only a small impact on the stock price. The opposite is true for high values of λ and thus low market depth.

Knowing $\hat{\lambda}$, the market depth expected by the insider $\dfrac{1}{\hat{\lambda}}$ is given by

$$\frac{1}{\hat{\lambda}} = \frac{2\sqrt{\sigma_u^2}}{\sigma_\theta^2} B_p. \tag{15}$$

The market depth expected by the informed trader depends on the amount of noise trading given by $\sqrt{\sigma_u^2}$, the variance of the risky asset σ_θ^2, the variance of the two signals S_H and S_L as well as the mixture weight p. The amount of noise trading and a high signal variance increase market liquidity, while an increase in the volatility of the asset pay off—which can be seen as an increase in the value of the insider's information—reduces liquidity as in Kyle (1985). However, the effect of an increasing asset variance is accommodated by an increasing signal variance. This means that while ceteris paribus a doubling in the amount of noise trading measured by its standard deviation σ_u doubles market liquidity, a doubling in the asset's variance does not halve market depth as it simultaneously increases B_p, $\frac{\partial B_p}{\partial \sigma_\theta} > 0$. The actual market depth in the economy is random and depends on ω_L and ω_H, hence aggregate demand. Overall market depth in the economy is given by $E[1/\lambda]$. Due to Jensen's inequality, overall market depth in the economy exceeds the market depth expected by the insider when conducting his maximization,

$$\frac{2\sqrt{\sigma_u^2}}{\sigma_\theta^2} B_p < E\left[\frac{1}{\lambda}\right].$$

The ex-ante expected profit of the informed trader (unconditional on S) is given by

$$E[\pi] = p\frac{1}{2}\sqrt{\sigma_u^2} B_p \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} + (1-p)\frac{1}{2}\sqrt{\sigma_u^2} B_p \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)}.$$

The informed's ex-ante expected profit is also strongly affected by the standard deviation of noise traders' demand. It increases with the variance of the asset's pay off σ_θ, as it measures the informational advantage of the insider. Furthermore, $E[\pi]$ decreases with the amount of noise inherited in the informed trader's signal as this noise distorts his informational advantage. By comparison, the insider's ex-post maximized profit conditional on S_i is of the form $\frac{(E[\theta|S_i]-\hat{\theta})^2}{4\hat{\lambda}}$, which is either

$$E[\pi|S_H] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2)^2}B_p\left(S-\hat{\theta}\right)^2 \quad \text{or}$$
$$E[\pi|S_L] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2)^2}B_p\left(S-\hat{\theta}\right)^2.$$

2.2 Comparison with benchmark

In this section, we compare the results of the static equilibrium derived in the previous section with a benchmark setting in which the type of signal is common knowledge.[4] Hence, the market maker knows the underlying distributions of S and X. This can be seen as the classical Kyle (1985) setting with two regimes and the insider receives a noisy signal about the fundamental value of the asset rather than the asset value itself. All involved parties know the prevailing regime that they are currently in. A part of the variables defining the equilibrium as well as the key statistics of the model are signal-dependent. In comparison, the respective variables in the benchmark setting are all constants and not affected by the value of the signal. We have to bear this in mind when comparing the equilibrium solutions of the two models. It can be shown that for reasonable differences between σ_{ϵ_H} and σ_{ϵ_L}, B_{ω_H} is a convex function. Hence, in the following paragraph $E\left[B_{\omega_H}\right]$ is replaced by $B_{E[\omega_H]}$. This means that the comparison is conducted by using the prior component weights of the mixture p and not their Bayesian updates ω_H as it makes the calculations much simpler and will not alter the direction of the results. The equilibrium price given a heterogeneous signal structure is either above or below the price without a heterogeneous signal, depending on the type of the signal.[5] The overall expected informativeness of the price is unaffected by the heterogeneous signal structure and in both cases it is unaffected by the amount of noise trading like in Kyle (1985). However, given the two different signal regimes, the average volatility over- or understates the volatility in the respective single regimes. Market depth in the informationally-restricted model is not constant, but rather a function of actual aggregate demand. Overall, it is lower than in the benchmark model. Nevertheless, this does

[4]Common knowledge implies that the informed trader as well as the market maker have access to the signal type and take the information into account. Regardless whether they know the signal or its type, the noise traders ignore it and act for reasons outside of the model, as already stated.

[5]This ambiguity is the root of resulting over- and under-reaction occurring in a setting comprising more than one period described in section 3.3.1.

not hold looking at signal-dependent market depth. In the regime with high uncertainty, market depth is on average lower, whereas in the regime with lower uncertainty market depth is on average higher than in the benchmark setting. In general, the unconditional expected profit in equilibrium assumes that asymmetric information regarding the signal structure exceeds that without asymmetric information about the signal structure. This is true unless the difference in the signal variances becomes marginal. The expected profit of the insider further increases with the variance of the asset as well as the amount of noise trading. By contrast, an increase in the noisiness of the signal reduces profit. The results are the same when analyzing maximized profits.

Price Depending on the signal, the equilibrium price is either higher or lower than the equilibrium price of the benchmark model.[6] The prevailing mechanism is that given a signal with high uncertainty, the equilibrium price exceeds the price of the benchmark model. In the regime with low signal variance, the equilibrium price is below that of the benchmark model. The reason for this behavior is that the equilibrium price can be seen as a weighted average of the prices in the two regimes of the benchmark model and thus it has to be somewhere in between the two. Technically speaking, when looking at the linear pricing rule of the market maker $P(X) = \hat{\theta} + \lambda X$, with $\lambda_{Ben_{S_H}} = \frac{1}{2}\frac{\sigma_\theta^2}{\sqrt{\sigma_u^2(\sigma_\theta^2+\sigma_{\epsilon_H}^2)}}$, $\lambda_{Ben_{S_L}} = \frac{1}{2}\frac{\sigma_\theta^2}{\sqrt{\sigma_u^2(\sigma_\theta^2+\sigma_{\epsilon_L}^2)}}$ and $E[\lambda_{Equi}] = \frac{1}{2}\frac{\sigma_\theta^2}{\sqrt{\sigma_u^2 B_p}}$, it holds that $\lambda_{Ben_{S_H}} < \hat{\lambda}_{Equi} < \lambda_{Ben_{S_L}}$.

Informativeness of the price system As aggregated demand is known to all agents—insider as well as market maker, the informativeness of the price system in the economy corresponds to the informativeness of aggregate demand X. In other words, the equilibrium price is a sufficient statistic of aggregate demand. It is measured by the variance of the fundamental value of the asset given the information revealed in the economy $Var[\theta|X]$. Unconditional on the signal S_i and not knowing the prevailing signal regime, there is no difference from the benchmark model and one half of the informed's private information is incorporated in aggregate demand. This is due to the fact that the variance of a normal mixture is simply a weighted average of the variances of the mixture components assessed by their mixture weights. However, when differentiating between the two signal regimes, the informativeness of the economy compared to the benchmark model changes substantially. For an uninformed agent, the price system over-rates the variance in the low volatility regime and under-rates the uncertainty inherited in the high volatility regime.

Market depth In line with expectations, the additional uncertainty in the static equi-

[6]This pattern is the basic source of under- and over-reaction induced by the model if there is more than one trading period.

librium compared to the benchmark setting reduces overall market depth as long as the difference between the two regimes is not marginal. It can be shown that $p\frac{1}{\lambda_{Ben_{S_H}}} + (1 - p)\frac{1}{\lambda_{Ben_{S_L}}} > \frac{1}{\hat{\lambda}_{Equi}}$ for all p as long as $\sigma_{\epsilon_H} - \sigma_{\epsilon_L}$ is not marginally small.[7] However, depending on the different regimes, compared to the benchmark, the market is more liquid in the state of lower uncertainty and vice versa. This is advantageous to the informed agent, as the additional market depth helps the insider to exploit his informational advantage more aggressively compared to the benchmark setting. The natural mechanism that market makers reduce market liquidity when confronted with a better-informed insider is thus dampened by the asymmetric information about the signal structure. Given the properties of B_p, it is easy to see that

$$E\left[\frac{1}{\lambda_{Equi}}\right] > \frac{1}{\hat{\lambda}_{Equi}} > \frac{1}{\lambda_{Ben_{S_L}}} = \frac{2\sqrt{\sigma_u^2}}{\sigma_\theta^2}\sqrt{\sigma_\theta^2 + \sigma_{\epsilon_L}^2} \ \forall\ 0 < p < 1.$$

Profit The unconditional (on the signal) expected profit of the insider in the benchmark model is given by $E[\pi_H] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{\sqrt{\sigma_\theta^2+\sigma_{\epsilon_H}^2}}$ and $E[\pi_L] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{\sqrt{\sigma_\theta^2+\sigma_{\epsilon_L}^2}}$. Hence, overall unconditional expected profit in the benchmark model writes $pE[\pi_H] + (1-p)E[\pi_L]$, compared to $pE[\pi_H]\frac{B_p}{\sqrt{\sigma_\theta^2+\sigma_{\epsilon_H}^2}} + (1-p)E[\pi_L]\frac{B_p}{\sqrt{\sigma_\theta^2+\sigma_{\epsilon_L}^2}}$ in our model. While unconditional profit increases with the amount of noise trading σ_u^2, as well as the value of information, σ_θ^2, it is a decreasing function of the variance of the noise term, σ_{ϵ_i}. These properties hold both for the benchmark as well as in our model. However, the overall expected profit of the insider in our static equilibrium exceeds the overall expected profit of the insider in the benchmark model for all $0 < p < 1$, as long as the difference between the two noise terms is not marginal. Again, the classical pattern that benchmark profits are higher in the regime with higher signal uncertainty while equilibrium profits are higher in the regime with lower signal uncertainty is confirmed. Nevertheless, overall profits are higher in our model compared to the benchmark case. This is intuitive as in our model the insider should be able to monetize his additional information. The gain in profit that he receives from being able to trade more aggressively in the low-uncertainty setting always more than outweighs the limitation that he faces in the regime given higher uncertainty. The same holds for maximized profits, which in the benchmark setting are given by

$$E[\pi|S_H] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2)^2}\sqrt{\sigma_\theta^2+\sigma_{\epsilon_H}^2}(S-\hat{\theta})^2 \text{ and } E[\pi|S_L] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2)^2}\sqrt{\sigma_\theta^2+\sigma_{\epsilon_L}^2}(S-\hat{\theta})^2. \tag{16}$$

[7] The difference in between the two regimes would have to be $< 1 * 10^{-9}$ to give an indication of the magnitude.

Compared to

$$E[\pi|S_H] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2)^2}B_p(S-\hat{\theta})^2 \text{ and } E[\pi|S_L] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2)^2}B_p(S-\hat{\theta})^2, \tag{17}$$

if the signal structure is not common knowledge. The profits are still proportional to market depth, like in Kyle (1985), although they are standardized by the squared signal variance. This leads to a strong emphasis on the expected profit in the low volatility regime.

To summarize, the equilibrium price given a heterogeneous signal structure is either above or below the price without a heterogeneous signal, depending on the type of the signal.[8] The overall expected informativeness of the price is unaffected by the heterogeneous signal structure and in both cases it is unaffected by the amount of noise trading, like in Kyle (1985). However, given the two different signal regimes, the average volatility over- or understates the volatility in the respective single regimes. Market depth in the informationally-restricted model is not constant, but rather a function of actual aggregate demand. Overall, it is lower than in the benchmark model. Nevertheless, this does not hold looking at signal-dependent market depth. In the regime with high uncertainty, market depth is on average lower whereas in the regime with lower uncertainty market depth is on average higher than in the benchmark setting. In general, the unconditional expected profit in equilibrium assuming asymmetric information regarding the signal structure exceeds that without asymmetric information about the signal structure. This is true unless the difference in the signal variances becomes marginal. The expected profit of the insider further increases with the variance of the asset as well as the amount of noise trading. By contrast, an increase in the noisiness of the signal reduces profit. The results are the same when analyzing maximized profits. The results above prove that knowing whether the signal regime that the economy actually faces is valuable information that pays to possess and is also exploited by the insider.

3 The two-period model

In this section, we expand the model by an additional period where the signal regime is revealed after the first period. After solving for the equilibrium of the two-period model, we examine potential price dynamics in such an equilibrium and put the results under further scrutiny by conducting a numerical analysis.

[8] This ambiguity is the root of resulting over- and under-reaction occurring in a setting comprising more than one period described in section 3.3.1.

3.1 Derivation of equilibrium

We start out by deriving the equilibrium solution and characterizing the equilibrium price in the two-period setting using backward induction.

3.1.1 Environment

In the two-period model, the signal type of the informed trader is revealed after the first period. The structure of the first period is equal to the single-auction setting. In the second period after the revelation of the signal type, additional noise trader demand enters the market and a new round of trading starts. Figure 2 shows the timeline of events. The informed agent does not receive any new signal. However, he is now optimizing his profit over two periods, taking into account the pricing rule as well as the new information set of the market maker, the new public information that enters the market after the first period. The market maker still sets the price according to his pricing rule, now incorporating all information available at each point in time. This means that after the type of the signal is revealed after period one, the market maker can distinguish between the two distributions and will do so. He revises his first-period pricing rule to correctly incorporate the action of the informed trader in the first period in period two prices. This leads to two possible states of the world: one based on the high-variance regime in the first period and the second based on the low-variance regime. Additionally, this separation in the second period inherits different levels of trading aggressiveness of the informed trader, which causes different βs for the two mixture components of the distribution of aggregate demand. The value function of the informed trader writes

$$V_t(x_t) = E_t\left[\sum_{t=1}^{2}(\theta - P_t)x_t|S_i\right], \tag{18}$$

where

$$E_t[\pi|S_i] = E_t[(\theta - P_t)x_t|S_i] \tag{19}$$

is the expected profit of the informed trader in period t. The model is solved by backward induction.

