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Skill-Biased Technological Change and the Real Exchange Rate*

Matthias Gubler Christoph Sax

Abstract

We sketch a model that shows how skill-biased technological change may reverse the classic Balassa-Samuelson effect, leading to a negative relationship between the productivity in the tradable sector and the real exchange rate. In a small open economy, export goods are produced with capital, high-skilled and low-skilled labor, and traded for imported consumption goods. Non-tradable services are produced with low-skilled labor only. A rise in the productivity of capital has two effects: (1) It may reduce the demand for labor in the tradable sector if the substitutability of low-skilled labor and capital in the tradable sector is high; and (2) it increases the demand for non-tradables and its labor input. Overall demand for low-skilled labor declines if the labor force of the tradable sector is large relative to the labor force of the non-tradable sector. This leads to lower wages and thus to lower prices and a real exchange rate depreciation.

JEL Classifications: F16, F31, F41, J24
Keywords: Real Exchange Rate, Balassa-Samuelson Hypothesis, Skill-biased Technological Change, General Equilibrium

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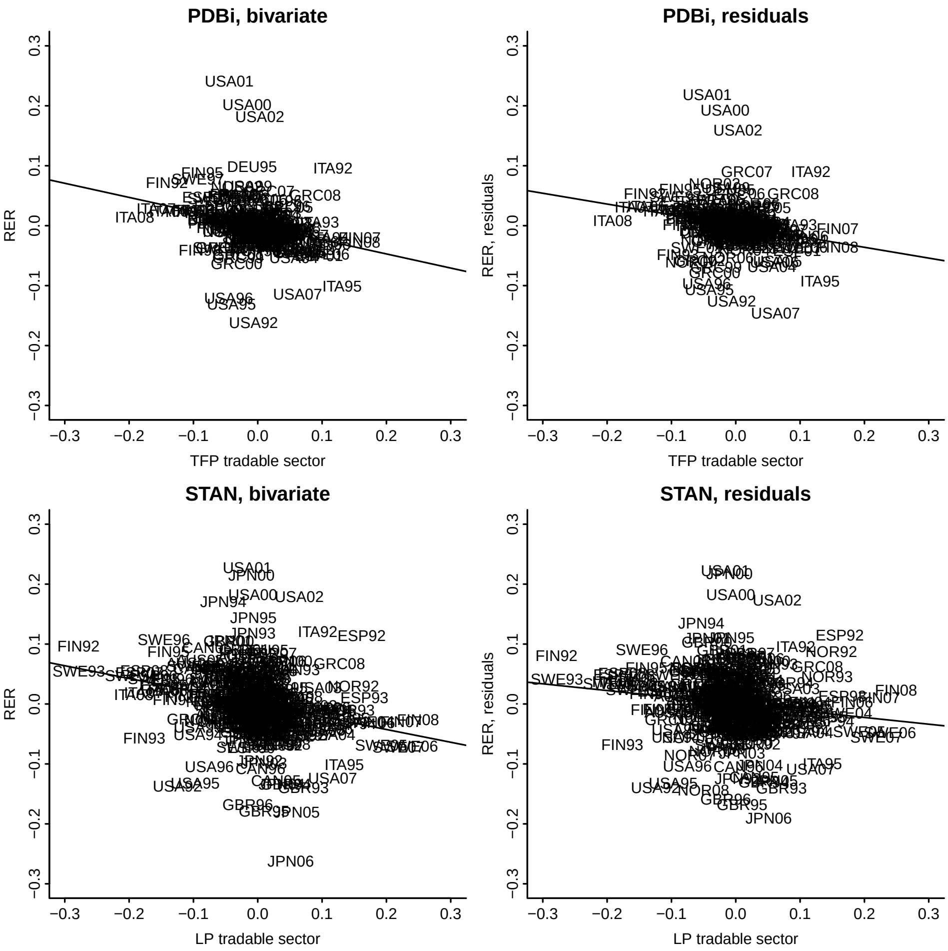

Figure 1: Tradable Productivity and the Real Exchange Rate since 1992

1 Introduction

The Balassa-Samuelson (BS) hypothesis states that price level differences between countries, expressed in the same currency, can be ascribed to different productivity differentials between the non-tradable and tradable sector. Through wage adjustments in the non-tradable sector, an increase in the productivity of tradables leads to an appreciation of the real exchange rate, while an increase in the productivity of non-tradables has the opposite effect. The hypothesis was simultaneously developed by Balassa (1964) and Samuelson (1964), but has a research precedent in the work of Harrod (1933). It is one of the most widespread explanations for structural deviations from purchasing power parity (Dornbusch, 1985).

There are a number of studies that find evidence supporting the BS hypothesis (see, e.g., De Gregorio and Wolf, 1994; Chinn and Johnston, 1996 or MacDonald and Ricci, 2007) by using panel data on sectoral total factor productivity (TFP). However, all of these studies rely on the discontinued OECD International Sectoral Database (ISDB). When performing a similar analysis with contemporary data, taken from the newly released OECD Productivity Database (PDBi), Gubler and Sax (2011) cannot confirm the hypothesis.

For the last two decades, they find a robust negative relationship between the productivity in the tradable sector and the real exchange rate in the long run, in contrast to BS. Earlier results supporting the BS hypothesis seem to depend strongly on the choice of the data set.[1] The findings of Gubler and Sax (2011) are confirmed once the TFP values are substituted by labor productivity (LP) values from the OECD Structural Analysis (STAN) database. Figure (1) illustrates the negative relationship. The left panel contains the productivity of tradables in relation to the real exchange rate adjusted by country-specific and time-specific effects. For the right panel, the real exchange rate is additionally adjusted by the productivity of non-tradables and the terms of trade. Both estimations with LP data from the STAN database and TFP data from the PDBi show a significant negative relationship.[2]

The fact that there is a robust negative relationship between tradable productivity and the real exchange rate is puzzling. According to the BS hypothesis, a higher productivity in the tradable sector is expected to be associated with a stronger real exchange rate. What causes this puzzle?

This paper presents a static general-equilibrium model with skill-biased technological change (SBTC). Inspired by the work of De Gregorio and Wolf (1994) and Autor and Dorn (2009), it provides an explanation for the negative relationship between the productivity in the tradable sector and the real exchange rate.

Our model shares its basic structure with the model of De Gregorio and Wolf (1994): There is a tradable goods industry that trades its single out-

[1] The analysis indicates that the discrepancy in the results cannot be ascribed to the change in the sample period.

[2] A detailed analysis reveals that this reversal is mainly driven by the manufacturing sector, i.e., the higher the productivity in manufacturing, the lower is a country's relative price level.

put good for a single imported good, which is consumed together with a domestically produced non-tradable service.

Furthermore, our model introduces two types of labor, along the lines suggested by Autor and Dorn (2009): low-skilled and high-skilled workers. High-skilled labor is used exclusively in the tradable sector, while low-skilled labor moves freely between the tradable and the non-tradable sector. In the non-tradable sector, low-skilled labor is the only factor of production.

In the tradable sector, low-skilled labor, together with capital, is used to produce an intermediate routine task good, which in turn is combined with high-skilled labor to produce the final tradable good. A key feature of the model is the substitutability of the two factors involved in the production of the intermediate routine task good, low-skilled labor and capital.

In order to analyze the reversion of the BS effect, our study assesses the effect of capital augmenting, i.e. Solow-neutral, technological change on the economy, and especially on the real exchange rate. Ongoing technological progress during the last two decades, particularly in information technology since the 1990s, makes this assumption plausible. Furthermore, Boskin and Lau (2000) identify capital augmenting technological change as the main driver of postwar economic growth of the G7 countries. Alternatively, a very similar effect occurs when the price of capital decreases.[3]

An increase in capital productivity has two effects on the real exchange rate, both operating through their impact on the demand for low-skilled labor. First, under certain conditions, a capital productivity improvement reduces the demand for low-skilled labor in the tradable sector. This is the *labor-repellent effect.* The demand diminishes as long as the elasticity of substitution between low-skilled labor and capital is high relative to the importance of the intermediate routine task good in the production of the final tradable good. We provide the necessary and sufficient condition for the effect to occur.

Second, a rise in capital productivity increases the demand for low-skilled workers in the non-tradable service sector. This is the *labor-attracting effect.* As an increase in capital productivity leads to higher income, consumers can increase their consumption of tradable imported goods. Limited consumer desire to substitute between tradable goods and non-tradable services also

[3]Autor and Dorn (2009) assume a steady fall in the price of capital in their analysis of wage dispersion in the United States.

increases the demand for non-tradable services, which in turn raises the demand of firms in the non-tradable sector for low-skilled workers.

Depending on whether the labor-repellent effect or the labor-attracting effect is stronger, it is possible that overall demand for low-skilled workers diminishes, i.e., the rise in demand for low-skilled workers in the non-tradable sector does not offset the fall in the demand for these workers to produce tradable goods. Consequently, the wage for low-skilled labor drops in the general equilibrium, because it is assumed that the overall labor force is fixed and the labor market clears. Finally, the price level of the economy decreases. Thus, an increase in tradable productivity may be connected to a lower price level, and leads to an opposite BS effect.

Whether the labor-repellent effect or the labor-attracting effect dominates depends crucially on the fraction of low-skilled labor used in the production of tradable goods. In order to ensure that the labor-repellent effect outweighs the labor-attracting effect for a given wage rate, the labor force in the tradable sector must be large compared to the labor force in the non-tradable sector.

The remainder of this paper is organized as follows. We present the structure of the model in Section 2. In Section 3, we discuss the demand for low-skilled labor in the tradable sector. Section 4 derives the demand for low-skilled workers in the non-tradable sector. Overall demand for low-skilled labor is described in Section 5. Section 6 outlines the general equilibrium. Section 7 concludes.

2 Structure of the Economy

The basic structure of the economy is build along the lines suggested by De Gregorio and Wolf (1994): In a small open economy, there are two sectors, each producing a homogeneous good, the tradable exported good, Y_x, and the non-tradable service, Y_n. The tradable good is entirely traded for the imported good, Y_m, at a given world price, p_x. Households gain utility from the consumption of the imported good, Y_m, and the non-tradable service, Y_n. Capital is specific to the tradable sector and assumed to be completely mobile between countries. Low-skilled workers can move between sectors but not between countries. In the following sections, we specify the model in detail.

2.1 Production of Tradables and Non-Tradables

In our model, the production of tradables differs in two ways from the model proposed by De Gregorio and Wolf (1994): First, there is a second type of labor, high-skilled labor, L_h, that is specific to the tradable industry. Second, low-skilled labor, L_x, and capital, K, are close substitutes. Both differences are reflected in a modified production function for tradables that is borrowed from Autor and Dorn (2009):

$$Y_x = L_h^{1-\beta} \left(\underbrace{[(a_r\, L_x)^\mu + (a_k\, K)^\mu]^{\frac{1}{\mu}}}_{\text{routine task good}} \right)^\beta . \tag{1}$$

This Cobb-Douglas production function with $0 < \beta < 1$ nests a Constant Elasticity of Substitution (CES) function, which produces an intermediate routine task good. As capital and low-skilled labor are, by assumption, close substitutes, the elasticity of substitution, $\epsilon = 1/(1-\mu)$, is larger than 1, implying $0 < \mu < 1$. The intermediate routine task good is combined with high-skilled labor to produce the final exported good. $a_r > 0$ and $a_k > 0$ represent exogenous productivity parameters for low-skilled labor and capital, respectively. Note that the productivity parameter for high-skilled labor is normalized to unity, and so a_r and a_k may be interpretated as relative productivity terms.

The production of non-tradables, Y_n, is described by a linear production function in low-skilled labor, L_n, (De Gregorio and Wolf, 1994; Autor and Dorn, 2009):

$$Y_n = a_n\, L_n, \tag{2}$$

where $a_n > 0$ denotes exogenous low-skilled labor productivity in the non-tradable sector.

2.2 Capital and Labor Markets

In our model, we assume that capital is completely mobile between countries. Moreover, the economy is too small to affect the world price of capital. Therefore, firms in the tradable sector can adjust their capital input at a given price $r > 0$.

High-skilled labor is used exclusively in the tradable sector, while low-skilled workers are mobile between the tradable and the non-tradable sector. In the non-tradable sector, low-skilled labor is the only factor of production. We assume that the supply of both low-skilled labor, $\bar{L}_l$, and high-skilled labor, $\bar{L}_h$, is fixed and no transformation from $\bar{L}_l$ to $\bar{L}_h$ is possible. Furthermore, labor cannot move between countries.

2.3 Consumption

Households gain utility from the consumption of the imported good, Y_m, and the non-tradable service, Y_n, according to a CES utility function (De Gregorio and Wolf, 1994; Autor and Dorn, 2009):

$$U = \left({Y_n}^{\phi} + {Y_m}^{\phi}\right)^{\frac{1}{\phi}}, \tag{3}$$

where $\epsilon_c = 1/(1-\phi)$ is the elasticity of substitution between the two consumption goods. We assume that imported goods and non-tradable services are complements, and therefore, $\epsilon_c < 1$, implying $\phi < 0$.

2.4 Prices, Wages, and the Real Exchange Rate

As there is only one international currency, the real exchange rate (RER) between two countries is defined as the ratio of the consumer price index (CPI) of the home country, i, to the CPI of the foreign country, j:

$$\text{RER}_{ij} = \frac{\text{CPI}_i}{\text{CPI}_j}, \tag{4}$$

where the CPI is a weighted average of the goods and services that are consumed domestically, i.e., imported goods and non-tradable services:

$$\text{CPI} = \gamma\, p_n + (1-\gamma)\, p_m, \tag{5}$$

where $\gamma = 1/(1+s_c)$ denotes the share of C_n in total consumption and $s_c = C_m/C_n > 0$ is the fraction of imported goods to non-tradable services.

Without loss of generality, we set the price of the imported good equal to one ($p_m = 1$). Therefore, all prices are expressed in units of the imported good. The normalization has two advantages: First, the price of the exported good, p_x, directly reflects the terms of trade. Second, as the consumer price index is expressed in units of the imported good, Equation (5) simplifies to

$\text{CPI} = \gamma\, p_n + (1-\gamma)$. Given that γ is held constant[4], the price of the non-tradable service, p_n, determines the CPI and the real exchange rate. Finally, p_n is determined by the profit maximizing conditions in the non-tradable service industry: $w = a_n\, p_n$, where w denotes the wage rate and a_n denotes exogenous labor productivity in the non-tradable sector.[5]

In the following, we skip the steps from w to p_n to the real exchange rate, focusing on the behavior of w in the general equilibrium. Once the equilibrium wage rate, w^*, is known, the determination of the equilibrium price of the non-tradable service, p_n^*, the equilibrium consumer price index, CPI*, and the equilibrium real exchange rate, RER*, is straightforward:

$$w^* \Longrightarrow p_n^* \Longrightarrow \text{CPI}^* \Longrightarrow \text{RER}^*$$

3 Low-Skilled Labor Demand in the Tradable Sector

As the supply of low-skilled labor is fixed, the wage rate, w, is determined by the demand for low-skilled workers. Overall demand for low-skilled labor consists of two components: the demand for low-skilled workers in the tradable sector and the demand for low-skilled workers in the non-tradable sector. An exogenous shock, like an increase in capital productivity, may affect the demand for this input factor both in the tradable export sector and in the non-tradable service sector. We will discuss low-skilled labor demand in the tradable sector first.

Given the production function in Equation (1), the profit function is:

$$\pi = p_x\, Y_x - w\, L_x - r\, K - w_h\, L_h, \tag{6}$$

where w and w_h are the real wages of low-skilled labor and high-skilled labor, respectively. Hereafter, low-skilled labor is generally referred to as labor. r denotes the given world real interest rate for capital. As is the case for any production function with constant returns to scale, the optimal capital intensity in a CES production function does not depend on the level of production. As shown in Appendix A.1, the optimal capital intensity, s, depends only on relative productivities, a_k/a_r, relative factor prices, w/r,

[4] Usually for price index calculations, the weights are held constant. However, in our model, the weights are adjusting to price changes and amplify the effect of p_n on the CPI.