3.1.2 Optimization

The Bellman equation writes

$$\max_{x_1} \; E[(\theta - P_1)x_1 + V_1|S_i]. \tag{20}$$

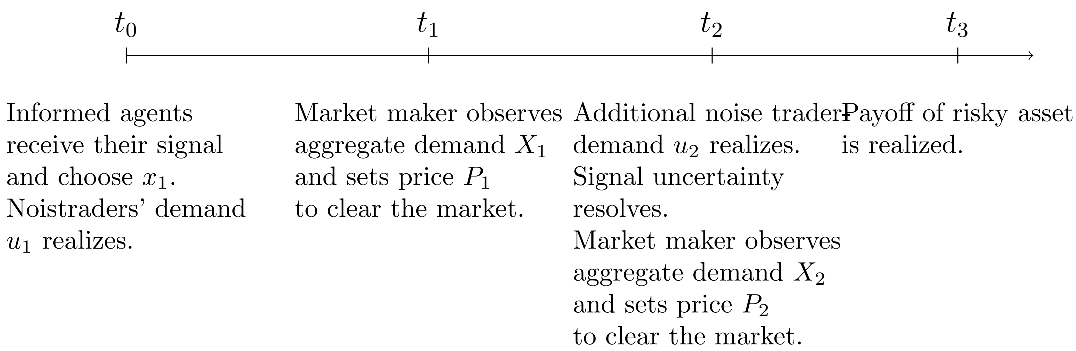

Figure 2: The time line shows the sequence of events in the two-period model. At $t = 0$, the informed agents receive their private signal S_i and posit their market order x. Further noise traders' demand u is realized. At $t = 1$, financial markets open, the market maker observes aggregate demand X_1 and sets a price P_1 to clear the market. In the next step at $t = 2$, the uncertainty about the signal structure $i = H/L$ is revealed, additional noise traders, u_2, enter the market and informed agents place their order for the second round of trading, x_2. Market makers observe aggregate demand X_2 and set the price P_2 to clear the market. Uncertainty is resolved at $t = 3$ and the pay off of the risky asset θ materializes.

The conjectured demands of the informed trader are still assumed to be linear functions in each period. They are given by $x_1 = \beta_1(E[\theta|S_i] - \hat{\theta})$ and $x_2 = \beta_2(E[\theta|S_i] - P_1^*)$. The pricing rule of the market maker for $t = 1$ is the same as in the single-auction equilibrium and of the form

$$P_1 = E[\theta|X_1] = \hat{\theta} + \lambda_1 X_1. \tag{21}$$

The pricing rule in t=2 is still linear, but it incorporates all information that the market maker possesses at this point in time. In the second period, the signal type is publicly revealed and the part of the market maker's information set stemming from period one becomes more precise. The market maker's pricing rule in $t = 2$ can be written as

$$P_2 = E[\theta|X_1, X_2] = P_{1,i}^* + \lambda_{2i} X_2, \tag{22}$$

where $P_{1,i}^*$ is the revised price of the first period, which incorporates the additional information about the signal type. It is given by

$$P_{1,i}^* = E_i[\theta|X_1] = \hat{\theta} + \lambda_{1,i}^* X_1. \tag{23}$$

Solving the model by backward induction, the maximization of the final period writes

$$\max_{x_2} \; E[(\theta - P_2)x_2|S_i, P_1, X_1]. \tag{24}$$

Plugging the pricing rule 22 into 24 gives

$$\max_{x_2} \; E\left[\left(\theta - P_{1,i}^* - \lambda_2(x_2 + u_2)\right) x_2|S_i, P_{1,i}^*\right],$$

yielding the FOC

$$E\left[(\theta - P^*_{1,i} - 2\lambda_2 x_2 - \lambda_2 u_2 | S_i, P^*_{1,i}\right] = 0. \tag{25}$$

Solving for x_2 yields the demand of the strategic trader in $t = 2$ depending on the signal type:

$$x_{2,i} = \frac{1}{2\lambda_{2,i}} \left(E\left[(\theta|S_i\right] - P^*_{1,i}\right). \tag{26}$$

The second-order condition is given by $2\lambda_{2,i} > 0$.
Knowing that the value function of a risk-neutral agent is quadratic, one can utilize 26 and plug it into the Bellman equation 20:

$$\max_{x_1}\ E\left[(\theta - P_1)x_1 + (\theta - P_{2,i}(x_2^*))x_2^* | S_i\right], \tag{27}$$

resulting in the FOC

$$E\left[\theta|S_i\right] - \hat{\theta} - 2\lambda_1 x_1 - \frac{\lambda_{1,i}}{2\lambda_{2,i}}\left(E\left[\theta|S_i\right] - \hat{\theta} - \lambda^*_{1,i}x_1\right) = 0. \tag{28}$$

The first-period demand of the informed agent is given by

$$x_{1,i} = \frac{2\lambda_{2,i} - \lambda^*_{1,i}}{4\lambda_1\lambda_{2,i} - {\lambda^*_{1,i}}^2}\left(E\left[(\theta|S_i\right] - \hat{\theta}\right). \tag{29}$$

The second-order condition is given by $\frac{4\lambda_1\lambda_{2,i} - {\lambda^*_{1,i}}^2}{2\lambda_{2,i}} > 0$.

3.1.3 Equilibrium

In the two-period model, the β coefficients in equilibrium are given by:

$$\beta_{1,i} = \frac{2\lambda_{2,i} - \lambda^*_{1,i}}{4\lambda_1\lambda_{2,i} - {\lambda^*_{1,i}}^2}, \qquad \beta_{2,i} = \frac{1}{2\lambda_{2,i}}. \tag{30}$$

It becomes immediately clear that the revelation of the uncertainty regime after period one leads to different βs depending on the signal and hence a different distribution of aggregate demand compared to the static model.
As the model is symmetric and for reasons of simplicity, the coefficients in the high-uncertainty regime are stated in the following. The coefficients of the low-uncertainty regime can be obtained analogously by simply replacing the subscript H with L. A detailed description of all equilibrium coefficients as well as the distributional details of the model are given in appendix B.3.

The equilibrium coefficients of the two-period model write

$$\beta_{1H} = \frac{2\lambda_{2H} - \lambda_{1H}^*}{4\lambda_1\lambda_{2H} - {\lambda_{1H}^*}^2}, \qquad \beta_{2H} = \frac{1}{2\lambda_{2H}}, \tag{31}$$

with

$$\beta_{2H} = \frac{\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)}{\sigma_\theta^2}\sqrt{\frac{{\beta_{1H}}^2\sigma_\theta^4 + \sigma_{u_1}^2(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)}{2{\beta_{1H}}^2\sigma_\theta^4{\sigma_{\epsilon_H}}^2 + \sigma_{u_1}^2(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)^2}} \tag{32}$$

and

$$\lambda_{2H} = \frac{\sigma_\theta^2}{2\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)}\sqrt{\frac{2{\beta_{1H}}^2\sigma_\theta^4{\sigma_{\epsilon_H}}^2 + \sigma_{u_1}^2(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)^2}{{\beta_{1H}}^2\sigma_\theta^4 + \sigma_{u_1}^2(\sigma_\theta^2 + {\sigma_{\epsilon_H}}^2)}}, \tag{33}$$

$$\lambda_1 = (1-\omega_H)\frac{\beta_{1L}\sigma_\theta^4}{{\beta_{1L}}^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + \omega_H\frac{\beta_{1H}\sigma_\theta^4}{{\beta_{1H}}^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}, \tag{34}$$

$$\lambda_{1H}^* = \frac{\beta_{1H}\sigma_\theta^4}{{\beta_{1H}}^2\sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}. \tag{35}$$

The second-order conditions guarantee $\beta_{2H} > 0$ as well as $\beta_{1H} > 0$. This abstracts the informed agent from destabilizing prices in the first period to make extreme profits in the second period. Given that $\lambda_{1H}^* < \lambda_1$, it is also obvious that β_{1H} is smaller than the respective β of a model without two different uncertainty regimes. The informed agent does not trade as aggressively on his information in the high-uncertainty case as he would without having the uncertainty about the signal regime in the first period. In the case of the low-uncertainty regime, exactly the opposite is true and the informed agent trades more aggressively than in the case without signal uncertainty. Compared to the static model, there exist different signal-dependent trading intensities.

In the first period, the following conditions hold in equilibrium. $\lambda_{1H} < \lambda_1 < \lambda_{1L}$, which implies according to equation 30 that $\beta_{1H} < \beta_{1H}^*$ and $\beta_{1L} > \beta_{1L}^*$ with $\beta_{1,i}^*$ denoting the value of β in the respective regime without the heterogeneous signal structure. This implies that compared to a model without uncertainty regarding the information regime, the strategic trader trades more aggressively in the low-variance regime than he would in the high-variance regime. Furthermore, it holds that $\beta_{1H} < \beta_{1L}$ for $1 \leq \sigma_{\epsilon_L} < \sigma_{\epsilon_H}$ and $\frac{\partial \beta_{1,i}}{\partial \sigma_{\epsilon,i}} > 0$, which is puzzling at first sight.

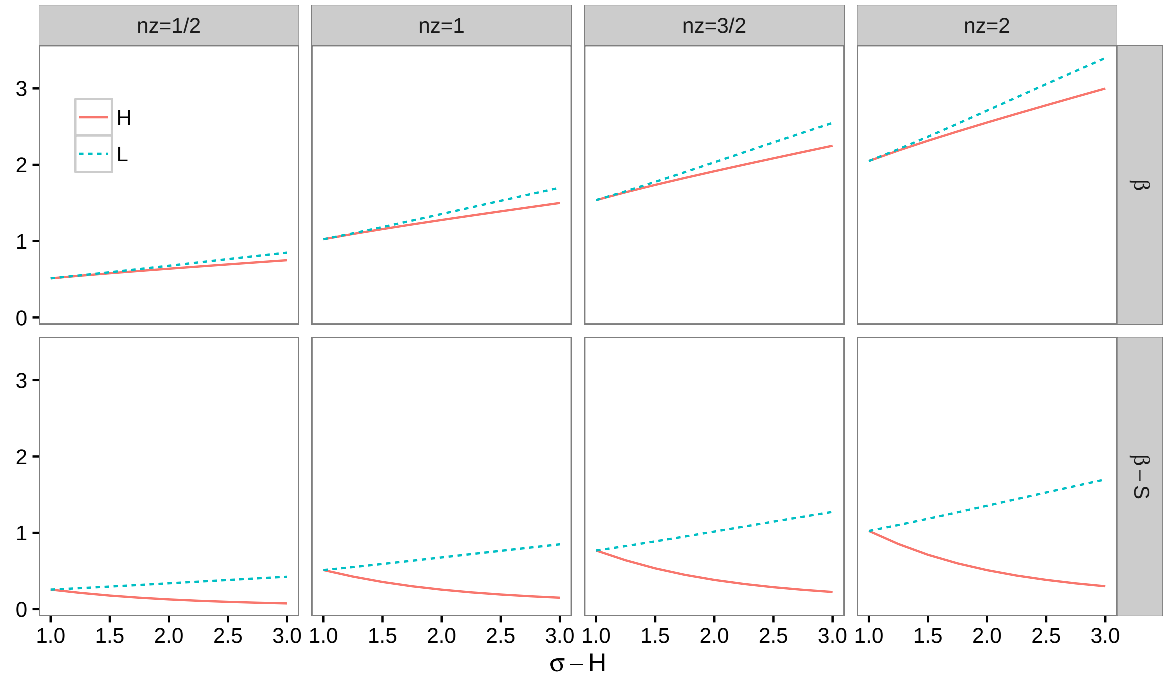

Figure 3: development of β and $\beta - S$ subject to an increase in $\sigma_{\epsilon_H} = \sigma - H$ over the first period of the model and for four different levels of noise trading $\sigma_u = nz = \{1/2, 1, 3/2, 2\}$. σ_{ϵ_L} is fixed at 1.

3.2 Discussion and comparison with benchmark

We first want to introduce a new variable, namely $\beta - S_i$. β measures the trading activity of the informed agent given his expectation about the terminal value of the risky asset $E[\theta|S_i]$ as well as his expectation about the price of the respective period $E[P|S_i]$. In the following, it is also referred to as the informed agent's trading aggressiveness. In comparison, $\beta - S_i$ is a statistic measuring the insider's trading activity towards the pure signal; hence, β scaled by the respective update factor F_i. Therefore, $\beta - S_i$ decreases with the variance of noise trading, while β exhibits an opposite reaction and increases with the variance of noise trading. β only captures the fact that an increasing variance of noise trading makes it much easier for the insider to hide, while $\beta - S_i$ also captures the fact that increasing σ_{ϵ_i} reduces the precision of the signal.

Figure 3 shows the development of β_{1L} and β_{1H} as well as $\beta - S_{1L}$ and $\beta - S_{1H}$ for different levels of signal noise in the high-variance regime $\sigma_{\epsilon_H} = \sigma - H \in [1, 3]$. The signal noise in the low-variance regime is held constant at the level of 1, $\sigma_{\epsilon_L} = 1$. The amount of noise trading ranges from 1/2 to 3, $\sigma_u = nz \in [1/2, 3]$. The informed agent's trading aggressiveness increases with noise traders' demand in both regimes, H and L. The higher the noise trading, the more that the strategic trader is able to act on his informational advantage without revealing too much of his private information. Kyle (1985, p. 1316) refers to this phenomenon that "the noise traders provide camouflage" for the insider.

Furthermore, β increases with the variance of the signal's noise component in the high-uncertainty regime, σ_{ϵ_H}. The economic intuition explaining this behavior is very similar to the noise trader effect already stated: the more uncertainty that is incorporated in the economy, the lower the risk of the informed revealing too much information to the market maker by his actions.

The most striking fact is the path of β_{1L}. It not only increases with σ_{ϵ_H}, but also constantly above β_{1H} with a steadily growing spread, $\beta_{1L} > \beta_{1H} \quad \forall \quad 1 \leq \sigma_{\epsilon_L} < \sigma_{\epsilon_H}$. This is true despite σ_{ϵ_L} being held constant at the level of one. The technical explanation of this pattern is entrenched in equation 30. The economic reason is the informed agent's advantage in profitability when trading on the low-variance signal compared to the high-variance signal. Hence, the informed agent acts more decisively on the more informative signal, whereas he relatively scales back his reaction to the signal with the higher uncertainty. As the market maker is unable to distinguish between the two regimes, the informed trader exploits his informational advantage in the favorable regime and in return sacrifices some profit in the comparatively less favorable regime. The basic idea behind this trade balancing between regimes is once again to avoid revealing too much information—he skews his trading towards the favorable regime). The mechanism becomes even more obvious looking at $\beta - S_{1,i}$. As already mentioned, $\beta - S_{1,i}$ measures the trading aggressiveness of the informed regarding the spread between the unconditional expectation of the asset's pay off and the pure signal, $S - \hat{\theta}$. Compared to $\beta_{1,i}$, which measures the trading activity taking into account the difference between the conditional expectation of the asset's pay off and the unconditional expectation $(E[\theta|S_i] - \hat{\theta})$, $\beta - S_{1,i}$ incorporates the effect of a higher signal variance directly into the trading behavior of the informed agent. It shows the combined effect that an increase in signal noise in the high-uncertainty regime has on the trading behavior of the agent when observing a certain signal S. The graph shows a decreasing trading intensity in the high-variance regime, as the indirect effect of an increase in β_{1H} is over-compensated by the direct effect of an increasing σ_{ϵ_H} on signal precision. The opposite is true in the low-variance regime. As σ_{ϵ_L} is held constant at one, the effect is purely governed by the increase of β_{1L}. Nonetheless, trading activity with respect to a certain signal in the low-uncertainty regime increases with σ_{ϵ_H}. If the informed receives a signal in the low-variance regime, he reacts by scaling up his trading activity, while in the high-variance regime he cuts back. The described pattern is consistent for all mixture weights. However, the more weight that is placed on the high-variance regime, the more aggressive the trading of the informed agent becomes in the low-variance regime.