[5] The profit maximizing conditions in the non-tradable sector are derived in Section 4.

and on the elasticity of substitution, μ:

$$s = \frac{K^*}{L_x^*} = \left(\frac{{a_k}^{\mu}\, w}{{a_r}^{\mu}\, r}\right)^{\frac{1}{1-\mu}}. \tag{7}$$

Intuitively, an increase in a_k or a decrease in r makes capital more attractive, causing firms to substitute capital for labor. On the other hand, an increase in a_r or a decrease in w makes labor more attractive, causing firms to substitute labor for capital.

Because s does not depend on the level of L_x and K, we substitute sL_x for K in Equation (1) and replace Y_x in Equation (6) to obtain:

$$\pi = p_x\, L_h^{1-\beta} ({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})^{\frac{\beta}{\mu}}\, {L_x}^{\beta} - w\, L_x - r\, s\, L_x - w_h\, L_h. \tag{8}$$

As the supply of high-skilled labor is fixed to $\bar{L}_h$, firms will employ all high-skilled workers and optimize over the number of low-skilled workers.[6]

Thus, the first order condition with respect to L_x is:

$$\beta\, p_x\, \bar{L}_h^{1-\beta} ({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})^{\frac{\beta}{\mu}}\, {L_x}^{\beta-1} = w + r\, s, \tag{9}$$

and has a straightforward interpretation: The left-hand side is the marginal revenue of L_x, taking into account that an increase in L_x also implies a higher K. The right-hand side represents the marginal costs of L_x and the additional amount of K associated with it.

Solving for L_x reveals the optimal demand for labor in the tradable sector:

$$L_x = \bar{L}_h \left(\frac{\beta\, p_x\, ({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})^{\frac{\beta}{\mu}}}{w + r\, s}\right)^{\frac{1}{1-\beta}}. \tag{10}$$

Note that capital intensity, s, is itself a function of the parameters a_k, a_r, r, w and μ.

Figure (2) displays the relationship between the wage and low-skilled labor demand in the tradable sector for two values of a_k. As the proof in Appendix A.2 demonstrates, the demand for L_x is decreasing in w for all values in the specified parameter space. Intuitively, there are two reasons behind the decreasing relationship: First, there is a *substitution effect*: An

[6]Given the fixed supply $\bar{L}_h$, the wage of high-skilled workers is determined for every level of Y_x. However, for our analysis, the wage of high-skilled workers is irrelevant.

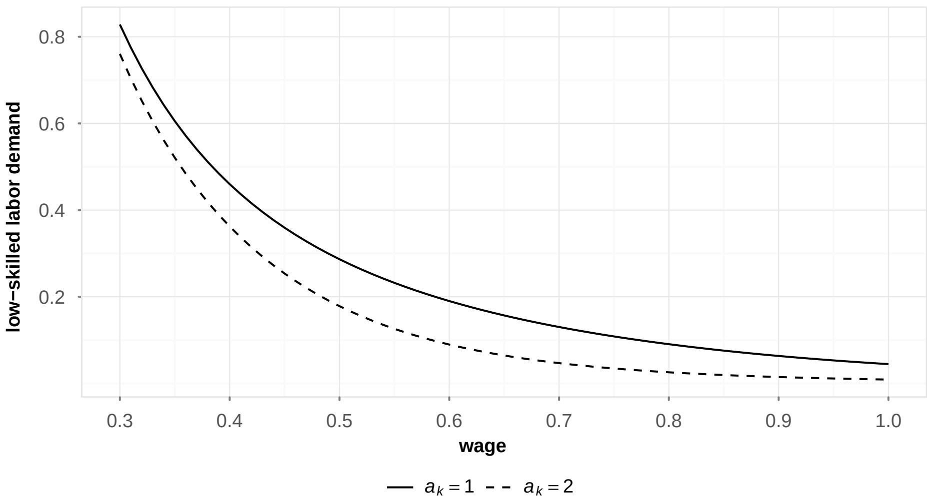

Figure 2: The figure shows a numerical example for low-skilled labor demand in the tradable sector with low capital productivity ($a_k = 1$, solid line), and a numerical example with high capital productivity ($a_k = 2$, dashed line). The other parameters are $a_r = 1$, $\mu = 0.8$, $\beta = 0.5$, $r = 1$, $p_x = 1$, $L_h = 0.3$.

increase in w leads to an increase in s, as firms substitute capital for labor. Second, there is an *income effect*. Even if there was no substitution effect, firms would reduce the number of workers as the optimal level of overall production decreases.

What is the impact of the productivity of capital, a_k, on L_x? Again, an increase in a_k has two effects: Through the substitution effect, a_k negatively affects L_x. With a_k increasing, s rises as firms substitute capital for labor. However, the income effect works in the opposite direction. With an enhanced capital productivity, the optimal production of the final good increases, causing firms to increase their demand for labor. Overall, the impact of a_k on L_x is ambiguous. As shown in Appendix A.3, the impact of a_k on L_x crucially depends on the relation between μ and β. If and only if $\mu > \beta$, an increase in a_k leads to a decrease in L_x. In Figure (2), with μ being larger than β, an increase in a_k gives rise to a decrease in L_x, while in Figure (3), with μ being smaller than β, an increase in a_k leads to an increase in L_x.

Intuitively, in order to observe a *labor-repellent effect* in the tradable sector, the substitution effect must be high relative to the income effect. If β is small, the routine task intermediate good has only a limited importance

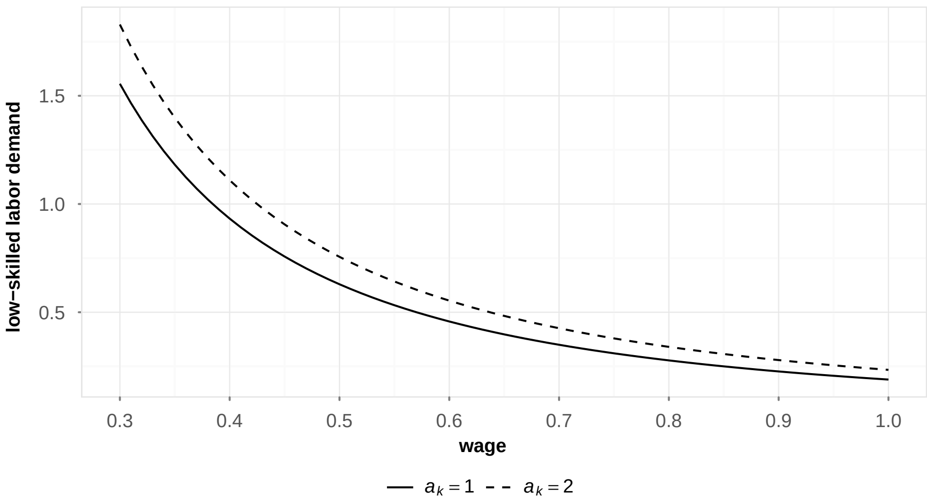

Figure 3: The figure shows a numerical example for low-skilled labor demand in the tradable sector with low capital productivity ($a_k = 1$, solid line), and a numerical example with high capital productivity ($a_k = 2$, dashed line). The other parameters are $a_r = 1$, $\mu = 0.3$, $\beta = 0.5$, $r = 1$, $p_x = 1$, $L_h = 0.3$.

in the production of the final good, and the increase in overall production is small. Thus, β scales the size of the income effect, while μ determines the substitution effect.

The effect of a change in the price of capital, r, on L_x is comparable to the effect of a_k on L_x, but works in the opposite direction. Through the substitution effect, a decrease in r leads to a lower demand for labor; through the income effect, it increases the demand for labor. Overall, if and only if $\mu > \beta$, a decrease in r leads to a decrease in L_x, as shown in Appendix A.4.

Remember that a negative impact of a_k on L_x is a necessary precondition in order to observe the reversed BS effect. Figure (4) depicts the *negative* elasticity of L_x with respect to a_k, $\epsilon_{Lx,ak}$. As soon as the elasticity becomes positive, it is below the surface and not shown. The percentage change in L_x is plotted on the vertical axis, w and μ on the horizontal axes. Figure (4) demonstrates the interplay of μ and w on the elasticity of L_x. From this, the following observations can be made:

First, the elasticity is negative only for values of $\mu > \beta$, as explained above. Second, for values of $\mu > \beta$, there is a transient peak as one moves from low to high values of w: Low wages are associated with low absolute elasticity values, as the number of workers that are substituted by an increase

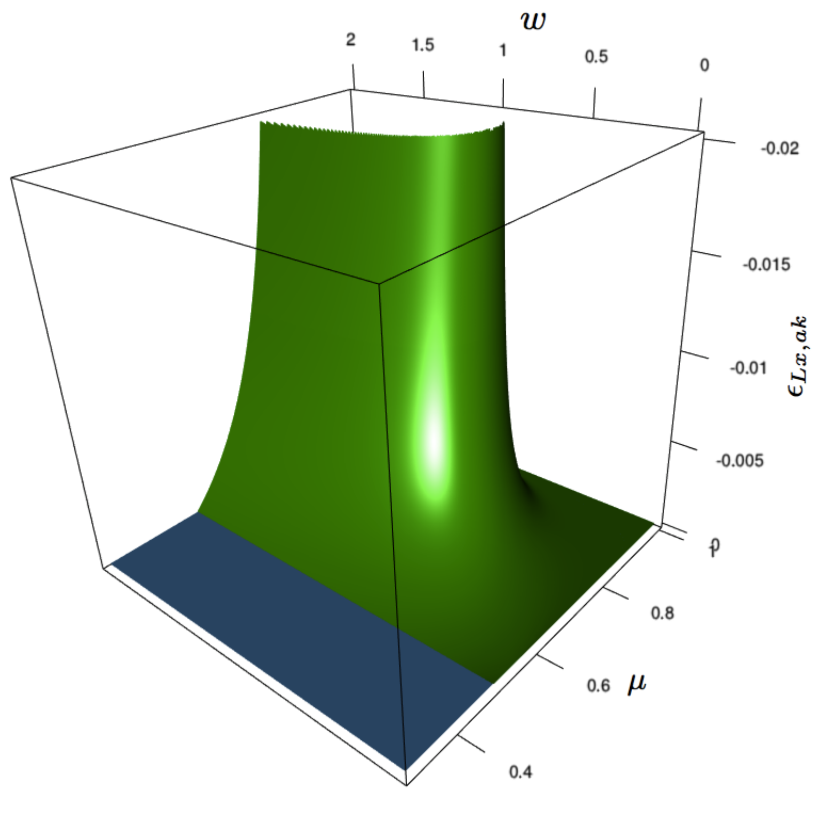

Figure 4: The figure shows the elasticity of low-skilled labor demand in the tradable sector with respect to capital productivity, $\epsilon_{Lx,ak}$, as a function of μ, where $1/(1-\mu)$ is the elasticity of substitution between low-skilled labor and capital, and the wage rate, w. The other parameters are $a_r = 1$, $\beta = 0.5$, $r = 1$, $p_x = 1$.

Table 1: The effect of the parameters on L_x

Parameter	Substitution Eff.	Income Effekt	Overall, $\beta < \mu$	Overall, $\beta > \mu$
w	$-$	$-$	$-$	$-$
r	$+$	$-$	$+$	$-$
a_k	$-$	$+$	$-$	$+$
a_r	$+$	$+$	$+$	$+$
L_h			$+$	$+$
p_x			$+$	$+$

in a_k is very small relative to the size of the labor force in the sector. An increase in w leads to a stronger negative elasticity, but the marginal effect converges to zero, because most of the production of the intermediate routine good is done by capital. Third, if $\mu \to 1$, either labor *or* capital is employed in the production of the intermediate good. At the point where the capital fraction, s, is equal to one, all workers are substituted by capital. Thus, the parameters determining s also determine the shape of the 'elasticity hill'. Note that the figure is truncated at $\epsilon_{Lx,ak} = -0.2$. Therefore, the figure does not show the large negative values for $\mu \to 1$, in order to facilitate the interpretation of the figure.

There are two other parameters that influence L_x. Increasing the supply of high-skilled workers, $\bar{L}_h$, simply scales up the production. The elasticity of L_x with respect to $\bar{L}_h$ is one. As the price of the imported good, p_m is normalized to unity, p_x denotes the terms of trade and has a similar effect as $\bar{L}_h$. A one percent increase in p_x leads to an increase in L_x by $1/(1-\beta)$ percent. Both parameters, $\bar{L}_h$ and p_x, do not affect the elasticity of L_x with respect to a_k, as can be seen from Equation (29) in Appendix A.3. The impact of all parameters on L_x is summarized in Table (1).

The analysis reveals that the relation between the substitution effect determined by μ and the income effect determined by β is crucial for observing the reverse BS effect. Whether or not the condition is fulfilled is an empirical question. Krusell et al. (2000) estimate the substitution elasticity, $1/(1-\mu)$, between capital and unskilled labor in a four-factor production function to be about 1.7. However, their data set covers both the tradable and non-tradable sector of the U.S. economy, and the sample period ends in 1992. We think that this value is substantially larger for the tradable sector

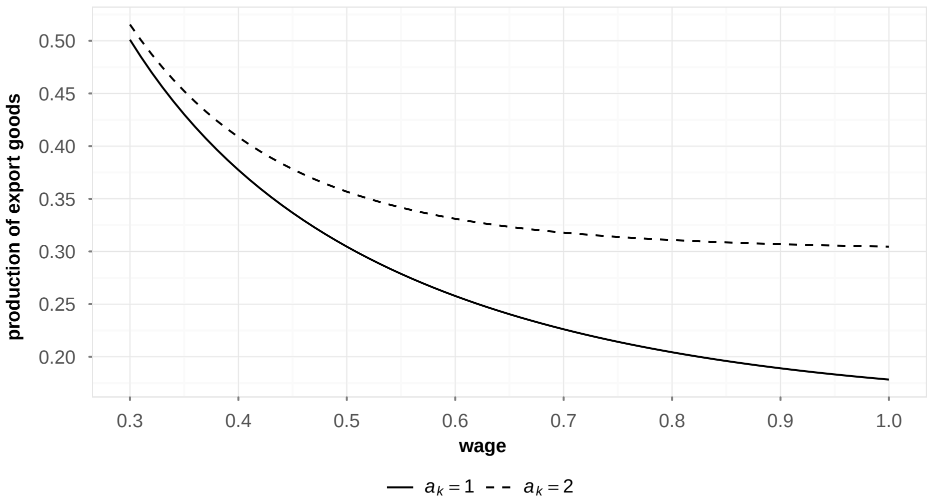

Figure 5: The figure shows a numerical example for the production of exported goods with low capital productivity ($a_k = 1$, solid line), and a numerical example with high capital productivity ($a_k = 2$, dashed line). The other parameters are $a_r = 1$, $\mu = 0.8$, $\beta = 0.5$, $r = 1$, $p_x = 1$, $L_h = 0.3$.

during recent decades, in particular for manufacturing, the largest subsector. Additionally, the income share of capital and low-skilled labor has been falling over the last 30 years, as the relative wage and employment share of high-skilled labor has been rising (Autor and Dorn, 2009).

4 Low-Skilled Labor Demand in the Non-Tradable Sector

The demand for labor in the non-tradable sector is the second component of overall labor demand. An increase in capital productivity affects the demand for labor in the non-tradable sector by increasing the production of exported goods, and by increasing the amount of imported goods available to consumers. Since the elasticity of substitution of the consumers is limited, a higher amount of the imported good leads to an increase in the demand for non-tradable services, which in turn increases the demand for labor in this sector. We analyze this mechanism step by step.

4.1 Production of Tradable Goods and International Trade

An expression for the production of the exported good, Y_x, can be obtained by inserting the low-skilled labor demand function of the export sector (given in Equation 10) into the production function (given in Equation 1). The resulting function decreases in w, but converges to a constant level associated with capital as the only input in the production of the intermediate routine task good. Figure (5) shows the relationship between the wage and the production of exported goods for two values of a_k. For any given wage, the production of Y_x is increasing in a_k. However, for very low levels of w, almost no capital is employed, and an increase in a_k has only a small effect on production.