Figure 4 shows the development of the complete range of coefficients of the model, $\beta, \beta - S$ and λ, for both periods as well as regimes. In the second period β, as the first

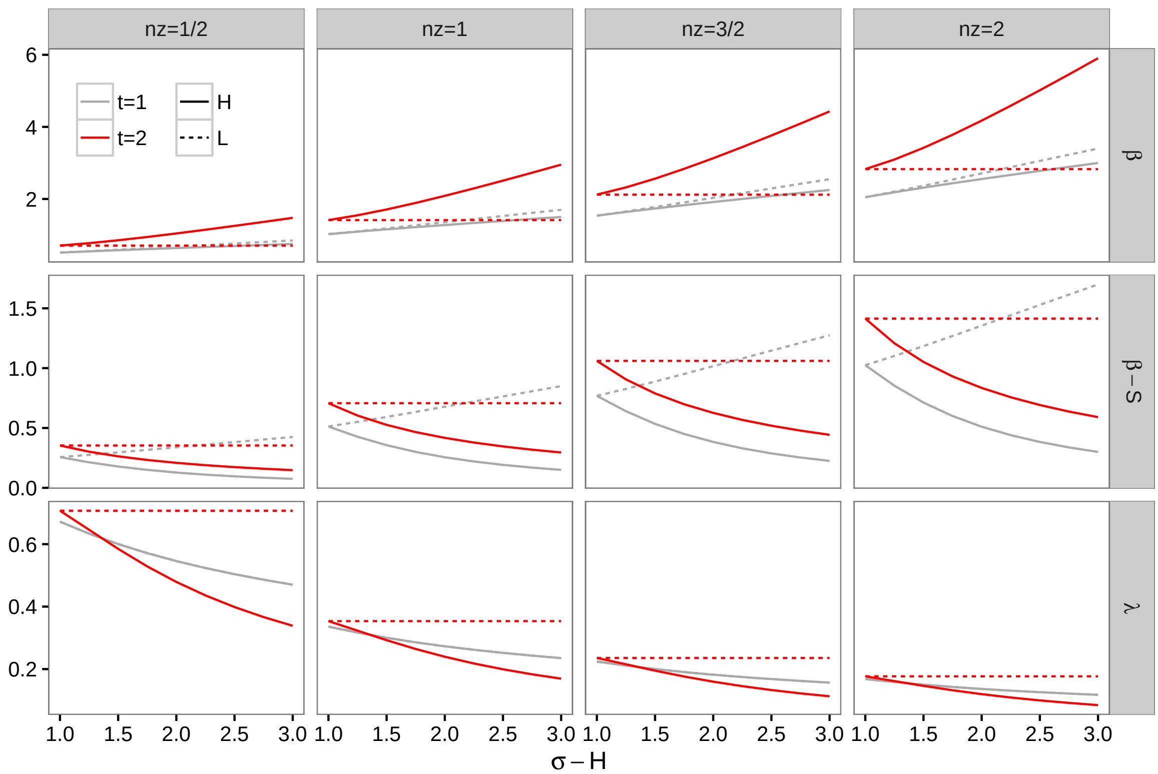

Figure 4: development of the main coefficients of the model, β, $\beta - S$ and λ subject to an increase in $\sigma_{\epsilon_H} = \sigma - H$ over both periods and for four different levels of noise trading $\sigma_u = nz = \{1/2, 1, 3/2, 2\}$. σ_{ϵ_L} is fixed at 1.

period, the level of noise trading increases in both regimes. Furthermore, β_{2H} increases with σ_{ϵ_H} and constantly exceeds its first period peer β_{1H}. By contrast, β_{2L} remains rather constant, as its sole exposure to σ_{ϵ_H} is via β_{1L}. The enhanced trading activity in the second period is in line with the economics of the model. Trading activity/intensity between the periods is linked by the informativeness of prices. If the informed agent trades more aggressively in the first period, he is penalized by facing a worse price in the second period. In the last period of trading, the informed agent no longer faces this trade off. Therefore, in a two-period model, the informed will exploit his information more actively with higher trading intensity in the second period. When taking a closer look at the graph, the stated mechanism is confirmed with respect to β_H but not in line with the development of β_L. The reason is an intertemporal shift of trading activity from the second period to the first in the low-uncertainty regime. The informed agent increases his trading intensity in the first period to exploit his additional informational advantage concerning the state of the uncertainty regime. For this purpose, he sacrifices trading opportunities in the second period when the state of the regime is revealed and his informational advantage is no longer as strong. This mechanism is also depicted by the evolution of λ. In the second period, $\lambda_{2,i}$ is simply given by 1/2 the reciprocal of $\beta_{2,i}$ and hence the trading activity of the informed agent has a strong and direct influence on market depth. In period one, this mechanism is much more involved, leading to an increase in trading activity as β_{1L} has a smaller impact on market depth in the first period of the low-variance regime. The informed agent makes use of this fact by increasing his trading activity in the low-variance regime of the first period.

3.3 Over- and under-reaction

In the following, we analyze the equilibrium price dynamics of the model and further elaborate on their main characteristics by conducting a numerical analysis.

3.3.1 Price dynamics

The above-stated two-period equilibrium supports under- as well as over-reaction in prices, without giving rise to arbitrage opportunities as prices are fully rational given the respective information sets of the agents. The market maker quotes a price that is exactly his conditional expectation of the pay off of the risky asset given his information. His information set comprises observed aggregate demand. Like in the static model, prices are a linear function of aggregate demand. In the first period, when observing aggregate demand, the market maker does not know which signal informed demand is based on. Hence, the price is set as a linear combination of the two possible states of the world. In either case, there exists a mispricing compared to the price, which would exist given the market maker had complete information about the signal regime. In the second period, the market maker learns about the kind of signal and is able to quote a price that incorporates all new information up to that point. Hence, the mispricing inherited from the first period resolves and causes the respective price pattern. It is possible to show that the market maker systematically over- or underestimates the price in the first period depending on which signal S_H or S_L the strategic trader observes. This means that the expected price dynamics exhibit momentum and reversal patterns in the two-period model.

Proposition 1. *In the above-stated sequential two-period equilibrium, there exist two distinct price movements between period one and two.*

(i) There is always over-reaction if informed demand is based on the signal with high uncertainty. This leads to a price reversal when the uncertainty regarding the signal regime is resolved.

(ii) There is always under-reaction if informed demand is based on the low-uncertainty signal. This leads to price momentum when the uncertainty regarding the signal regime is resolved.

Given the complete information set of the market maker after the second period, the equilibrium price of the second period $P_{2,i}$ writes as

$$\begin{aligned} P_{2H} &= E[\theta | X_{1H}, X_{2H}] = P^*_{1H} + \lambda_{2H} X_{2H}, \\ P_{2L} &= E[\theta | X_{1L}, X_{2L}] = P^*_{1L} + \lambda_{2L} X_{2L}, \end{aligned} \tag{36}$$

with $P^*_{1,i}$ being the hypothetical price in period one if the market maker had known the type of the signal and $X_{2,i}$ being the new aggregate demand in period two. The two hypothetical first-period prices for the two regimes are given by

$$\begin{aligned} P^*_{1H} &= E[\theta | X_{1H}] = \hat{\theta} + \lambda^*_{1H} X_{1H}, \\ P^*_{1L} &= E[\theta | X_{1L}] = \hat{\theta} + \lambda^*_{1L} X_{1L}, \end{aligned} \tag{37}$$

with

$$\lambda^*_{1H} = \frac{\beta_{1H} \sigma^4_\theta}{{\beta_{1H}}^2 \sigma^4_\theta + \sigma^2_u (\sigma^2_\theta + \sigma^2_{\epsilon_H})} \quad \text{and} \quad \lambda^*_{1L} = \frac{\beta_{1L} \sigma^4_\theta}{{\beta_{1L}}^2 \sigma^4_\theta + \sigma^2_u (\sigma^2_\theta + \sigma^2_{\epsilon_L})}.$$

The actual price in period one P_1 is given by

$$P_1 = E[\theta | X_1] = \hat{\theta} + \lambda_1 X_1, \tag{38}$$

with λ_1 being a weighted average of λ^*_{1H} and λ^*_{1L} written as

$$\lambda_1 = \omega_H \lambda^*_{1H} + (1-\omega_H)\lambda^*_{1L}. \tag{39}$$

The weights are determined by

$$\omega_H = \frac{p f_{X_{1H}}(X_1)}{(1-p) f_{X_{1L}}(X_1) + p f_{X_{1H}}(X_1)} \quad \text{and} \quad (1-\omega_H) = \frac{(1-p) f_{X_{1L}}(X_1)}{(1-p) f_{X_{1L}}(X_1) + p f_{X_{1H}}(X_1)}.$$

Knowing that $X_{2,i}$ is mean zero the expected price difference at the beginning of period two $\Delta P_i = P_{2,i} - P_1$ is defined by

$$\begin{aligned} E[\Delta P_i | X_1, i] &= E[P_{2,i} | X_1, i] - P_1 \\ &= P^*_{1,i} - P_1, \end{aligned} \tag{40}$$

the expected price movement can be characterized by the difference between the actual and the hypothetical first-period price. Simplifying the expression, the expected price difference in the respective regime writes

$$\begin{aligned} P^*_{1L} - P_1 &= \omega_H(\lambda^*_{1L} - \lambda^*_{1H}) X_1, \\ P^*_{1H} - P_1 &= (1-\omega_H)(\lambda^*_{1H} - \lambda^*_{1L}) X_1, \end{aligned}$$

and is purely governed by the difference between the values of λ^*_L and λ^*_H. It is easy to see that $P^*_{1H} < P_1 < P^*_{1L} \; \forall \; X > 0$ and $P^*_{1H} > P_1 > P^*_{1L} \; \forall \; X < 0$. Hence, one can on average observe an under-reaction, momentum, following the signal with the higher precision S_L and an over-reaction, reversal, following the signal containing more uncertainty, namely S_H. This pattern is also supported by the covariances of the price changes in the different regimes. Given $S = S_L$, $cov(\Delta P_1, \Delta P_2) > 0$, whereas if $S = S_H$, $cov(\Delta P_1, \Delta P_2) < 0$. These findings are summarized in proposition 2.

Proposition 2. *Depending on the signal:*

(i) Price changes/returns are positively correlated given the signal in the low-variance regime, $cov(\Delta P_{1L}, \Delta P_{2L}) > 0$ and thus exhibit momentum.

(ii) Price changes/returns are negatively correlated given the signal in the high-variance regime, $cov(\Delta P_{1H}, \Delta P_{2H}) < 0$ and thus exhibit reversal.

Proof. The proof of proposition 2 is given in appendix B.4. □

The general correlation pattern in the economy unconditional on the signal is governed by two effects: first, the variance of aggregate demand; and second, the mixture weight p. It is easy to see that the model creates signal-dependent momentum and reversal patterns

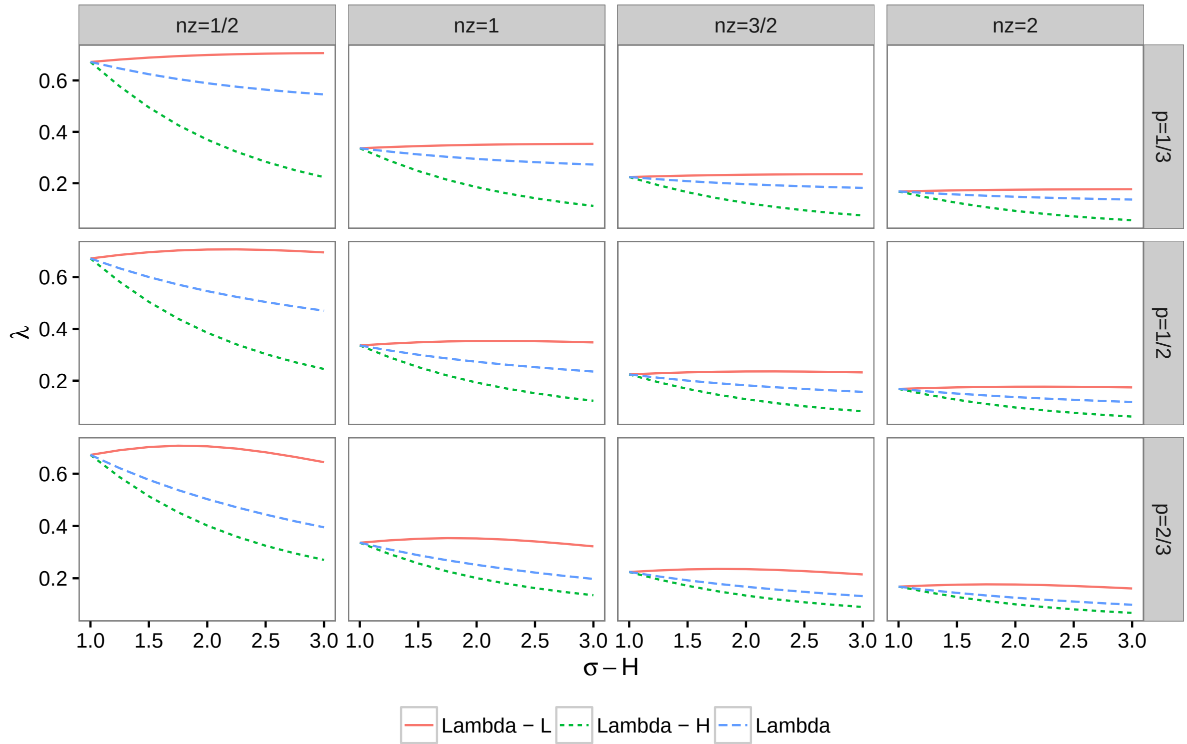

Figure 5: Development of λ_{1L}^*, λ_1 and λ_{1H}^* subject to an increase in $\sigma_{\epsilon_H} = \sigma - H$ for four different levels of noise trading $\sigma_u = nz = \{1/2, 1, 3/2, 2\}$ as well as different mixture weights $p = \{1/3, 1/2, 2/3\}$. σ_{ϵ_L} is fixed at 1.

and additionally allows for overall momentum or reversal in the economy depending on the ratio of high- to low-variance signals.

3.3.2 Numerical comparative statics

In order to develop a better understanding of the properties of the price movement and given that the direction of the effect as well as its magnitude are primarily governed by λ, we want to more closely elaborate on the properties of λ_1, λ_{1L}^* and λ_{1H}^*.