In the next step, Y_x is traded for the imported good, Y_m, at the price p_x. Thus, the 'production' of imports by domestic exporters is given by:

$$Y_m = p_x \, Y_x. \tag{11}$$

4.2 Consumers

Because consumers' utility is generated by a CES function (shown in Equation 3), the optimal consumption share, s_c, between the imported good and the non-tradable service is independent of the level of consumption. The same argument that applies to the optimal capital intensity, s, holds for s_c (see Appendix A.1).[7]

Given C_m, p_n and ϕ, the demand for non-tradable services, C_n, is determined by:

$$C_n = s_c \, C_m = \left(\frac{1}{p_n}\right)^{\frac{1}{1-\phi}} C_m. \tag{12}$$

The demand for C_n depends on the relative price of the two consumption goods (which is p_n, as p_m is normalized to unity), the elasticity of substitution, $\epsilon_c = 1/(1-\phi)$, and the demand for tradable goods, C_m.

4.3 Production of the Non-Tradable Service

The non-tradable sector uses labor as its only input factor and linearly transforms it into output. Profit maximizing implies that the wage rate is:

$$w = a_n \, p_n. \tag{13}$$

[7] For the Leontief utility function, a special case of the CES utility function, s_c is equal to one and independent of relative prices.

Substituting p_n in Equation (12) and using the market clearing conditions $Y_m = C_m$ and $Y_n = C_n$, we get:

$$Y_n = \left(\frac{a_n}{w}\right)^{\frac{1}{1-\phi}} Y_m. \tag{14}$$

When the demand for non-tradable services is determined, so is the demand for labor in the non-tradable sector. We make use of Equation (2) to obtain:

$$L_n = a_n{}^{\frac{\phi}{1-\phi}} \left(\frac{1}{w}\right)^{\frac{1}{1-\phi}} Y_m. \tag{15}$$

Because Y_x negatively depends on w, and because Y_m is the product of Y_x and p_x, an increase in w leads to lower imports and to a lower demand for L_n.

How is L_n affected by a_k? For any given wage, an increase in a_k increases the demand for non-tradable labor by raising the amount of imports due to the *labor-attracting effect* $(\partial L_n / \partial a_k > 0)$.

5 Total Demand for Low-Skilled Labor

As a final step, total demand for low-skilled labor is the sum of the demand for labor in both sectors:

$$L_l = L_x + L_n. \tag{16}$$

Figure (6) illustrates the relationship between the wage and total low-skilled labor demand for two values of a_k. The demand for L_l is decreasing in w for all values in the specified parameter space. This is not surprising, as both L_x and L_n are decreasing functions in w. An overview of the effects of the parameters on L_l is given in column (5) of Table (2).

The impact of a_k on L_l, however, is ambiguous. As it has been shown, $\mu > \beta$ is a necessary precondition in order to observe the labor-repellent effect in the tradable sector $(\partial L_x / \partial a_k < 0)$. On the other hand, in the non-tradable sector, an increase in a_k leads to the labor-attracting effect $(\partial L_n / \partial a_k > 0)$. If L_n is small compared to L_x, a_k has a negative impact on L_l. In Figure (6), the labor-repellent effect dominates for $w \lessapprox 0.8$, where an increase in a_k has a negative effect on L_l.

Thus, whether the marginal effect of a_k on L_l is positive or negative depends on the relative size of the labor force of the two sectors, L_n/L_x.

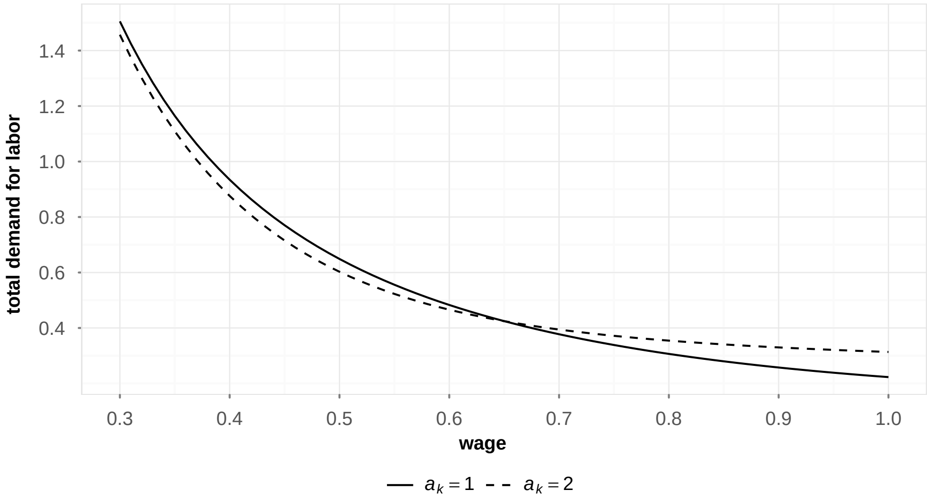

Figure 6: The figure shows a numerical example for low-skilled labor demand in the tradable and the non-tradable sector with a low capital productivity ($a_k = 1$, solid line), and a numerical example with high capital productivity ($a_k = 2$, dashed line). The other parameters are $\phi = -3$, $a_r = 1$, $a_n = 1$, $\mu = 0.8$, $\beta = 0.5$, $r = 1$, $p_x = 1$ and $L_h = 0.3$.

Table 2: The effect of the parameters on s, L_x, Y_x, L_n, L_l and L_n/L_x, given that $\beta < \mu$.

Parameter	s	L_x	Y_x	L_n	L_l	L_n/L_x
w	$+$	$-$	$-$	$-$	$-$	$+$
r	$-$	$+$	$-$	$-$	$-/+$ *	$-$
a_k	$+$	$-$	$+$	$+$	$-/+$ *	$+$
a_r	$-$	$+$	$+$	$+$	$+$	$-$
a_n	$\varnothing$	$\varnothing$	$\varnothing$	$-$	$-$	$-$
ϕ	$\varnothing$	$\varnothing$	$\varnothing$	$-$	$-$	$-$
L_h	$\varnothing$	$+$	$+$	$+$	$+$	$\varnothing$
p_x	$\varnothing$	$+$	$+$	$+$	$+$	$\varnothing$

* depending on L_n/L_x

Column (6) of Table (2) summarizes the effects of the parameters on L_n/L_x. All parameters have an unambiguous effect on L_n/L_x. L_n/L_x negatively depends on r and positively depends on a_k. This follows directly from columns (2) and (4). Because the effects of w and a_r on L_n/L_x are not obvious, proofs are given in Appendices A.5 and A.6. An increase in w leads to an increase in L_n/L_x, while a_r decreases L_n/L_x.

Two other parameters affect the relative labor force of the two sectors: An increase in non-tradable labor productivity, a_n, implies a lower L_n/L_x. Therefore, a_n is positively related with the probability that the labor-repellent effect dominates the labor-attracting effect. If a_n is very large, $L_n/L_x \to 0$, and the L_l-function converges to the L_x-function, as the labor-attracting effect of the non-tradable sector becomes irrelevant relative to the labor-repellent effect of the tradable sector.

The parameter controlling the elasticity of substitution in consumption, ϕ, has a negative but small impact on the relative labor force of the non-tradable sector, L_n/L_x. For reasonable values ($\phi < -2$), $\phi/(1-\phi)$ is already larger than 2/3. As ϕ decreases, the value converges towards one for a Leontief utility function (with $\phi \to -\infty$). In the numerical examples, a value of -3 has been chosen.