Figure 5 shows the development of the values of λ subject to an increase in the signal variance σ_{ϵ_H} for different values of noise trading and different mixture weights, while σ_{ϵ_L} is held constant at one. In the graph, the red and green line show the values of λ_{1L}^* and λ_{1H}^*. The blue line shows the value of λ_1, which the actual price in period one is based on. The difference between the red and blue line determines the magnitude of the under-reaction and thus is responsible for the momentum effect. The difference between the blue and green line depicts the strength of an over-reaction in the model and thus governs the reversal pattern. Ceteris paribus, the mispricing in either direction increases with σ_{ϵ_H}.

Furthermore, mispricing is the highest when noise trading is at a low level and it reduces with increasing levels of noise trading. This is in line with intuition. Given low levels of noise trading, aggregate demand is more informative for the market maker. Thus, the market maker reacts more strongly to his information, which increases the mispricing. Given high levels of noise trading, the market maker does not weight his information as heavily in his pricing rule. This is depicted in the steady decrease in the overall level of λ. Additionally, the difference between λ^*_{1L}, λ_1 and λ^*_{1H} decreases. The magnitude of the resulting mispricing deteriorates with an increasing level of noise trading.

A change in the mixture weights p in favor of a higher probability of the low-variance regime $p < 0.5$ shifts the blue line closer to the red one. This means that on average it is more probable that the economy experiences a scenario of under-reaction. Nevertheless, the magnitude of the price effect is smaller. At the same time, the probability of observing over-reaction reduces. Nonetheless, if an over-reaction occurs it has a more severe magnitude, as can be seen by the larger difference between the blue and the green line. The intuition is as follows. The market maker knows that it is more likely that he underestimates the price and reacts by trying to minimize this effect. However, when he is surprised by the other regime, the resulting price movement is more severe. The opposite is true for values of $p > 0.5$. In this case, over-reactions are more likely and smaller in magnitude while under-reactions do not occur as much. However, if they occur, they are of greater magnitude.

4 Extension to a sequential equilibrium with N periods

In this section we extend the two-period model to N trading periods, derive the solution to such a generalized sequential auction equilibrium and characterize its price dynamics.

4.1 The Model

In this section, we generalize the two-period setting to a model in which not only two but N rounds of trading take place sequentially. The structure is almost identical to the two-period setting. The signal type of the informed trader is again revealed after the first period; however, trading does not stop after the second period but rather continues up to period N. Overall, there are n auctions in this setting. The time at which the nth auction takes place is denoted as t_n. At each auction, new noise trader demand enters the market. Noise trader demand at time t_n, is denoted as u_n. In each period t_n, noise trader's demand is normally distributed with variance σ_u^2 and is independent over time, although the quantity traded by noise traders in one auction is independent of the quantity traded at other auctions. The distribution of u_n is given by $u_n \sim \mathcal{N}(0, \sigma^2_{u_n})$, with

$\sigma_{u_n}^2$ being constant for all n. Hence, we can write $\sigma_{u_n}^2 = \sigma_u^2 \; \forall \; n$. This corresponds to the original setting in Kyle (1985), assuming the time intervals between the different rounds of trading to be one, $\Delta t = t_n - t_{n-1} = 1$. Informed demand at time t_n is indicated as x_n, and corresponding aggregate demand of the nth auction writes X_n. The distributions of the signal as well as the liquidation value of the asset remain unchanged. The market clearing price at each auction set by the market maker is denoted P_n. The informed agent does not receive any new information throughout the n trading periods. Trade is structured in the same way as before.

The information set of the informed in the nth auction includes the signal regarding the liquidation value as well as all past prices set by the market maker up to the current point in time $\mathcal{F}_n^I = (P_1, ..., P_{n-1}, S_i)$. The informed trader still optimizes his profits taking into account the pricing rule of the market maker as well as the change in the information set of the market maker after the first period. However, he now optimizes not only over two but rather over n periods. The insider takes into account the impact of his actions not only on the price of the current auction but also on all future auctions.

The market maker still sets the price according to his linear pricing rule, again incorporating all information available at each point in time. After the revelation of the signal type at the end of period one, the market maker—analogous to the two-period setting—is able to distinguish between the two distributions and will do so. He revises his first-period pricing rule to correctly incorporate the action of the informed trader in period one in the prices that he sets in future periods. The information set of the market maker in period t_n includes all prices including P_n as well as past aggregate demand $\mathcal{F}_n^{MM} = (X_1, X_{1,i}..., X_{n-1,i}, X_{n,i})$. After period one, the economy exhibits two potential paths.

4.2 Optimization

The informed wants to maximize his expected profit. Hence, he wants to maximize

$$\pi_n = \sum_{k=n}^{N} (\theta - P_k)\, x_k, \qquad (n = 1, ..., N). \tag{41}$$

The maximization is conducted in two steps. First, the informed agent optimizes his demand path up to the end of the first period, when uncertainty regarding the risk regime is resolved. This first step is very similar to the original optimization in Kyle (1985). The Bellman equation for the periods $n = (2, ..., N)$ - which in the following are denoted as m - writes

$$\max_{x_m} \; E_m \left[U_m(x_m, P_m) + V_m(x_{m+1}, P_{m+1}) | S_i \right], \tag{42}$$

with U_m being the utility function of period two and V_m the respective value function, which is quadratic for risk-neutral agents,

$$U_m = (\theta - P_m)x_m,$$
$$V_m = \alpha_m(\theta - P_m)^2 + \delta_m.$$

Plugging in yields the following maximization problem

$$\max_{x_m} \ E_m \left[(\theta - P_m)x_m + \alpha_m(\theta - P_m)^2 + \delta_m|S_i\right]. \tag{43}$$

The pricing rule of the market maker and hence the price process of the economy for the periods $(m, ..., N)$ starting at $n = 2$ is given by the first-difference equation

$$P_{i,m} = P_{i,m-1} + \lambda_{i,m}X_m. \tag{44}$$

$P_{i,m-1}$ for $m = n = 2$ is equal to the revised price of the first period, which incorporates the additional information about the signal type, $P^*_{1,i}$. After plugging in 44 as expression for price, the maximization writes

$$\begin{aligned}\max_{x_m} \ E_m[\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))\,x_m \\ + \alpha_m\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))^2 + \delta_m|S_i].\end{aligned} \tag{45}$$

The resulting FOC regarding x_m is given by

$$\begin{aligned}E_m[\theta - P_{i,m-1} - 2\lambda_{i,m}x_m - \lambda_{i,m}u_m \\ - 2\lambda_{i,m}\alpha_m\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))\,|S_i] = 0.\end{aligned} \tag{46}$$

The resulting demand of the informed trader is

$$x_m = \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}(E_m[\theta|S_i] - P_{i,m-1}), \tag{47}$$

subject to the boundary condition $\alpha_N = \delta_N = 0$ and the second-order condition

$$\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) > 0. \tag{48}$$

In the next step, we maximize the first-period problem given the maximization up to period $m = 2$, taking into account the optimal behavior of the informed agent up to period 2. The Bellman equation writes

$$\max_{x_1} \; E\left[U_1(x_1, P_1) + U_m(x_m, P_m) + V_m(f(x_m, P_m))|S_i\right], \tag{49}$$

given

$$\begin{aligned}
U_1 =& (\theta - P_1)x_1 \\
U_m =& (\theta - P_m)x_m \\
V_m =& \alpha_m(\theta - P_m)^2 + \delta_m \\
P_m =& (P_{i,m-1} + \lambda_{i,m}X_m \\
P_{i,m-1} =& P^*_{1,i} = \hat{\theta} + \lambda^*_{1,i}X_1 \\
P_1 =& \hat{\theta} + \lambda_1 X_1.
\end{aligned}$$

Plugging into the maximization yields

$$\begin{aligned}
\max_{x_1} \; & E[(E[\theta|S_i] - \hat{\theta} - \lambda_1(x_1 + u_1))x_1 \\
& + ((E[\theta|S_i] - \hat{\theta} - \lambda^*_{1,i}(x_1 + u_1) - \lambda_{i,m}(x_m + u_m))x_m \\
& + \alpha_m((E[\theta|S_i] - \hat{\theta} - \lambda^*_{1,i}(x_1 + u_1) - \lambda_{i,m}(x_m + u_m))^2 + \delta_m].
\end{aligned} \tag{50}$$

Using the fact that

$$x_m = \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}(E_m[\theta|S_i] - \hat{\theta} - \lambda^*_{1,i}(x_1 + u_1)), \tag{51}$$

and calling

$$\psi = \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}, \tag{52}$$

$$\varphi = (E[\theta|S_i] - \hat{\theta}), \tag{53}$$

we can write

$$x_m = \psi(\varphi - \lambda^*_{1,i}(x_1 + u_1)). \tag{54}$$

Taking the first derivative regarding x_1 yields the FOC

$$\begin{aligned}
0 =& \varphi - 2\lambda_1 x_1 - 2\psi\lambda^*_{1,i}(\varphi - \lambda^*_{1,i}x_1) + 2\psi^2\lambda^*_{1,i}\lambda_{i,m}(\varphi - \lambda^*_{1,i}x_1) \\
& - 2\alpha_m\lambda^*_{1,i}\varphi + 2\alpha_m{\lambda^*_{1,i}}^2 x_1 + 4\alpha_m\psi\lambda^*_{1,i}\lambda_{i,m}(\varphi - \lambda^*_{1,i}x_1) \\
& - 2\alpha_m\psi^2\lambda^*_{1,i}\lambda^2_{i,m}(\varphi - \lambda^*_{1,i}x_1).
\end{aligned} \tag{55}$$

Solving for x_1 and plugging back in expressions 52 and 53 results in the first-period demand of the informed agent given by

$$x_1 = \frac{2\lambda_{(H/L),m}(1-\alpha_m\lambda_{(H/L),m}) - \lambda^*_{1(H/L)}}{4\lambda_1\lambda_{(H/L),m}(1-\alpha_m\lambda_{(H/L),m}) - {\lambda^*_{1(H/L)}}^2}(E[\theta|S_{H/L}] - \hat{\theta}), \tag{56}$$

subject to the second-order condition

$$\frac{4\lambda_1\lambda_{(H/L),m}(1-\alpha_m\lambda_{(H/L),m}) - {\lambda^*_{1(H/L)}}^2}{2\lambda_{(H/L),m}(1-\alpha_m\lambda_{(H/L),m})} > 0. \tag{57}$$

4.3 Market maker's filtering problem and price process

The price process—let's call it the market maker's filtering problem—looks as follows. The properties of the price process are very close to the those in the two-period model, with the difference being that it goes beyond the second period. First, we investigate the price process for the periods starting at m after the information about the signal regime is resolved. The derivation is in line with that in Kyle (1985), with the sole difference that being the informed agent observes a signal about the terminal value of the asset and not the liquidation value itself. First, we define the beliefs of the market maker regarding the insider's trading strategy.

$$x_{m,i} = \beta_{m,i}(E[\theta|S_{H/L}] - P_{i,m-1}) \tag{58}$$
$$X_{m,i} = \beta_{m,i}(E[\theta|S_{H/L}] - P_{i,m-1}) + u_m \tag{59}$$

Next, we define the market maker's information set by $\mathcal{F}^U_{m-1} = \{X_{1,i}, ..., X_{m-1,i}\}$. To enhance the readability, we will drop the subscript $i \in \{H, L\}$, which indicates the variance regime. This is without loss of generality, as at this stage of the model the market maker knows the type of the signal and all the calculations are conducted for either stage H or stage L .
We further define

$$\Sigma_{\theta,m-1} = VAR[\theta|\mathcal{F}^U_{m-1}]$$
$$\Sigma_{S,m-1} = VAR[S|\mathcal{F}^U_{m-1}]$$
$$\Sigma_{X,m-1} = VAR[X_m|\mathcal{F}^U_{m-1}]$$

Using the results 92, 93, 94, 95 and 96 from the two-period case in the appendix, one can show

$$\begin{aligned} E[X_m|\mathcal{F}^U_{m-1}] &= 0 \\ Cov[\theta, X_m|\mathcal{F}^U_{m-1}] &= \beta_m F \Sigma_{\theta,m-1} \\ \Sigma_{X,m-1} &= \beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}. \end{aligned}$$

Due to the linear structure of the equilibrium, the price P_m is given by

$$P_m = P_{m-1} + \lambda_m X_m,$$

and the market efficiency condition implies

$$P_m - P_{m-1} = \lambda_m X_m = E[\theta - P_{m-1}|\mathcal{F}^U_m].$$

Therefore, according to the projection theorem, λ_m is defined as

$$\lambda_m = \frac{\beta_m F \Sigma_{\theta,m-1}}{\Sigma_{X,m-1}} = \frac{\beta_m F \Sigma_{\theta,m-1}}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}},$$

and

$$\begin{aligned} \Sigma_{\theta,m} &= \Sigma_{\theta,m-1} - \frac{(\beta_m F \Sigma_{\theta,m-1})^2}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}}, \\ &= \frac{\beta_m^2 F^2 (\Sigma_{S,m-1} - \Sigma_{\theta,m-1})\Sigma_{\theta,m-1} + \Sigma_{\theta,m-1}\sigma^2_{u_m}}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}}, \end{aligned}$$

$$\begin{aligned} \Sigma_{S,m} &= \Sigma_{S,m-1} - \frac{(\beta_m F \Sigma_{S,m-1})^2}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}}, \\ &= \frac{\Sigma_{S,m-1}\sigma^2_{u_m}}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma^2_{u_m}}. \end{aligned}$$

Using the fact that

$$\begin{aligned} \Sigma_{S,m-1} &= \Sigma_{\theta,m-1} + VAR[\epsilon|\mathcal{F}^U_{m-1}], \\ VAR[\epsilon|\mathcal{F}^U_{m-1}] &= \Sigma_{\epsilon,m-1}, \end{aligned}$$

$$\Sigma_{\theta,m} = \frac{(\beta_m^2 F^2 \Sigma_{\epsilon,m-1} + \sigma_{u_m}^2)\Sigma_{\theta,m-1}}{\beta_m^2 F^2 \Sigma_{S,m-1} + \sigma_{u_m}^2},$$
$$\Sigma_{\theta,m} = \lambda_m * (\beta_m F \Sigma_{\epsilon,m-1} + \frac{\sigma_{u_m}^2}{\beta_m F}),$$

λ_m can be rewritten as

$$\lambda_m = \frac{\beta_m F \Sigma_{\theta,m}}{\beta_m^2 F^2 \Sigma_{\epsilon,m-1} + \sigma_{u_m}^2}. \tag{60}$$

For the first period, the coefficients of the model are

$$\lambda_{1H}^* = \frac{\beta_{1H} F_H \Sigma_\theta}{\Sigma_{X_H}} = \frac{\beta_{1H} F_H \Sigma_\theta}{\beta_{1H}^2 F_H^2 \Sigma_{S_H} + \sigma_{u_m}^2}, \tag{61}$$
$$\lambda_{1L}^* = \frac{\beta_{1L} F_L \Sigma_\theta}{\Sigma_{X_L}} = \frac{\beta_{1L} F_L \Sigma_\theta}{\beta_{1L}^2 F_L^2 \Sigma_{S_L} + \sigma_{u_m}^2}, \tag{62}$$

and the actual λ_1 is given by the weighted average, like in the two-period model

$$\lambda_1 = (1 - \omega_H)\frac{\beta_{1L} F_L \Sigma_\theta}{\Sigma_{X_L}} + \omega_H \frac{\beta_{1H} F_H \Sigma_\theta}{\Sigma_{X_H}}. \tag{63}$$

The price of the first period is given by

$$P_1 = E[\theta | X_1] = \hat{\theta} + \lambda_1 X_1. \tag{64}$$

The properties of the model between the first and second period - when the uncertainty of the signal regime resolves - are almost identical to those in the two-period case. The only difference is that the respective βs are now defined by the recursive equations 69, 79, 70 and 76 described in theorem 4.1. From period two onwards, for each signal regime the model follows the general solution of the original model of Kyle (1985) with a slight modification, given that the informed agent observes a signal about the liquidation value of the asset and not the value itself.

4.4 Nature of the equilibrium

The following theorem gives the equilibrium solution to the N-period model.

Theorem 4.1. *For the sequential model with n trading periods, a linear recursive equilibrium exists dependent on the signal regime. The coefficients defining the equilibrium can be divided along two sub-periods.*

For the periods ($m = 2, ..., N$), the equilibrium is defined by the following equations.

The solution is very close to the original equilibrium in Kyle (1985) and depends symmetrically on the uncertainty regime. [9]

$$x_m = \beta_m(E[\theta|S] - P_{m-1}), \tag{65}$$
$$\Delta P_m = P_m - P_{m-1} = \lambda_m X_m, \tag{66}$$
$$\Sigma_{\theta,m} = VAR[\theta|\mathcal{F}^U_m], \tag{67}$$
$$E[\pi_m|\mathcal{F}^I_m] = \alpha_{m-1}(E[\theta|S] - P_{m-1})^2 + \delta_{m-1}. \tag{68}$$

The constants solving the system of difference equations up to period $m = 2$ in the respective uncertainty regime are given by

$$\beta_{m,i} = \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}, \tag{69}$$
$$\lambda_{i,m} = \frac{\beta_{i,m} F\Sigma_{\theta,i,m}}{\beta^2_{i,m}F^2\Sigma_{\epsilon,m-1} + \sigma^2_{u_m}}, \tag{70}$$
$$\Sigma_{\theta,m} = \frac{\beta^2_m F^2\Sigma_{\epsilon,m-1} + \sigma^2_{u_m}}{\beta^2_m F^2\Sigma_{S,m-1} + \sigma^2_{u_m}}\Sigma_{\theta,m-1}, \tag{71}$$
$$\alpha_{m-1,i} = \frac{1}{4\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}, \tag{72}$$
$$\delta_{m-1,i} = \alpha_{m,i}\lambda^2_{i,m-1}\sigma^2_{u_m} + \delta_m. \tag{73}$$

For the first period, the equilibrium solution is given by

$$x_1 = \beta_{1,i}(E[\theta|S] - \hat{\theta}), \tag{74}$$
$$P_{1,i} = \hat{\theta} + \lambda_1 X_1, \tag{75}$$
$$\lambda_1 = \omega_H\lambda^*_{1H} + (1 - \omega_H)\lambda^*_{1L}, \tag{76}$$
$$\lambda^*_{1H} = \frac{\beta_{1H}F_H\Sigma_\theta}{\Sigma_{X_H}}, \tag{77}$$
$$\lambda^*_{1L} = \frac{\beta_{1L}F_L\Sigma_\theta}{\Sigma_{X_L}}, \tag{78}$$
$$\beta_{1,i} = \frac{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - \lambda^*_{1,i}}{,} 4\lambda_1\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2, \tag{79}$$
$$\alpha_{0,i} = \frac{\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - \lambda^*_{1,i} + \lambda_1}{4\lambda_1\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2}, \tag{80}$$
$$\delta_{0,i} = \alpha_{1,i}{\lambda^*_{1,i}}^2\sigma^2_{u_m} + \delta_1. \tag{81}$$

where $\lambda_{m,i}$ and $\beta_{m,i}$ are defined by 70 and 69 and $\beta_{1,i}$ and λ_1 by 79, 78 and 77.

Proof. The detailed derivation of theorem 4.1 is given in appendix C. □

[9] For notational convenience, the subscripts indicating the signal regime are skipped in some parts.

The price path in equilibrium up to period $m = 2$ is determined by 66. The price in period one is given by 75. Therefore, the expected price change from period one to period two is

$$\begin{aligned} E[\Delta P_{m=2,i}] &= E[P_{m=2,i}] - P_1 = \lambda_m X_m \\ &= P^*_{1,i} - P_1. \end{aligned} \tag{82}$$

Using 76, 78 and 77 this can be again written as

$$\begin{aligned} P^*_{1L} - P_1 &= \omega_H(\lambda^*_{1L} - \lambda^*_{1H})X_1, \\ P^*_{1H} - P_1 &= (1-\omega_H)(\lambda^*_{1H} - \lambda^*_{1L})X_1, \end{aligned} \tag{83}$$

with $\lambda^*_{1L} > \lambda^*_{1H}$. The above result confirms the price pattern stated in propositions 1 and 2 of the two-period model for the sequential equilibrium.

According to 68, ex-ante expected profit of the insider is given by

$$E[\pi|S_i] = \alpha_0(E[\theta|S_i] - P_0)^2 + \delta_0, \tag{84}$$

in each uncertainty regime. Inserting 80 and 81 results in

$$\begin{aligned} E[\pi|S_i] = &\frac{\lambda_{i,m}(1-\alpha_m\lambda_{i,m}) - \lambda^*_{1,i} + \lambda_1}{4\lambda_1\lambda_{i,m}(1-\alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2} F_i(S - \hat{\theta}) \\ &+ \frac{{\lambda^*_{1i}}^2 \sigma^2_{u_1}}{4\lambda_{i,m}(1-\alpha_m\lambda_{i,m})} + \delta_1, \end{aligned} \tag{85}$$

for $m = 2$ (see the appendix for the detailed calculation). Compared to a setting without scenario uncertainty, the ex-ante expected profit of the insider would write

$$\begin{aligned} E[\pi|S] = &\frac{\lambda_{i,m}(1-\alpha_m\lambda_{i,m})}{4\lambda^*_{1,i}\lambda_{i,m}(1-\alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2} F_i(S - \hat{\theta}) \\ &+ \frac{{\lambda^*_{1i}}^2 \sigma^2_{u_1}}{4\lambda_{i,m}(1-\alpha_m\lambda_{i,m})} + \delta_1. \end{aligned} \tag{86}$$

Looking at expressions 85 and 86, it becomes clear that the information asymmetry about the heterogeneous information structure influences the expected ex-ante utility of the informed investor in two dimensions. First, it directly enters in α_0, in the numerator via the difference between $\lambda^*_{1i} - \lambda_1$ and in the denominator via the product $\lambda_1\lambda_{i,m}$. Second, it influences the whole expression and all future periods by the different values of $\beta_{1,i}$.

5 Conclusion

In this paper, we have shown that patterns of security price over- and under-reaction can exist in a simple strategic trade model with risk-neutral agents without inducing any biases on the behavior of these agents. The result is achieved by turning the focus away from the conduct and interaction of the agents, rather targeting the signal structure itself. Most empirical literature like Chan (2003) or Gutierrez and Kelley (2008) as well as theoretical models like Holden and Subrahmanyam (2002), Cespa and Vives (2012) or Andrei and Cujean (2017) consider public news as a homogeneous information signal. We suggest that different types of information exist and the price reaction to new information depends on these types. More specifically, the price reaction is determined by the uncertainty incorporated in the different types of information. Following these lines, we have established a model that produces under-reaction given news with low variance and thus lower uncertainty and over-reaction given news with high variance and thus higher uncertainty. This is a very general setup and its behavior is in line with the empirical findings of Forrer (2015), as well as the words of Fama (1998, p. 284) that

> "Models dealing with predictability must specify mechanisms in such a way that the same investors underreact to some types of events and overreact to others."

Furthermore, our model is not at odds with the prominent behavioral explanations of momentum and reversal given by Daniel et al. (1998) and Barberis et al. (1998), but rather it is complementary. Those biases seem to amplify the price patterns of the model. The model provides an alternative source concerning how momentum and reversal can evolve while agents make fully rational trading decisions.

For future research, it would be worthwhile to investigate how the equilibrium outcome and the price process in the sequential equilibrium would be influenced if the information asymmetry regarding the signal characteristics were resolved in later periods. We believe that it should be possible to prolong the period of under- and over-reaction up to a certain extent.

References

Andrei, D., & Cujean, J. (2017). Information percolation, momentum and reversal. *Journal of Financial Economics*, *123*(3), 617–645.

Barberis, N., Shleifer, A., & Vishny, R. (1998). A model of investor sentiment. *Journal of Financial Economics*, *49*(3), 307.343.

Brunnermeier, M. K. (2005). Information leakage and market efficiency. *Review of Financial Studies*, *18*(2), 417–457.

Cespa, G., & Vives, X. (2012). Dynamic trading and asset prices: Keynes vs. hayek. *The Review of Economic Studies*, *79*(2), 539-580.

Chan, W. S. (2003). Stock price reaction to news and no-news: Drift and reversal after headlines. *Journal of Financial Economics*, *70*(2), 223–260.

Daniel, K., Hirshleifer, D., & Subrahmanyam, A. (1998). Investor psychology and security market under- and overreactions. *Journal of Finance*, *53*(6), 1839–1885.

Fama, E. (1998). Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics*, *49*(3), 283–306.

Forrer, R. (2015). Three essays on quantifying the impact of qualitative information on stock prices. *Dissertation, University of Basel*.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *The American Economic Review*, *70*(3), 393–408.

Gutierrez, R. C., & Kelley, E. (2008). The long-lasting momentum in weekly returns. *The Journal of Finance*, *63*(1), 415–447.

Holden, C. W., & Subrahmanyam, A. (2002). News events, information acquisition, and serial correlation. *The Journal of Business*, *75*(1), 1-32.

Hong, H., & Stein, J. C. (2007). Disagreement and the stock market. *The Journal of Economic Perspectives*.

Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*.

Kyle, A. S. (1985). Continous auctions and insider trading. *Econometrica*, *53*(6), 1315-1336.

Appendices

A The static model

Derivation of P_1

$$
\begin{aligned}
P_1 &= E[\theta|X] = \omega_L E[\theta|X_L] + \omega_H E[\theta|X_H] \\
P_1 &= \hat{\theta} + \omega_L \frac{\beta F_L \sigma_\theta^2}{\beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2} X + \omega_H \frac{\beta F_H \sigma_\theta^2}{\beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2} X \\
P_1 &= \hat{\theta} + \left(\omega_L \frac{\beta F_L \sigma_\theta^2}{\beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2} + \omega_H \frac{\beta F_H \sigma_\theta^2}{\beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2} \right) X \\
P_1 &= \hat{\theta} + \left(\omega_L \frac{\beta \sigma_\theta^4}{\beta^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + \omega_H \frac{\beta \sigma_\theta^4}{\beta^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} \right) X
\end{aligned}
$$

Defining the coefficients $E[\lambda]$ and β in equilibrium in terms of parameters of the distributions by solving for β and plugging in $E[\lambda] = \frac{1}{2\beta}$.

$$
\begin{aligned}
\frac{1}{2\beta} &= E\left[\omega_L \frac{\beta F_L \sigma_\theta^2}{\beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2} + \omega_H \frac{\beta F_H \sigma_\theta^2}{\beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2} \right] \\
0 &= E\left[\omega_L \frac{\beta^2 F_L \sigma_\theta^2}{\beta^2 F_L^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_u^2} + \omega_H \frac{\beta^2 F_H \sigma_\theta^2}{\beta^2 F_H^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_u^2} \right] - \frac{1}{2} \\
0 &= (1-p)\frac{\beta^2 F_L \sigma_\theta^2}{\beta^2 F_L \sigma_\theta^2 + \sigma_u^2} + p \frac{\beta^2 F_H \sigma_\theta^2}{\beta^2 F_H \sigma_\theta^2 + \sigma_u^2} - \frac{1}{2} \\
0 &= (1-p)\frac{\beta^2 \sigma_\theta^4}{\beta^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + p \frac{\beta^2 \sigma_\theta^4}{\beta^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} - \frac{1}{2}
\end{aligned}
\tag{87}
$$

Solving equation 87 for β means solving a fourth-order polynomial, which has four solutions. As according to the second-order condition β has to be real and positive, three of the four solutions can be ruled out immediately. The one surviving is

$$
\beta = \frac{\sqrt{\sigma_u^2}}{\sqrt{2}\sigma_\theta^2} \sqrt{(2p-1)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2) + \sqrt{\left((\sigma_{\epsilon_L}^2 + \sigma_\theta^2) + (\sigma_{\epsilon_H}^2 + \sigma_\theta^2)\right)^2 - 4(p-p^2)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)^2}},
$$

knowing $\hat{\lambda}$, the market depth expected by the insider $\frac{1}{\hat{\lambda}}$ is given by

$$
\frac{1}{\hat{\lambda}} = \frac{\sqrt{2\sigma_u^2}}{\sigma_\theta^2} \sqrt{(2p-1)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2) + \sqrt{\left((\sigma_{\epsilon_L}^2 + \sigma_\theta^2) + (\sigma_{\epsilon_H}^2 + \sigma_\theta^2)\right)^2 - 4(p-p^2)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)^2}},
$$

and for $p = 1/2$

$$
\frac{1}{\hat{\lambda}} = \frac{2\sqrt{\sigma_u^2}}{\sigma_\theta^2} \sqrt[4]{(\sigma_{\epsilon_L}^2 + \sigma_\theta^2)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)}.
$$

The ex-ante expected profits of the insider (unconditional on S) are given by

$$E[\pi] = \frac{1}{2}\frac{\sqrt{\sigma_u^2}}{\sigma_\theta^2}B_p\left(\frac{p\sigma_\theta^4}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2)} + \frac{(1-p)\sigma_\theta^4}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2)}\right),$$
$$E[\pi] = p\frac{1}{2}\sqrt{\sigma_u^2}B_p\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2)} + (1-p)\frac{1}{2}\sqrt{\sigma_u^2}B_p\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2)}.$$