Figure (7) shows a numerical example that illustrates the reversed BS effect. The figure depicts *negative* values of the elasticity of L_l with respect to a_k, $\epsilon_{Ll,ak}$. As in Figure (4), positive elasticities are below the surface. Again, the percentage change in L_l is plotted on the vertical axis, w and μ on the horizontal axes. We make the following observations:

First, because $\mu > \beta$ is a necessary precondition, negative elasticities can only be observed for $\mu > \beta$. Second, for $\mu > \beta$ the elasticity is negative if $w \to 0$. Intuitively, at low values of w, the intermediate good is produced almost exclusively by labor; an increase in a_k thus has almost no income effect, but a strong substitution effect. As w increases, firms will substitute capital for labor. For some values of w, this leads to a transient increase in the absolute value of the elasticity, as the marginal effect becomes larger relative to the remaining labor force. On the other hand, L_n/L_x increases in w. At some point, the labor-attracting effect of the non-tradable sector dominates the labor-repellent effect, and the elasticity becomes positive, leading to the standard BS result. Third, if $\mu \to 1$, all labor will be substituted at the point where $s = 1$. Comparable to the analysis of L_x in Figure (4), the parameters

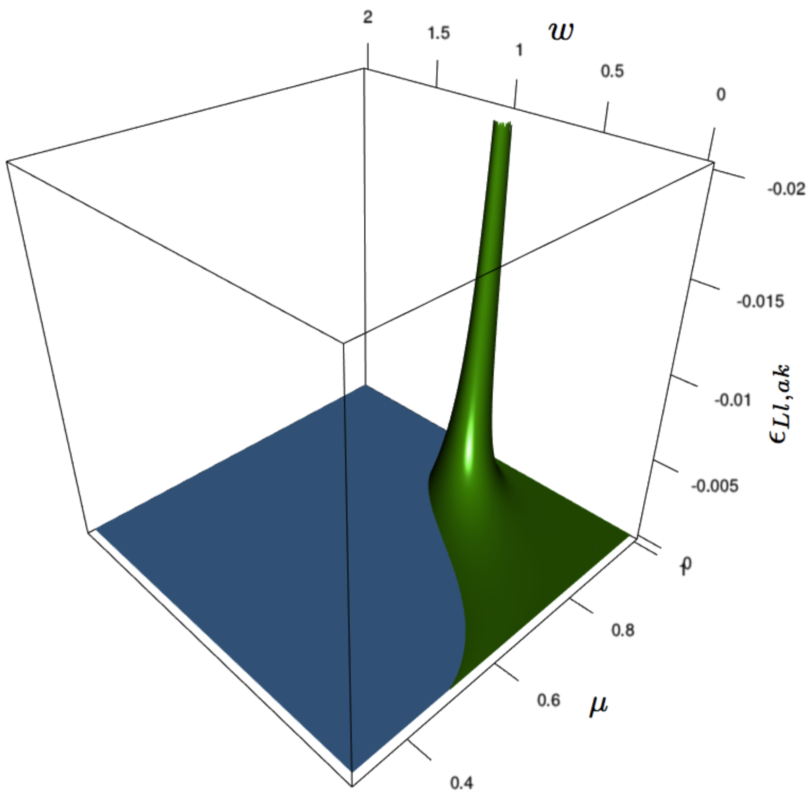

Figure 7: The figure shows the elasticity of low-skilled labor demand in the tradable sector with respect to capital productivity, $\epsilon_{Ll,ak}$, as a function of μ, where $1/(1-\mu)$ is the elasticity of substitution between low-skilled labor and capital, and the wage rate, w. The other parameters are $a_r = 1$, $\beta = 0.5$, $r = 1$, $p_x = 1$.

determining s also determine the shape of the 'elasticity hill'.

As in the case of L_x, there is a one-to-one relationship between $\bar{L}_h$ and L_n. Therefore, the elasticity of L_l with respect to $\bar{L}_h$ is also equal to one. While an increase in $\bar{L}_h$ proportionally increases the marginal effect of a_k on L_l, the elasticity of L_l with respect to a_k is not affected by $\bar{L}_h$. Therefore, $\bar{L}_h$ has no impact on the elasticity function shown in Figure (7). Similarly, p_x does not affect the the elasticity function of L_l with respect to a_k. As in the case of L_x, the elasticity of L_n with respect to p_x is $1/(1-\beta)$. Thus, an increase in p_x does not change the relative size of the sectors.

6 General Equilibrium and the Real Exchange Rate

Recall from Section 2.2 that the supply of labor is fixed and equal to $\bar{L}_l$. Therefore, in equilibrium, the wage rate is determined by setting supply equal to demand:

$$\bar{L}_l = L_l(w^*) = L_x(w^*) + L_n(w^*), \tag{17}$$

where w^* denotes the equilibrium wage rate for low skilled labor. As L_l is decreasing in w, there is a unique solution for w^*. This leads to a positive and monotone relationship between L_l and w^*.

As discussed in Section 2.4, there is a direct link from w^* to the equilibrium price of the non-tradable service, p_n^*, the equilibrium consumer price index, CPI^*, and the equilibrium real exchange rate, RER*.

If an improvement in capital productivity, a_k, diminishes the overall labor demand, L_l, it also decreases w^*, p_n^*, the CPI* and RER*. Therefore, a fall in L_l is sufficient to observe the opposite BS effect. As stated in the previous section, an improvement in a_k decreases L_l, if (1) the substitution effect dominates the income effect in the production of tradable goods, and (2) the labor force of the tradable sector is large relative to the labor force of the non-tradable sector.

7 Summary and Conclusions

We sketch a model that shows how skill-biased technological change may reverse the classic BS effect, leading to a negative relationship between the productivity in the tradable sector and the real exchange rate. In order to find such a relationship, the demand for low-skilled labor in the whole economy must fall in response to a rise in capital productivity. With a fixed supply of labor, this lowers the wage rate of low-skilled workers, and hence, the overall price level and the real exchange rate.

An increase in the productivity of capital has two effects on low-skilled labor demand: (1) a *labor-attracting* effect in the non-tradable sector and (2) a (potential) *labor-repellent* effect in the tradable sector. First, an increase in productivity leads to a higher income level in the whole economy. As consumers spend additional income in both the non-tradable and the tradable sector, the demand for low-skilled workers in the non-tradable sector increases. Second, an increase in the productivity of capital potentially decreases the demand for labor in the tradable sector. Such a negative effect occurs if and only if the substitution elasticity between low-skilled labor and capital is high relative to the importance of the routine task good in the production of the final good.

In order to observe the opposite BS effect, the labor-repellent effect in the tradable sector must outweigh the labor-attracting effect in the non-tradable sector. For the labor-repellent effect to dominate the labor-attracting effect, the low-skilled labor force in the tradable sector must be large compared to the non-tradable sector labor force. If the labor force in the tradable sector is small relative to the non-tradable sector, the labor-repellent effect is dominated by the labor-attracting effect, and the classic BS effect occurs.

Several testable hypotheses can be derived from our model: According to our model, the opposite BS effect should be observed in countries where (1) capital productivity enhancement dominates low-skilled labor productivity gains, (2) the income share of high-skilled labor is high (low β), (3) capital is substitutable for low-skilled labor, and (4) the labor force of the tradable sector is large relative to the labor force of the non-tradable sector.

In the United States, for example, employment in low-skilled occupations in industry and agriculture has strongly decreased in the last decades (Autor and Dorn, 2009). Gains in productivity, thus, have led to a decrease in demand for low-skilled workers in the tradable industry. At the same time, an increase in the demand for non-tradable services has led to a strong increase in non-tradable service occupations between 1980 and 2005 (Autor and Dorn, 2009). While the wage growth in these service occupations was stronger than the wage growth in similarly low-skilled occupations in industry, the overall effect on relative wages for low-skilled workers was clearly negative. According to our model, this has reduced the relative costs of non-tradable goods, leading to a depreciation of the real exchange rate, in line with the empirical findings of Gubler and Sax (2011). As we have stated, the occurrence of the reversed BS effect is temporary. Today, productivity gains in US-industry may well lead to the traditional BS effect and a real exchange apreciation. This is because the share of low-skilled labor in industry has become small. Productivity enhancements thus are likely to increase tradable production (a large income effect) while only a small number of workers are laid off (a small substitution effect).

We do not expect the revese BS effect to be of major importance in emerging economies such as China. This is because low-skilled labor is still the dominant factor in tradable production. At current Chinese wage rates, capital intensity in the production of routine tasks is low. An increase in capital productivity would have only a very small negative effect on the low-skilled labor force in the tradable sector.

Of course, at this stage, the model provides only one possible explanation for the empirical finding of an opposite BS effect in the tradable sector. Ultimately, the model needs to be tested empirically. Future research should include further exploration of the hypotheses and their validation.

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A Mathematical Appendix

A.1 Optimal Capital Intensity s

We start with the production function:

$$Y_x = L_h^{1-\beta}[(a_r\, L_x)^\mu + (a_k\, K)^\mu]^{\frac{\beta}{\mu}}. \tag{18}$$

The first order conditions with respect to L_x and K are:

$$\frac{L_h^{1-\beta}\beta\, p_x\,(a_r\, L_x^*)^\mu\,((a_r\, L_x^*)^\mu + (a_k\, K^*)^\mu)^{\frac{\beta}{\mu}-1}}{L_x^*} = w \tag{19}$$

and

$$\frac{L_h^{1-\beta}\beta\, p_x\,(a_k\, K^*)^\mu\,((a_r\, L_x^*)^\mu + (a_k\, K^*)^\mu)^{\frac{\beta}{\mu}-1}}{K^*} = r. \tag{20}$$

Dividing Equation (19) by Equation (20) yields:

$$\frac{(a_r\, L_x^*)^\mu\, K^*}{(a_k\, K^*)^\mu\, L_x^*} = \frac{w}{r}. \tag{21}$$

Define $s = K^*/L_x^*$ and solve for s to obtain Equation (7).