B The two-period model

B.1 Optimization

As the structure of the problem is identical for both uncertainty regimes, the optimization is valid for both regimes, indicated by the subscript $i \in \{H, L\}$. Knowing that the value function for a risk-neutral agent is quadratic, one can utilize the maximized demand of period two given in expression 26 and plug it into the maximization problem of period one.

$$\max_{x_1}\ E\left[(\theta - P_1)x_1 + (\theta - P_{2,i}(x_2^*))x_2^*|S_i\right] \tag{88}$$

$$\begin{aligned}&\max_{x_1}\ E\left[\left((\theta-\hat{\theta}-\lambda_1(x_1+u_1)\right)x_1 + \left(\theta - P_{1,i}^* - \lambda_{2,i}(x_2^*+u_2)\right)x_2^*|S_i\right]\\ &\max_{x_1}\ E\left[\left((\theta-\hat{\theta}-\lambda_1(x_1+u_1)\right)x_1 + \frac{1}{4\lambda_{2,i}}\left(\theta-\hat{\theta}-\lambda_{1,i}^*(x_1+u_1)\right)^2 - \lambda_{2,i}u_2x_2^*|S_i\right].\end{aligned} \tag{89}$$

Resulting in the FOC

$$\begin{aligned}&E\left[\left(\theta-\hat{\theta}-\lambda_1 u_1 - 2\lambda_1 x_1\right) + \frac{1}{2\lambda_{2,i}}\left(\theta-\hat{\theta}-\lambda_{1,i}^*(x_1+u_1)\right)(-\lambda_{1,i})|S_i\right] = 0\\ &E\left[\theta|S_i\right] - \hat{\theta} - 2\lambda_1 x_1 - \frac{\lambda_{1,i}}{2\lambda_{2,i}}\left(E\left[\theta|S_i\right] - \hat{\theta} - \lambda_{1,i}^* x_1\right) = 0,\end{aligned} \tag{90}$$

and yielding

$$x_{1,i} = \frac{2\lambda_{2,i} - \lambda_{1,i}^*}{4\lambda_1\lambda_{2,i} - {\lambda_{1,i}^*}^2}\left(E\left[(\theta|S_i\right] - \hat{\theta}\right).$$

The second-order condition is given by

$$\frac{4\lambda_1\lambda_{2,i} - {\lambda_{1,i}^*}^2}{2\lambda_{2,i}} > 0 \tag{91}$$

B.2 Distributions

The signal distribution in the two-period setting is as follows:
First Period

$$x_{1H} \sim \mathcal{N}(0, {\beta_{1H}}^2 {F_H}^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)) \quad and \quad x_{1L} \sim \mathcal{N}(0, {\beta_{1L}}^2 {F_L}^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2))$$

$$X_{1H} = x_{1H} + u_1 = \beta_{1H} F_H (S_H - \hat{\theta}) + u_1 \quad and \quad X_{1L} = x_{1L} + u_1 = \beta_{1L} F_L (S_L - \hat{\theta}) + u_1$$
$$X_{1H} \sim \mathcal{N}(0, {\beta_{1H}}^2 {F_H}^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_{u_1}^2) \quad and \quad X_{1L} \sim \mathcal{N}(0, {\beta_{1L}}^2 {F_L}^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_{u_1}^2).$$

Given the distributions of X_{1H} and X_{1L}, aggregate demand in the first period X_1 and θ are distributed jointly normal,

$$\begin{matrix} \theta \\ X_{1H} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \beta_{1H} F_H \sigma_\theta^2 \\ \beta_{1H} F_H \sigma_\theta^2 & {\beta_{1H}}^2 {F_H}^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_{u_1}^2 \end{pmatrix}\right],$$
$$\begin{matrix} \theta \\ X_{1L} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \beta_{1L} F_L \sigma_\theta^2 \\ \beta_{1L} F_L \sigma_\theta^2 & {\beta_{1L}}^2 {F_L}^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_{u_1}^2 \end{pmatrix}\right].$$

The variance of θ given $X_{1,i}$ is

$$Var[\theta | X_{1,i}] = \sigma_\theta^2 - \frac{(\beta_{1,i} F_i \sigma_\theta^2)^2}{{\beta_{1,i}}^2 {F_i}^2(\sigma_\theta^2 + \sigma_{\epsilon_i}^2) + \sigma_{u_1}^2} = \frac{{\beta_{1,i}}^2 {F_i}^2 \sigma_\theta^2 \sigma_{\epsilon_i}^2 + \sigma_{u_1}^2 \sigma_\theta^2}{{\beta_{1,i}}^2 {F_i}^2(\sigma_\theta^2 + \sigma_{\epsilon_i}^2) + \sigma_{u_1}^2} \tag{92}$$

Given the distributions of X_{1H} and X_{1L}, aggregate demand in the first period X_i and S_i is distributed jointly normal,

$$\begin{matrix} S_H \\ X_{1H} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 + \sigma_{\epsilon_H}^2 & \beta_{1H} F_H (\sigma_\theta^2 + \sigma_{\epsilon_H}^2) \\ \beta_{1H} F_H \sigma_\theta^2 & {\beta_{1H}}^2 {F_H}^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_{u_1}^2 \end{pmatrix}\right],$$
$$\begin{matrix} S_L \\ X_{1L} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 + \sigma_{\epsilon_L}^2 & \beta_{1L} F_L (\sigma_\theta^2 + \sigma_{\epsilon_L}^2) \\ \beta_{1L} F_L \sigma_\theta^2 & {\beta_{1L}}^2 {F_L}^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_{u_1}^2 \end{pmatrix}\right].$$

The joint distribution of X_i and S_i is

$$\begin{matrix} S_H \\ X_{1H} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 + \sigma_{\epsilon_H}^2 & \beta_{1H} \sigma_\theta^2 \\ \beta_{1H} F_H \sigma_\theta^2 & {\beta_{1H}}^2 {F_H}^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) + \sigma_{u_1}^2 \end{pmatrix}\right],$$
$$\begin{matrix} S_L \\ X_{1L} \end{matrix} \sim \mathcal{N}\left[\begin{pmatrix} \hat{\theta} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 + \sigma_{\epsilon_L}^2 & \beta_{1L} \sigma_\theta^2 \\ \beta_{1L} F_L \sigma_\theta^2 & {\beta_{1L}}^2 {F_L}^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) + \sigma_{u_1}^2 \end{pmatrix}\right].$$

According to the distributions stated above, the conditional variance $Var[S_i|X_{1,i}]$ is given by

$$\begin{aligned}
Var[S_i|X_{1,i}] &= \sigma_\theta^2 + \sigma_{\epsilon_i}^2 - \frac{(\beta_{1,i}F_i(\sigma_\theta^2+\sigma_{\epsilon_i}^2))^2}{\beta_{1,i}{}^2F_i{}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)+\sigma_{u_1}^2} \\
&= \frac{\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)}{\beta_{1,i}{}^2F_i{}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)+\sigma_{u_1}^2} = \frac{\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)^2}{\beta_{1,i}{}^2\sigma_\theta^4+\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)}.
\end{aligned} \tag{93}$$

The next distribution to be evaluated is the joint distribution of $F(\theta|X_{1,i})$ and $F(X_{2,i}|X_{1,i})$. The first two moments of $F(X_{2,i}|X_{1,i})$ after the signal type is revealed are given by

$$\begin{aligned}
E[X_{2,i}|X_{1,i}] &= E\left[\beta_{2,i}(E[\theta|S_i]-p_{1,i}^*)+u_2|X_{1,i}\right] \\
&= E\left[\beta_{2,i}(E[\theta|S_i]-p_{1,i}^*)|X_{1,i}\right] + E\left[u_2|X_{1,i}\right] \\
&= \beta_{2,i}\underbrace{(E[\theta|X_{1,i}]-p_{1,i}^*)}_{=0} + \underbrace{E\left[u_2|X_{1,i}\right]}_{=0} \\
E[X_{2,i}|X_{1,i}] &= 0
\end{aligned} \tag{94}$$

and

$$\begin{aligned}
VAR[X_{2,i}|X_{1,i}] &= VAR\left[\beta_{2,i}(E[\theta|S_i]-p_{1,i}^*)+u_2|X_{1,i}\right] \\
&= VAR\left[\beta_{2,i}(E[\theta|S_i]-p_{1,i}^*)|X_{1,i}\right] + VAR\left[u_2|X_{1,i}\right] \\
&= \underbrace{\beta_{2,i}{}^2VAR\left[E[\theta|S_i]|X_{1,i}\right]}_{\beta_{2,i}{}^2F_i^2VAR[S|X_{1,i}]} + \sigma_{u_2}^2 \\
&= \beta_{2,i}{}^2F_i^2\frac{\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)^2}{\beta_{1,i}{}^2\sigma_\theta^4+\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)} + \sigma_{u_2}^2 \\
VAR[X_{2,i}|X_{1,i}] &= \beta_{2,i}{}^2\frac{\sigma_{u_1}^2\sigma_\theta^4}{\beta_{1,i}{}^2\sigma_\theta^4+\sigma_{u_1}^2(\sigma_\theta^2+\sigma_{\epsilon_i}^2)} + \sigma_{u_2}^2.
\end{aligned} \tag{95}$$

The covariance conditional on the first period's information $COV[X_{2,i},\theta|X_{1,i}]$ is given by

$$\begin{aligned}
COV[\beta_{2,i}E[\theta|S_i],\theta|X_{1,i}] &= COV[\beta_{2,i}(\hat{\theta}+F_i(S-\hat{\theta}),\theta|X_{1,i}] \\
&= \underbrace{COV[\beta_{2,i}\hat{\theta},\theta|X_{1,i}]}_{=0} + COV[\beta_{2,i}F_i(S-\hat{\theta}),\theta|X_{1,i}] \\
&= COV[\beta_{2,i}F_iS,\theta|X_{1,i}] \\
&= COV[\beta_{2,i}F_i\theta,\theta|X_{1,i}] + \underbrace{COV[\beta_{2,i}F_i\epsilon_i,\theta|X_{1,i}]}_{=0} \\
&= \beta_{2,i}F_i\left(E[\theta^2|X_{1,i}]-E[\theta|X_{1,i}]^2\right) \\
COV[\beta_{2,i}E[\theta|S_i],\theta|X_{1,i}] &= \beta_{2,i}F_iVAR[\theta|X_{1,i}].
\end{aligned} \tag{96}$$

Knowing these parameters and given that $F(\theta|X_{1,i})$ and $F(X_{2,i}|X_{1,i})$ are jointly normal distributed, one can calculate the price in period two, which is the expected liquidation value of the risky asset θ conditional on the information set available to the market maker in period two;

hence, $E[\theta|X_{1,i}, X_{2,i}, P_{1,i}]$. Applying again the projection theorem for jointly normal variables yields

$$E[\theta|X_{1,i}, X_{2,i}, P_{1,i}] = P_{1,i} + \lambda_{2,i} X_{2,i}. \tag{97}$$

B.3 Equilibrium coefficients two-period setting

with $\lambda_{2,i}$ being

$$\begin{aligned}\lambda_{2,i} &= \frac{COV\left[\beta_{2,i} E[\theta|S_i], \theta|X_{1,i}\right]}{VAR[X_{2,i}|X_{1,i}]} \\ &= \frac{\beta_{2,i} F_i VAR[\theta|X_{1,i}]}{{\beta_{2,i}}^2 F_i^2 VAR[S|X_{1,i}] + \sigma_{u_2}^2},\end{aligned}$$

knowing $\beta_{2,i} = \frac{1}{2\lambda_{2,i}}$, one can solve for $\beta_{2,i}$, yielding

$$\begin{aligned}{\beta_{2,i}}^2 &= \frac{\sigma_{u_2}^2 \left({\beta_{1,i}}^2 \sigma_\theta^4 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2))\right)}{\sigma_\theta^4 \left(2{\beta_{1,i}}^2 F_i^2 {\sigma_{\epsilon_i}}^2 + \sigma_{u_1}^2\right)} \\ \beta_{2,i} &= \frac{\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)}{\sigma_\theta^2} \sqrt{\frac{{\beta_{1,i}}^2 \sigma_\theta^4 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)}{2{\beta_{1,i}}^2 \sigma_\theta^4 {\sigma_{\epsilon_i}}^2 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)^2}},\end{aligned}$$

then $\lambda_{2,i}$ is given by

$$\lambda_{2,i} = \frac{\sigma_\theta^2}{2\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)} \sqrt{\frac{2{\beta_{1,i}}^2 \sigma_\theta^4 {\sigma_{\epsilon_i}}^2 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)^2}{{\beta_{1,i}}^2 \sigma_\theta^4 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_i}}^2)}}.$$

Second-period coefficients for the case $S = S_L$

$$\beta_{2L} = \frac{\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)}{\sigma_\theta^2} \sqrt{\frac{{\beta_{1L}}^2 \sigma_\theta^4 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)}{2{\beta_{1L}}^2 \sigma_\theta^4 {\sigma_{\epsilon_L}}^2 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)^2}},$$

and

$$\lambda_{2L} = \frac{\sigma_\theta^2}{2\sqrt{\sigma_{u_2}^2}(\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)} \sqrt{\frac{2{\beta_{1L}}^2 \sigma_\theta^4 {\sigma_{\epsilon_L}}^2 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)^2}{{\beta_{1L}}^2 \sigma_\theta^4 + \sigma_{u_1}^2 (\sigma_\theta^2 + {\sigma_{\epsilon_L}}^2)}},$$

$$\lambda_{1H}^* = \frac{\beta_{1H}\sigma_\theta^4}{{\beta_{1H}}^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} \qquad \lambda_{1L}^* = \frac{\beta_{1L}\sigma_\theta^4}{{\beta_{1L}}^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)},$$

$$\lambda_1 = (1 - \omega_H)\frac{\beta_{1L}\sigma_\theta^4}{{\beta_{1L}}^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} + \omega_H \frac{\beta_{1H}\sigma_\theta^4}{{\beta_{1H}}^2 \sigma_\theta^4 + \sigma_u^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)}.$$