A.2 L_x is Decreasing in w

Proposition. *For the parameters a_k, a_r, r, w, $p_x > 0$, $0 < \mu < 1$ and $0 < \beta < 1$: L_x is a decreasing function of w.*

Proof. Taking the logarithm of both sides of Equation (10) and differentiating with respect to w yields:

$$\frac{\partial \log L_x}{\partial w} = \frac{1}{1-\beta}\left[\frac{\beta}{\mu}\,\frac{{a_k}^\mu \mu s^{\mu-1}\frac{\partial s}{\partial w}}{{a_k}^\mu\, s^\mu + {a_r}^\mu} - \frac{1 + r\frac{\partial s}{\partial w}}{w + r\, s}\right]. \tag{22}$$

The derivative of s with respect to w is:

$$\frac{\partial s}{\partial w} = \frac{\left(\frac{{a_k}^\mu\, w}{{a_r}^\mu\, r}\right)^{\frac{1}{1-\mu}}}{w\,(1-\mu)} = \frac{s}{w\,(1-\mu)}. \tag{23}$$

Substituting this result in Equation (22) yields:

$$\frac{\partial \log L_x}{\partial w} = \frac{1}{1-\beta}\left[\frac{\beta\, {a_k}^{\mu}\, s^{\mu}}{w\,(1-\mu)\,({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})} - \frac{r\, s + (1-\mu)w}{w(1-\mu)(w + r\, s)}\right]. \tag{24}$$

For $\partial L_x/\partial w < 0$, the term in the square bracket must be negative, therefore:

$$\frac{\beta\, {a_k}^{\mu}\, s^{\mu}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} < \frac{r\, s + (1-\mu)w}{w + r\, s}. \tag{25}$$

We multiply both sides of Equation (25) by $(w + r\, s)$ and $({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})$ to obtain:

$$\beta\, {a_k}^{\mu}\, s^{\mu}(w + r\, s) < (r\, s + (1-\mu)w)({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}). \tag{26}$$

Subtracting $(\beta\, {a_k}^{\mu}\, s^{\mu}(w + r\, s))$ on both sides and rearranging yields:

$$\begin{aligned} 0 < (1-\mu)w\, {a_r}^{\mu} + r\, s\, {a_r}^{\mu} + (1-\beta)\, {a_k}^{\mu}\, s^{1+\mu}\, r \\ + (1-\beta)\, {a_k}^{\mu}\, s^{\mu}\, w - {a_k}^{\mu}\, s^{\mu}\, w\, \mu. \end{aligned} \tag{27}$$

We replace ${a_k}^{\mu}$ by $(s^{1-\mu}\, {a_r}^{\mu}\, r)/w$ (see Equation 7) in the last term of the right-hand side and rearrange to obtain:

$$\begin{aligned} 0 < (1-\mu)w\, {a_r}^{\mu} + (1-\beta)\, {a_k}^{\mu}\, s^{1+\mu}\, r + (1-\beta)\, {a_k}^{\mu}\, s^{\mu}\, w \\ + (1-\mu) r\, s\, {a_r}^{\mu}. \end{aligned} \tag{28}$$

Since $0 < \beta < 1$ and $0 < \mu < 1$, the right-hand side is positive and $\partial L_x/\partial w < 0$.

□

A.3 Necessary and Sufficient Condition for $\partial L_x/\partial a_k < 0$

Proposition. *For the parameters a_k, a_r, r, w, $p_x > 0$, $0 < \mu < 1$ and $0 < \beta < 1$: $1 > \mu > \beta$ is a necessary and sufficient condition for $\frac{\partial \log L_x}{\partial a_k} < 0$.*

Proof. Taking the logarithm of both sides of Equation (10) and differentiating with respect to a_k, one gets:

$$\frac{\partial \log L_x}{\partial a_k} = \frac{1}{1-\beta}\left[\frac{\beta}{\mu}\,\frac{{a_k}^{\mu}\mu s^{\mu-1}\frac{\partial s}{\partial a_k} + \mu {a_k}^{\mu-1} s^{\mu}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} - \frac{r\frac{\partial s}{\partial a_k}}{w + r\, s}\right]. \tag{29}$$

The derivative of s with respect to a_k is:

$$\frac{\partial s}{\partial a_k} = \frac{\mu\left(\frac{{a_k}^{\mu}\, w}{{a_r}^{\mu}\, r}\right)^{\frac{1}{1-\mu}}}{a_k\,(1-\mu)} = \frac{\mu\, s}{a_k\,(1-\mu)}. \tag{30}$$

Substituting this result in Equation (29) yields:

$$\frac{\partial \log L_x}{\partial\, a_k} = \frac{1}{1-\beta}\left[\frac{\beta}{\mu}\,\frac{\frac{{a_k}^{\mu-1}\mu^2 s^{\mu}}{1-\mu} + \mu {a_k}^{\mu-1} s^{\mu}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} - \frac{\mu\, r\, s}{a_k(1-\mu)(w + r\, s)}\right]. \tag{31}$$

$\frac{\partial \log L_x}{\partial\, a_k} < 0$ only holds if the square bracket in Equation (31) is negative:

$$\frac{\beta}{\mu}\,\frac{\frac{{a_k}^{\mu-1}\mu^2 s^{\mu}}{1-\mu} + \mu {a_k}^{\mu-1} s^{\mu}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} < \frac{\mu\, r\, s}{a_k(1-\mu)(w + r\, s)}. \tag{32}$$

After multiplying both sides of Equation (32) by $(1-\mu)({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})$, we obtain:

$$\frac{\beta}{\mu}\,({a_k}^{\mu-1}\mu^2 s^{\mu} + (1-\mu)\mu {a_k}^{\mu-1} s^{\mu}) < \frac{\mu\, r\, s({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})}{a_k(w + r\, s)}. \tag{33}$$

After some manipulations and cancelling μ on both sides, we get:

$$\frac{\beta}{\mu}\, {a_k}^{\mu} s^{\mu-1} < \frac{r({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})}{(w + r\, s)}. \tag{34}$$

Multiplying both sides of Equation (34) by $(w + r\, s)$ yields:

$$\frac{\beta}{\mu}\, w\, {a_k}^{\mu} s^{\mu-1} + \frac{\beta}{\mu}\, r\, {a_k}^{\mu} s^{\mu} < r\, {a_k}^{\mu} s^{\mu} + r\, {a_r}^{\mu}. \tag{35}$$

We replace ${a_r}^{\mu}$ by $({a_k}^{\mu}\, w)/(s^{1-\mu}\, r)$ (see Equation 7) in the last term of the right-hand side to get:

$$\frac{\beta}{\mu}\, w\, {a_k}^{\mu} s^{\mu-1} + \frac{\beta}{\mu}\, r\, {a_k}^{\mu} s^{\mu} < r\, {a_k}^{\mu} s^{\mu} + w\, {a_k}^{\mu} s^{\mu-1}. \tag{36}$$

We subtract $(r\, {a_k}^{\mu} s^{\mu} + w\, {a_k}^{\mu} s^{\mu-1})$ from both sides and rearrange to obtain:

$$\left[\frac{\beta}{\mu} - 1\right]\, (w\, {a_k}^{\mu} s^{\mu-1} + r\, {a_k}^{\mu} s^{\mu}) < 0. \tag{37}$$

Therefore, if and only if $\mu > \beta$, Equation (37) holds, and thus $\frac{\partial \log L_x}{\partial a_k} > 0$. □

A.4 Necessary and Sufficient Condition for $\partial L_x / \partial r > 0$

Proposition. *For the parameters a_k, a_r, r, w, $p_x > 0$, $0 < \mu < 1$ and $0 < \beta < 1$, $1 > \mu > \beta$ is a necessary and sufficient condition for $\frac{\partial \log L_x}{\partial r} > 0$.*

Proof. In order to draw on Proof A.2, we show that $1 > \mu > \beta$ is a necessary and sufficient condition for $\frac{\partial \log K}{\partial w} > 0$. Without loss of generality we can redefine s as $s = L_x/K$. Then, the same result applies to $\frac{\partial \log L_x}{\partial r}$.