Plugging all coefficients into equation 30, β_{1L} and β_{1H} are implicitly defined by the following system of equations:

$$\beta_{1L} = \frac{\left(\sigma_\theta^2 + \sigma_{\epsilon\mathrm{L}}^2\right)\left(\frac{1}{\sqrt{\sigma_{\mathrm{u2}}^2}\sqrt{\frac{\sigma_\theta^4\beta_{1L}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2\right)}{2\sigma_\theta^4\beta_{1L}^2\sigma_{\epsilon\mathrm{L}}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2\right)^2}}} - \frac{\sigma_\theta^2\beta_{1L}}{\frac{\sigma_\theta^4\beta_{1L}^4}{\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2}+\sigma_{\mathrm{u1}}^2}\right)}{\frac{2\sigma_\theta^4\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2\right)\left(\frac{p\beta_{1H}}{\sigma_\theta^4\beta_{1H}^4+\sigma_\theta^2\sigma_{\mathrm{u1}}^2+\sigma_{\mathrm{u1}}^2\sigma_{\epsilon\mathrm{H}}^2}+\frac{\beta_{1L}-p\beta_{1L}}{\sigma_\theta^4\beta_{1L}^4+\sigma_\theta^2\sigma_{\mathrm{u1}}^2+\sigma_{\mathrm{u1}}^2\sigma_{\epsilon\mathrm{L}}^2}\right)}{\sqrt{\sigma_{\mathrm{u2}}^2}\sqrt{\frac{\sigma_\theta^4\beta_L^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2\right)}{2\sigma_\theta^4\beta_{1L}^2\sigma_{\epsilon\mathrm{L}}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2\right)^2}}} - \frac{\sigma_\theta^6\beta_{1L}^2}{\left(\frac{\sigma_\theta^4\beta_{1L}^4}{\sigma_\theta^2+\sigma_{\epsilon\mathrm{L}}^2}+\sigma_{\mathrm{u1}}^2\right)^2}} \tag{98}$$

$$\beta_{1H} = \frac{\left(\sigma_\theta^2 + \sigma_{\epsilon\mathrm{H}}^2\right)\left(\frac{1}{\sqrt{\sigma_{\mathrm{u2}}^2}\sqrt{\frac{\sigma_\theta^4\beta_{1H}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2\right)}{2\sigma_\theta^4\beta_{1H}^2\sigma_{\epsilon\mathrm{H}}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2\right)^2}}} - \frac{\sigma_\theta^2\beta_{1H}}{\frac{\sigma_\theta^4\beta_{1H}^4}{\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2}+\sigma_{\mathrm{u1}}^2}\right)}{\frac{2\sigma_\theta^4\left(\sigma_h^2+\sigma_\theta^2\right)\left(\frac{p\beta_{1H}}{\sigma_\theta^4\beta_{1H}^4+\sigma_\theta^2\sigma_{\mathrm{u1}}^2+\sigma_{\mathrm{u1}}^2\sigma_{\epsilon\mathrm{H}}^2}+\frac{\beta_{1L}-p\beta_{1L}}{\sigma_\theta^4\beta_{1L}^4+\sigma_\theta^2\sigma_{\mathrm{u1}}^2+\sigma_{\mathrm{u1}}^2\sigma_{\epsilon\mathrm{L}}^2}\right)}{\sqrt{\sigma_{\mathrm{u2}}^2}\sqrt{\frac{\sigma_\theta^4\beta_{1H}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2\right)}{2\sigma_\theta^4\beta_{1H}^2\sigma_{\epsilon\mathrm{H}}^2+\sigma_{\mathrm{u1}}^2\left(\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2\right)^2}}} - \frac{\sigma_\theta^6\beta_{1H}^2}{\left(\frac{\sigma_\theta^4\beta_{1H}^4}{\sigma_\theta^2+\sigma_{\epsilon\mathrm{H}}^2}+\sigma_{\mathrm{u1}}^2\right)^2}}. \tag{99}$$

Expressions 98 and 99 can now be solved numerically for β_{1L} and β_{1H}.

B.4 Price behavior

Knowing that $X_{2,i}$ is mean zero, the expected price difference at the beginning of period two $\Delta P_{,i} = P_{2,i} - P_1$ is defined by

$$\begin{aligned} E[\Delta P_{,i}|X_1, i] &= E[P_{2,i}|X_{1,i}] - P_1 \\ &= E[P_{1,i}^* + \lambda_{2,i}X_{2,i}|X_{1,i}] - P_1 \\ &= P_{1,i}^* - P_1. \end{aligned} \tag{100}$$

The expected price movement can be characterized by the difference between the actual and the hypothetical first-period price. Simplifying the expression, the expected price difference in the respective regime writes

$$\begin{aligned} P_{1L}^* - P_1 &= \omega_H(E[\theta|X_{1L}] - E[\theta|X_{1H}]) = \omega_H(\lambda_{1L}^* - \lambda_{1H}^*)X_1 \\ P_{1H}^* - P_1 &= (1-\omega_H)(E[\theta|X_{1H}] - E[\theta|X_{1L}]) = (1-\omega_H)(\lambda_{1H}^* - \lambda_{1L}^*)X_1. \end{aligned} \tag{101}$$

Knowing the price differences given by 101, we proceed with the proof of proposition 2.

Proof. Proposition 2
First define:

$\Delta P_{1,i} = P_1 - P_0$ and $\Delta P_{2,i} = P_{2,i} - P_1$

$$\begin{aligned} cov(\Delta P_{1L}\Delta P_{2L}) &= E[(P_1 - P_0)(P_{2L} - P_1)] - E[P_1 - P_0]\underbrace{E[P_{2L} - P_1]}_{0} \\ &= E[P_1(P_{2L} - P_1)] = E[P_1 P_{2L} - P_1^2] = E[P_1 P_{2L}] - E[P_1^2] \\ &= E[\hat{\theta}^2] + \lambda_1\lambda_{1L}^* E[X_{1L}^2] + \lambda_{2L} E[P_1 X_{2L}] - E[\hat{\theta}^2] - \lambda_1\lambda_1 E[X_{1L}^2] \\ &= \lambda_1\lambda_{1L}^* E[X_{1L}^2] + \lambda_{2L}\left(\beta_{2L} E[P_1(E[\theta|S_L] - P_{1L}^*)]\right) - \lambda_1^2 E[X_{1L}^2] \\ &= \lambda_1\lambda_{1L}^* E[X_{1L}^2] + \lambda_{2L}\beta_{2L}\left(\lambda_1\beta_{1L}F_L^2 Var[S_L] - \lambda_1\lambda_{1L}^* E[X_{1L}^2]\right) - \lambda_1^2 E[X_{1L}^2] \\ &= \frac{1}{2}\lambda_1\lambda_{1L}^* E[X_{1L}^2] + \frac{1}{2}\lambda_1\beta_{1L}F_L^2 Var[S_L] - \lambda_1^2 E[X_{1L}^2] \\ &= \lambda_1(\frac{1}{2}\lambda_{1L}^* - \lambda_1)E[X_{1L}^2] + \frac{1}{2}\lambda_1\beta_{1L}F_L^2 Var[S_L] \end{aligned} \tag{102}$$

For $cov(\Delta P_{1L}\Delta P_{2L}) > 0$ equation 102 implies

$$(\lambda_1 - \frac{1}{2}\lambda_{1L}^*) < \frac{1}{2}\frac{\beta_{1L}F_L^2 Var[S_L]}{E[X_{1L}^2]}, \tag{103}$$

using $\lambda_1 = (1-p)\lambda_{1L}^* + p\lambda_{1H}^*$ one gets

$$\begin{aligned} ((\frac{1}{2} - p)\lambda_{1L}^* + p\lambda_{1H}^*) &< \frac{1}{2}\frac{\beta_{1L}F_L^2 Var[S_L]}{E[X_{1L}^2]}, \\ ((\frac{1}{2} - p)\lambda_{1L}^* + p\lambda_{1H}^*) &< \frac{1}{2}\frac{\beta_{1L}F_L^2 Var[S_L]}{E[X_{1L}^2]}. \end{aligned} \tag{104}$$

Knowing

$$\frac{\beta_{1L}F_L^2 Var[S_L]}{E[X_{1L}^2]} = \lambda_{1L}^*,$$

the inequality reduces to

$$\begin{aligned} p\lambda_{1H}^* &< p\lambda_{1L}^* \\ \lambda_{1H}^* &< \lambda_{1L}^* \end{aligned} \tag{105}$$

which is true by definition.

As the problem is symmetric, $cov(\Delta P_{1H}\Delta P_{2H})$ follows analogously

$$
\begin{aligned}
cov(\Delta P_{1H}\Delta P_{2H}) &= E[(P_1 - P_0)(P_{2H} - P_1)] - E[P_1 - P_0]\underbrace{E[P_{2H} - P_1]}_{0} \\
&= E[P_1(P_{2H} - P_1)] = E[P_1 P_{2H} - P_1^2] = E[P_1 P_{2H}] - E[P_1^2] \\
&= E[\hat{\theta}^2] + \lambda_1 \lambda_{1H}^* E[X_{1H}^2] + \lambda_{2H} E[P_1 X_{2H}] - E[\hat{\theta}^2] - \lambda_1 \lambda_1 E[X_{1H}^2] \\
&= \lambda_1 \lambda_{1H}^* E[X_{1H}^2] + \lambda_{2H}\left(\beta_{2H} E[P_1(E[\theta|S_H] - P_{1H}^*)]\right) - \lambda_1^2 E[X_{1H}^2] \\
&= \lambda_1 \lambda_{1H}^* E[X_{1H}^2] + \lambda_{2H}\beta_{2H}\left(\lambda_1 \beta_{1H} F_H^2 Var[S_H] - \lambda_1 \lambda_{1H}^* E[X_{1H}^2]\right) - \lambda_1^2 E[X_{1H}^2] \\
&= \frac{1}{2}\lambda_1 \lambda_{1H}^* E[X_{1H}^2] + \frac{1}{2}\lambda_1 \beta_{1H} F_H^2 Var[S_H] - \lambda_1^2 E[X_{1H}^2] \\
&= \lambda_1(\frac{1}{2}\lambda_{1H}^* - \lambda_1)E[X_{1H}^2] + \frac{1}{2}\lambda_1 \beta_{1H} F_H^2 Var[S_H]
\end{aligned}
\tag{106}
$$

For $cov(\Delta P_{1H}\Delta P_{2L}) < 0$ equation 106 implies

$$
\lambda_1 > \frac{1}{2}\frac{\beta_{1H} F_H^2 Var[S_H]}{E[X_{1H}^2]} + \frac{1}{2}\lambda_{1H}^*. \tag{107}
$$

Knowing

$$
\frac{\beta_{1H} F_H^2 Var[S_H]}{E[X_{1H}^2]} = \lambda_{1H}^*, \tag{108}
$$

the inequality reduces to

$$
\lambda_{1H}^* < \lambda_1 \qquad \text{which implies} \qquad \lambda_{1H}^* < \lambda_{1L}^*. \tag{109}
$$

□

C Sequential equilibrium

C.1 Optimization

Optimization for the periods $(m, ..., N)$

Plugging U_m, V_m and $P_{i,m-1}$ into the maximization yields

$$
\begin{aligned}
&\max_{x_m}\ E_m[\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))\,x_m \\
&\qquad\qquad + \alpha_m\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))^2 + \delta_m | S_i] \\
&\max_{x_m}\ E_m[\,(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m)))\,x_m \\
&\qquad\qquad + \alpha_m((\theta - P_{i,m-1})^2 - 2\lambda_{i,m}(x_m + u_m)(\theta - P_{i,m-1}) \\
&\qquad\qquad + \lambda_{i,m}^2 (x_m + u_m)^2) + \delta_m | S_i].
\end{aligned}
\tag{110}
$$

The resulting FOC regarding x_m is:

$$\begin{aligned} 0 &= E_m[\theta - P_{i,m-1} - 2\lambda_{i,m}x_m - \lambda_{i,m}u_m \\ &\quad + 2\alpha_m\left(\theta - (P_{i,m-1} + \lambda_{i,m}(x_m + u_m))\right)(-\lambda_{i,m})|S_i] \\ 0 &= E_m[\theta|S_i] - P_{i,m-1} - 2\lambda_{i,m}x_m \\ &\quad + 2\alpha_m(E_m[\theta|S_i] - P_{i,m-1})(-\lambda_{i,m}) + 2\alpha_m\lambda_{i,m}^2x_m \end{aligned} \tag{111}$$

Solving for x_m:

$$\begin{aligned} 2\lambda_{i,m}x_m - 2\alpha_m\lambda_{i,m}^2x_m &= (E_m[\theta|S_i] - P_{i,m-1}) \\ &\quad - 2\alpha_m\lambda_{i,m}(E_m[\theta|S_i] - P_{i,m-1}) \\ x_m 2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) &= (1 - 2\alpha_m\lambda_{i,m})(E_m[\theta|S_i] - P_{i,m-1}) \end{aligned} \tag{112}$$

$$x_m = \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}(E_m[\theta|S_i] - P_{i,m-1}) \tag{113}$$

The SOC is given by:

$$2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) > 0 \tag{114}$$

Optimization for the first period, given the result from period $(m - N)$

$$\begin{aligned} &\max_{x_1}\ E[(\theta - \hat{\theta} - \lambda_1(x_1 + u_1))x_1 + (\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1) - \lambda_{i,m}(x_m + u_m))x_m \\ &\qquad + \alpha_m(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1) - \lambda_{i,m}(x_m + u_m))^2 + \delta_m|S_i] \\ &\max_{x_1}\ E[(\theta - \hat{\theta} - \lambda_1(x_1 + u_1))x_1 + (\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1) - \lambda_{i,m}(x_m + u_m))x_m \\ &\qquad + \alpha_m(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))^2 - 2\alpha_m(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))\lambda_{i,m}(x_m + u_m) \\ &\qquad + \alpha_m(\lambda_{i,m}(x_m + u_m))^2 + \delta_m|S_i] \end{aligned} \tag{115}$$

$$\begin{aligned} &\max_{x_1}\ E[(\theta - \hat{\theta} - \lambda_1(x_1 + u_1))x_1 + (\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))x_m - \lambda_{i,m}(x_m^2 + x_mu_m) \\ &\qquad + \alpha_m(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))^2 - 2\alpha_m\lambda_{i,m}(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))x_m \\ &\qquad - 2\alpha_m\lambda_{i,m}(\theta - \hat{\theta} - \lambda_{1,i}^*(x_1 + u_1))u_m \\ &\qquad + \alpha_m\lambda_{i,m}^2(x_m^2 + 2x_mu_m + u_m^2) + \delta_m|S_i] \end{aligned} \tag{116}$$

Taking expectations:

$$
\begin{aligned}
\max_{x_1}\ & E[(\theta-\hat{\theta}-\lambda_1(x_1+u_1))|S_i]x_1 + E[(\theta-\hat{\theta}-\lambda^*_{1,i}(x_1+u_1))x_m|S_i] \\
& - E[\lambda_{i,m}(x_m^2+x_m u_m)|S_i] + E[\alpha_m(\theta-\hat{\theta}-\lambda^*_{1,i}(x_1+u_1))^2|S_i] \\
& - 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}(x_1+u_1))x_m|S_i] \\
& - 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}(x_1+u_1))u_m|S_i] \\
& + E[\alpha_m\lambda_{i,m}^2(x_m^2+2x_m u_m+u_m^2)|S_i] + E[\delta_m|S_i] \\
\max_{x_1}\ & E[(\theta-\hat{\theta}-\lambda_1 x_1)|S_i]x_1 + E[(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)x_m|S_i] \\
& - E[\lambda_{i,m}x_m^2|S_i] + E[\alpha_m(\theta-\hat{\theta})^2|S_i] - 2E[\alpha_m\lambda^*_{1,i}x_1(\theta-\hat{\theta})|S_i] \\
& + E[\alpha{\lambda^*_{1,i}}^2(x_1+u_1)^2|S_i] \qquad (117) \\
& - 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)x_m|S_i] \\
& + E[\alpha_m\lambda_{i,m}^2(x_m^2+u_m^2)|S_i] + E[\delta_m|S_i] \\
\max_{x_1}\ & E[(\theta-\hat{\theta}-\lambda_1 x_1)|S_i]x_1 + E[(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)x_m|S_i] \\
& - E[\lambda_{i,m}x_m^2|S_i] + E[\alpha_m(\theta-\hat{\theta})^2|S_i] - 2E[\alpha_m\lambda^*_{1,i}x_1(\theta-\hat{\theta})|S_i] \\
& + E[\alpha_m{\lambda^*_{1,i}}^2x_1^2|S_i] - 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)x_m|S_i] \\
& + E[\alpha_m\lambda_{i,m}^2x_m^2|S_i] \\
& + E[\alpha_m\lambda_{i,m}^2u_m^2|S_i] + E[\alpha_m{\lambda^*_{1,i}}^2u_1^2|S_i] + E[\delta_m|S_i]
\end{aligned}
$$

Using the fact that

$$x_m = \frac{(1-2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1-\alpha_m\lambda_{i,m})}(E_m[\theta|S_i]-\hat{\theta}-\lambda^*_{1,i}(x_1+u_1)), \qquad (118)$$

and defining

$$
\begin{aligned}
\psi &= \frac{(1-2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1-\alpha_m\lambda_{i,m})}, \\
\varphi &= (E[\theta|S_i]-\hat{\theta}),
\end{aligned}
$$

yields

$$x_m = \psi(\varphi-\lambda^*_{1,i}(x_1+u_1)). \qquad (119)$$

Plugging in for x_m:

$$\max_{x_1}\; E[(\theta-\hat{\theta}-\lambda_1 x_1)|S_i]x_1 + E[(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}(x_1+u_1)))|S_i]$$
$$- E[\lambda_{i,m}(\psi(\varphi-\lambda^*_{1,i}(x_1+u_1)))^2|S_i] + E[\alpha_m(\theta-\hat{\theta})^2|S_i]$$
$$- 2E[\alpha_m\lambda^*_{1,i}x_1(\theta-\hat{\theta})|S_i] + E[\alpha_m{\lambda^*_{1,i}}^2 x_1^2|S_i]$$
$$- 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}(x_1+u_1)))|S_i]$$
$$+ E[\alpha_m\lambda^2_{i,m}(\psi(\varphi-\lambda^*_{1,i}(x_1+u_1)))^2|S_i]$$
$$+ \alpha_m\lambda^2_{i,m}E[u_m^2|S_i] + \alpha_m{\lambda^*_{1,i}}^2 E[u_1^2|S_i] + E[\delta_m|S_i]$$
$$\max_{x_1}\; E[(\theta-\hat{\theta}-\lambda_1 x_1)|S_i]x_1 + E[(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}x_1))|S_i] \tag{120}$$
$$- E[\lambda_{i,m}(\psi(\varphi-\lambda^*_{1,i}x_1))^2|S_i] + E[\alpha_m(\theta-\hat{\theta})^2|S_i]$$
$$- 2E[\alpha_m\lambda^*_{1,i}x_1(\theta-\hat{\theta})|S_i] + E[\alpha_m{\lambda^*_{1,i}}^2 x_1^2|S_i]$$
$$- 2E[\alpha_m\lambda_{i,m}(\theta-\hat{\theta}-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}x_1))|S_i]$$
$$+ E[\alpha_m\lambda^2_{i,m}(\psi(\varphi-\lambda^*_{1,i}x_1))^2|S_i]$$
$$- E[\lambda_{i,m}(\psi\lambda^*_{1,i}u_1)^2|S_i] + E[\alpha_m\lambda^2_{i,m}(\psi\lambda^*_{1,i}u_1)^2|S_i]$$
$$+ \alpha_m\lambda^2_{i,m}E[u_m^2|S_i] + \alpha_m{\lambda^*_{1,i}}^2 E[u_1^2|S_i] + E[\delta_m|S_i]$$

Taking expectations, rearranging and collecting terms:

$$\max_{x_1}\; (\varphi-\lambda_1 x_1)x_1 + (\varphi-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}x_1))$$
$$- \lambda_{i,m}(\psi(\varphi-\lambda^*_{1,i}x_1))^2$$
$$- 2\alpha_m\lambda^*_{1,i}x_1\varphi + \alpha_m{\lambda^*_{1,i}}^2 x_1^2$$
$$- 2\alpha_m\lambda_{i,m}(\varphi-\lambda^*_{1,i}x_1)(\psi(\varphi-\lambda^*_{1,i}x_1))$$
$$+ \alpha_m\lambda^2_{i,m}(\psi(\varphi-\lambda^*_{1,i}x_1))^2$$
$$- \lambda_{i,m}(\psi\lambda^*_{1,i})^2\sigma^2_{u_1} + \alpha_m\lambda^2_{i,m}(\psi\lambda^*_{1,i})^2\sigma^2_{u_1}$$
$$+ \alpha_m\lambda^2_{i,m}\sigma^2_{u_m} + \alpha_m{\lambda^*_{1,i}}^2\sigma^2_{u_1} + E[\delta_m|S_i] + \alpha_m E[(\theta-\hat{\theta})^2|S_i] \tag{121}$$
$$\max_{x_1}\; (\varphi-\lambda_1 x_1)x_1 + \psi(\varphi-\lambda^*_{1,i}x_1)^2$$
$$- \lambda_{i,m}\psi^2(\varphi-\lambda^*_{1,i}x_1)^2$$
$$- 2\alpha_m\lambda^*_{1,i}x_1\varphi + \alpha_m{\lambda^*_{1,i}}^2 x_1^2$$
$$- 2\alpha_m\lambda_{i,m}\psi(\varphi-\lambda^*_{1,i}x_1)^2$$
$$+ \alpha_m\lambda^2_{i,m}\psi^2(\varphi-\lambda^*_{1,i}x_1)^2$$
$$- \lambda_{i,m}(\psi\lambda^*_{1,i})^2\sigma^2_{u_1} + \alpha_m\lambda^2_{i,m}(\psi\lambda^*_{1,i})^2\sigma^2_{u_1}$$
$$+ \alpha_m\lambda^2_{i,m}\sigma^2_{u_m} + \alpha_m{\lambda^*_{1,i}}^2\sigma^2_{u_1} + E[\delta_m|S_i] + \alpha_m E[(\theta-\hat{\theta})^2|S_i]$$

Taking the first derivative regarding x_1 yields the FOC:

$$
\begin{aligned}
0 =& \varphi - 2\lambda_1 x_1 + 2\psi(\varphi - \lambda^*_{1,i}x_1)(-\lambda^*_{1,i}) \\
&- 2\lambda_{i,m}\psi^2(\varphi - \lambda^*_{1,i}x_1)(-\lambda^*_{1,i}) - 2\lambda^*_{1,i}\varphi \\
&+ 2\alpha_m {\lambda^*_{1,i}}^2 x_1 - 4\alpha_m\lambda_{i,m}\psi(\varphi - \lambda^*_{1,i}x_1)(-\lambda^*_{1,i}) \\
&+ 2\alpha_m\lambda^2_{i,m}\psi^2(\varphi - \lambda^*_{1,i}x_1)(-\lambda^*_{1,i}) \\
0 =& \varphi - 2\lambda_1 x_1 - 2\psi\lambda^*_{1,i}(\varphi - \lambda^*_{1,i}x_1) + 2\psi^2\lambda^*_{1,i}\lambda_{i,m}(\varphi - \lambda^*_{1,i}x_1) \\
&- 2\alpha_m\lambda^*_{1,i}\varphi + 2\alpha_m{\lambda^*_{1,i}}^2 x_1 + 4\alpha_m\psi\lambda^*_{1,i}\lambda_{i,m}(\varphi - \lambda^*_{1,i}x_1) \\
&- 2\alpha_m\psi^2\lambda^*_{1,i}\lambda^2_{i,m}(\varphi - \lambda^*_{1,i}x_1)
\end{aligned}
\tag{122}
$$

Rearranging and collecting terms:

$$
\begin{aligned}
0 =& \varphi - 2\lambda_1 x_1 + 2{\lambda^*_{1,i}}^2(\psi\lambda_{i,m} - 1)(\alpha_m\psi\lambda_{i,m} - \alpha_m - \psi)x_1 \\
&- 2\lambda^*_{1,i}(\psi\lambda_{i,m} - 1)(\alpha_m\psi\lambda_{i,m} - \alpha_m - \psi)\varphi
\end{aligned}
\tag{123}
$$

Finally, solving for x_1 yields:

$$
x_1 = \frac{2\lambda^*_{1,i}(\psi\lambda_{i,m} - 1)(\alpha_m\psi\lambda_{i,m} - \alpha_m - \psi) - 1}{2{\lambda^*_{1,i}}^2(\psi\lambda_{i,m} - 1)(\alpha_m\psi\lambda_{i,m} - \alpha_m - \psi) - 2\lambda_1}\varphi. \tag{124}
$$

Substituting back

$$
\begin{aligned}
\psi &= \frac{(1 - 2\alpha_m\lambda_{i,m})}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})} \\
\varphi &= (E[\theta|S_i] - \hat{\theta})
\end{aligned}
$$

gives

$$
\begin{aligned}
x_1 &= \frac{\lambda^*_{1,i} + 2\lambda_{i,m}(\alpha_m\lambda_{i,m} - 1)}{{\lambda^*_{1,i}}^2 + 4\lambda_1\lambda_{i,m}(\alpha_m\lambda_{i,m} - 1)}(E[\theta|S_i] - \hat{\theta}) \\
x_1 &= \frac{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - \lambda^*_{1,i}}{4\lambda_1\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2}(E[\theta|S_i] - \hat{\theta}).
\end{aligned}
\tag{125}
$$

With the SOC:

$$
\begin{aligned}
0 &< 2\lambda_1 - 2{\lambda^*_{1,i}}^2(\psi\lambda_{i,m} - 1)(\alpha_m\psi\lambda_{i,m} - \alpha_m - \psi) \\
0 &< \frac{{\lambda^*_{1,i}}^2 + 4\lambda_1\lambda_{i,m}(\alpha_m\lambda_{i,m} - 1)}{2\lambda_{i,m}(\alpha_m\lambda_{i,m} - 1)} \\
0 &< \frac{4\lambda_1\lambda_{i,m}(1 - \alpha_m\lambda_{i,m}) - {\lambda^*_{1,i}}^2}{2\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}
\end{aligned}
\tag{126}
$$

C.2 Equilibrium coefficients N-period setting

Plugging in the optimized value of x_m:

$$x_m = \psi(\varphi - \lambda^*_{1,i}(x_1 + u_1)) \tag{127}$$

$$\begin{aligned} &E[\,(\theta - (P_{i,m-1} + \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1}) + u_m)))\,\psi(E[\theta|S_i] - P_{i,m-1}) \\ &\quad + \alpha_m\,(\theta - (P_{i,m-1} + \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1}) + u_m)))^2 + \delta_m|S_i] \end{aligned} \tag{128}$$

Taking expectations

$$\begin{aligned} &(E[\theta|S_i] - (P_{i,m-1} + \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1})))\,\psi(E[\theta|S_i] - P_{i,m-1}) \\ &+ \alpha_m E[(\theta - (P_{i,m-1} + \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1}) + u_m)))^2\,|S_i] + \delta_m \end{aligned} \tag{129}$$

$$\begin{aligned} &\psi(E[\theta|S_i] - P_{i,m-1})^2 - \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1}))^2 \\ &+ \alpha_m E[(\theta - (P_{i,m-1})^2 - 2\lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1})(\theta - P_{i,m-1}) \\ &+ {\lambda_{i,m}}^2(\psi(E[\theta|S_i] - P_{i,m-1})^2|S_i] + \alpha_m{\lambda_{i,m}}^2\sigma^2_{u_m} + \delta_m \\ &= \psi(E[\theta|S_i] - P_{i,m-1})^2 - \lambda_{i,m}(\psi(E[\theta|S_i] - P_{i,m-1}))^2 \\ &+ \alpha_m(E[\theta|S_i] - P_{i,m-1})^2 - 2\lambda_{i,m}\psi(E[\theta|S_i] - P_{i,m-1})^2 \\ &+ {\lambda_{i,m}}^2(\psi(E[\theta|S_i] - P_{i,m-1})^2|S_i] + \alpha_m{\lambda_{i,m}}^2\sigma^2_{u_m} + \delta_m, \end{aligned} \tag{130}$$

which can be rewritten as

$$\begin{aligned} &(\psi - \lambda_{i,m}\psi^2 + \alpha_m - 2\alpha_m\lambda_{i,m}\psi + \alpha_m + {\lambda_{i,m}}^2\psi^2)(E[\theta|S_i] - P_{i,m-1})^2 \\ &+ \alpha_m{\lambda_{i,m}}^2\sigma^2_{u_m} + \delta_m. \end{aligned} \tag{131}$$

This yields

$$\alpha_{m-1} = (\psi - \lambda_{i,m}\psi^2 + \alpha_m - 2\alpha_m\lambda_{i,m}\psi + \alpha_m + {\lambda_{i,m}}^2\psi^2),$$

which simplifies to

$$\alpha_{m-1} = \frac{1}{4\lambda_{i,m}(1 - \alpha_m\lambda_{i,m})}. \tag{132}$$

The expected utility of the informed in the first period is given by:

$$\alpha_0 = \frac{\lambda_{i,m}(1 - \alpha_m \lambda_{i,m}) - \lambda_{1,i}^* + \lambda_1}{4\lambda_1 \lambda_{i,m}(1 - \alpha_m \lambda_{i,m}) - {\lambda_{1,i}^*}^2} \tag{133}$$

The overall utility of the informed trader writes:

$$\begin{aligned} E[\pi|S_i] = & \frac{\lambda_{i,m}(1 - \alpha_m \lambda_{i,m}) - \lambda_{1,i}^* + \lambda_1}{4\lambda_1 \lambda_{i,m}(1 - \alpha_m \lambda_{i,m}) - {\lambda_{1,i}^*}^2} F_i(S - \hat{\theta}) \\ & + \frac{{\lambda_{1,i}^*}^2 \sigma_{u_1}^2}{4\lambda_{i,m}(1 - \alpha_m \lambda_{i,m})} + \delta_1 \end{aligned} \tag{134}$$