We take the logarithm of both sides of $K = sL_x$ and differentiate with respect to w to obtain:

$$\frac{\partial \log K}{\partial w} = \frac{\partial \log s}{\partial w} + \frac{\partial \log L_x}{\partial w}. \tag{38}$$

From Proof A.2 we know the result of $\frac{\partial \log L_x}{\partial w}$, given in Equation (24). The derivative of $\log s$ with respect to w yields:

$$\frac{\partial \log s}{\partial w} = \frac{1}{(1-\mu)\, w}. \tag{39}$$

Therefore, substituting the results of Equation (24) and Equation (39) in Equation (38) gives:

$$\begin{aligned}\frac{\partial \log K}{\partial w} = \frac{1}{(1-\mu)\, w} &+ \frac{1}{1-\beta}\, \frac{\beta\, {a_k}^{\mu}\, s^{\mu}}{w\,(1-\mu)\,({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})} \\ &- \frac{1}{1-\beta}\, \frac{r\, s + (1-\mu) w}{w(1-\mu)(w + r\, s)}.\end{aligned} \tag{40}$$

In order to get $\frac{\partial \log K}{\partial w} > 0$, it must be that:

$$\frac{1}{(1-\mu)\, w} + \frac{1}{1-\beta}\, \frac{\beta\, {a_k}^{\mu}\, s^{\mu}}{w\,(1-\mu)\,({a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu})} > \frac{1}{1-\beta}\, \frac{r\, s + (1-\mu) w}{w(1-\mu)(w + r\, s)}. \tag{41}$$

After some manipulations we obtain:

$$0 > {a_r}^{\mu}\; \left[(\beta - \mu)\,(w + rs)\right]. \tag{42}$$

Therefore, $1 > \mu > \beta$ is a necessary and sufficient condition for this equation to hold. □

A.5 L_n/L_x is Increasing in w

Proposition. *For the parameters a_k, a_r, r, w, $p_x > 0$, $\phi < 0$, $0 < \mu < 1$ and $0 < \beta < 1$: L_n/L_x is an increasing function of w.*

Proof. We show that $\frac{\partial \log L_x}{\partial w} < \frac{\partial \log L_n}{\partial w}$. From Proof A.2 we know $\frac{\partial \log L_x}{\partial w}$. Taking the logarithm of both sides of Equation (14) and differentiating with respect to w yields:

$$\frac{\partial \log L_n}{\partial w} = -\frac{1}{w(1-\phi)} + \frac{\partial \log Y_x}{\partial w}, \tag{43}$$

$$\frac{\partial \log L_n}{\partial w} = -\frac{1}{w(1-\phi)} + \beta\,\frac{\partial \log L_x}{\partial w} + \frac{\beta}{\mu}\,\frac{\partial \log\left(a_r^\mu + a_k^\mu s^\mu\right)}{\partial w}.$$

Therefore, the inequality $\frac{\partial \log L_x}{\partial w} < \frac{\partial \log L_n}{\partial w}$ is equal to:

$$\frac{\partial \log L_x}{\partial w} < -\frac{1}{w(1-\phi)} + \beta\,\frac{\partial \log L_x}{\partial w} + \frac{\beta}{\mu}\,\frac{\partial \log\left(a_r^\mu + a_k^\mu s^\mu\right)}{\partial w}. \tag{44}$$

By subtracting $\beta\,\frac{\partial \log L_x}{\partial w}$ from both sides and using the result from Equation (24), we get:

$$\frac{\beta\, {a_k}^\mu\, s^\mu}{w\,(1-\mu)\,({a_k}^\mu\, s^\mu + {a_r}^\mu)} - \frac{r\,s + (1-\mu)w}{w\,(1-\mu)(w + r\,s)} < -\frac{1}{w(1-\phi)} + \frac{\beta\, {a_k}^\mu\, s^\mu}{w\,(1-\mu)\,({a_k}^\mu\, s^\mu + {a_r}^\mu)}. \tag{45}$$

After some manipulations we obtain:

$$0 < \frac{\mu\, r\, s - w\,\phi - r\,s\,\phi - \mu\, w\,\phi}{1-\phi}. \tag{46}$$

This equation holds given the parameter value restrictions. □

A.6 L_n/L_x is Decreasing in a_r

Proposition. *For the parameters a_k, a_r, r, w, $p_x > 0$, $\phi < 0$, $0 < \mu < 1$ and $0 < \beta < 1$: L_n/L_x is a decreasing function of a_r.*

Proof. We show that $\frac{\partial \log L_x}{\partial a_r} > \frac{\partial \log L_n}{\partial a_r}$. Taking the logarithm of both sides of Equation (10) and differentiating with respect to a_r yields:

$$\frac{\partial \log L_x}{\partial a_r} = \frac{1}{1-\beta}\left[\frac{\beta}{\mu}\,\frac{\mu {a_r}^{\mu-1} + {a_k}^{\mu}\mu s^{\mu-1}\frac{\partial s}{\partial a_r}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} - \frac{r\frac{\partial s}{\partial a_r}}{w + r\, s}\right]. \tag{47}$$

The derivative of s with respect to a_r is:

$$\frac{\partial s}{\partial a_r} = \frac{-\mu\left(\frac{{a_k}^{\mu}\, w}{{a_r}^{\mu}\, r}\right)^{\frac{1}{1-\mu}}}{a_r\,(1-\mu)} = \frac{-\mu\, s}{a_r\,(1-\mu)}. \tag{48}$$

Substituting this result in Equation (47) yields:

$$\frac{\partial \log L_x}{\partial a_r} = \frac{1}{1-\beta}\left[\frac{\beta}{\mu}\,\frac{\mu {a_r}^{\mu-1} - \frac{{a_k}^{\mu}\mu^2 s^{\mu}}{a_r(1-\mu)}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} + \frac{r\, s\, \mu}{a_r(1-\mu)(w + r\, s)}\right]. \tag{49}$$

a_r affects Y_n only via the output of the exported good Y_x (see Equation 14). Hence, taking the logarithm of both sides of Equation (1) with K replaced by sL_x and differentiating with respect to a_r yields:

$$\frac{\partial \log Y_x}{\partial a_r} = \beta\,\frac{\partial \log L_x}{\partial a_r} + \frac{\beta}{\mu}\,\frac{\partial \log\left(a_r^{\mu} + a_k^{\mu} s^{\mu}\right)}{\partial a_r}. \tag{50}$$

Therefore, the inequality $\frac{\partial\, \log L_x}{\partial\, a_r} > \frac{\partial \log L_n}{\partial\, a_r}$ is equal to:

$$\frac{\partial \log L_x}{\partial a_r} > \beta\,\frac{\partial \log L_x}{\partial a_r} + \frac{\beta}{\mu}\,\frac{\partial \log\left(a_r^{\mu} + a_k^{\mu} s^{\mu}\right)}{\partial a_r}. \tag{51}$$

By subtracting $\beta\,\frac{\partial \log L_x}{\partial\, a_r}$ from both sides and using the result from Equation (49), we get:

$$\frac{\beta}{\mu}\,\frac{\mu {a_r}^{\mu-1} - \frac{{a_k}^{\mu}\mu^2 s^{\mu}}{a_r(1-\mu)}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}} + \frac{r\, s\, \mu}{a_r(1-\mu)(w + r\, s)} > \tag{52}$$

$$\frac{\beta}{\mu}\,\frac{\mu {a_r}^{\mu-1} - \frac{{a_k}^{\mu}\mu^2 s^{\mu}}{a_r(1-\mu)}}{{a_k}^{\mu}\, s^{\mu} + {a_r}^{\mu}}. \tag{53}$$

Therefore, we obtain:

$$\frac{r\, s\, \mu}{a_r(1-\mu)(w + r\, s)} > 0. \tag{54}$$

This equation holds, given the parameter value restrictions.

□