Expected Money Growth, Markov Trends and the Instability of Money Demand in the Euro Area

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This version
December 2007

Abstract

This paper analyzes the recently documented instability of money demand in the euro area in the framework of a Markov switching trend model. First, we consider a standard flexible price model with stable money demand, rational expectations, and an exogenous income-money ratio which follows a Markov trend. This framework, which implies an influence of expected future money on prices, leads to a cointegrating relationship between (log) prices and the (log of the) money-income ratio with a switching intercept term. Of course, this likely leads to a rejection of cointegration by standard tests and to the erroneous conclusion of an unstable money demand. Second, a more general model allowing for endogeneity and more general dynamics is estimated with Bayesian methods for euro area data from 1975-2003. This exercise provides support for our model and a stable demand for M3 in the euro area.
JEL classification: C11,C32,E41
Key words: Bayesian cointegration analysis, Markov trend, Markov chain Monte Carlo, money demand.

1 Introduction

The data since 2001 raise a substantial doubt about the existence of a stable long run money demand function for M3 in the euro area, which seemed to be firmly established with data of the 1980s and 1990s (see for instance Bruggeman, Donati and Warne, 2003, Carstensen, 2004, Kugler and Kaufmann, 2005). This finding is usually attributed to permanent changes in money demand due to structural changes induced by the ongoing European unification process during the last two decades and by the increased uncertainty and/or low interest rates during the last three years. The latter problem can be accounted

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‡The paper was presented at the joint Bundesbank – University of Bonn Workshop on "What Central Banks Can Learn from Money and Credit Aggregates" held in Eltville in October 2005. We would like to thank the participants for comments and in particular Joao Miguel Sousa for a valuable and helpful discussion. The paper contains the views of the authors and not necessarily those of the OeNB.

for by including (non stationary) measures of uncertainty in the cointegrating relationship (e.g. Carstensen, 2004) or using a nonlinear specification for the money demand function.

This paper proposes an alternative explanation for the instability result of euro area M3 demand based on Markov switching trends. Assuming rational expectation formation, we investigate the effects of introducing a Markov switching trend in the money-income ratio (log of money minus log of income) on the cointegrating relationship between prices, money and income. This approach is based on the corresponding analysis in Hall, Psaradakis and Sola (1997) who, within the framework of the permanent income hypothesis, analyze the implications of a Markov switching trend in income for the cointegrating relationship between consumption and income. We consider a standard flexible price model with a stable money demand, rational expectations and an exogenous money-income ratio which follows a Markov trend. This framework, which implies an influence of expected future money on prices, leads to a cointegrating relationship between (log) prices and the (log of the) money-income ratio with a switching intercept term. Of course, this likely leads to rejections of cointegration by standard tests and to the erroneous conclusion of an unstable money demand. Second, a more general model allowing for endogeneity and more general dynamics is estimated for euro area data over the period 1975-2003, applying Bayesian methods used by Paap and van Dijk (2003) for the permanent income model. This exercise provides support for our model and the long-run stability of the demand for M3 in the euro area.

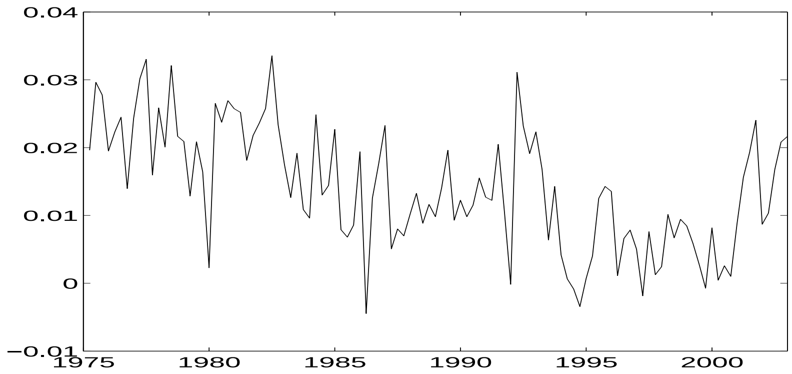
The smooth transition equilibrium correction model in Sarno et al. (2003) is another framework to estimate and test the presence of nonlinearities in the money demand function. Their approach, however, is based on a stable linear cointegration relationship in money, prices, income and interest rates. The non-linearity occurs in the adjustment towards the long-run equilibrium. In our investigation, the non-linearity enters the cointegration relationship while the dynamics are assumed to be linear. We think that this latter specification is consistent with the evidence obtained so far for the euro area data, which documents that changes concern mainly the long-run properties of the money demand.

The content of the paper is organized as follows: Some empirical facts concerning the growth of money relative to real income in the euro area are presented in section 2. Section 3 contains the analysis of the theoretical flexible price model assuming a Markov trend for the money-income ratio. The econometric model is outlined in section 4 and the Bayesian estimation is outlined in section 5. The empirical results for the euro area are presented in section 6. Section 7 concludes.

2 Money and real income in the euro area

Figure 1 displays the growth rate of the money-real income ratio in the euro area from 1975 to 2003. We observe periods during which money has been growing on quarterly basis at around 2% faster than real income (1979-83, 1993, 2001-03) and periods during which money growth has been moderately higher than real income (1984-92, 1994-2000). These different episodes reflect among other things the stance of monetary policy. In particular, we have a relatively restrictive monetary policy on the way to monetary union in the years 1994-1998. By contrast, monetary policy had been very expansive during the 1970s and the early 1980s in some EMU countries in trying to avoid high unemployment by tolerating rather high inflation rates in the aftermath of the two oil crises. The monetary expansions

Figure 1: Growth rate of the money-income ratio in the euro area 1975-2003.



in 1993 and during 2001-2003 were supporting the attempt to avoid strong negative real effects of the EMS crisis and the terrorist attack in September 2001, respectively. In the present context, the exact reason for these different regimes in the growth rate of the money-real income ratio is not important. What is important is that the euro area witnessed prolonged period of relatively slow or flat money growth. Ultimately, this may influence the expectations concerning future money growth. For instance, in contrast to the 1970s and 1980s, the high money growth rates after September 11, 2001 may have been perceived as only transitory and therefore had no full effect on prices.

These different episodes may lead to breaks in the relation between inflation and money growth or in the relation between the price level, real income and the money stock. We explore this possibility theoretically and empirically in the present paper. First, we consider a simple theoretical model with restrictive assumptions concerning the real interest rate and the dynamics in the next section. In the subsequent section, we use a more general empirical model based on less restrictive assumptions and estimate it with euro area data to assess the empirical relevance of the theoretical predictions.

3 Expected money growth, Markov trends and prices

In this section we adopt the approach developed by Hall, Psaradakis and Sola (1997) for the permanent income model to investigate the relationship between money, income and prices under price flexibility and rational expectations. Consider the money demand function

$$m_t - p_t = y_t - b(E_t p_{t+1} - p_t) + \xi_t \tag{1}$$

where m_t, p_t and y_t are the log money stock, log price level and log income (measured by real GDP), respectively. ξ_t is a white noise random error and E_t means conditional expectations given information at time t. The expected inflation term reflects the influence of the nominal interest rate and b is therefore the interest rate semi elasticity of money demand. For the moment, we assume that the real interest rate is constant and we omit the resulting constant term for the sake of simplicity. The empirical econometric model will allow for a stationary real interest rate. Similarly, we omit a deterministic trend in velocity of money in this section, but in our empirical work this possibility is taken into account. Now let us assume that prices are fully flexible and money and output are exogenous. Solving the money demand equation for p_t yields

$$p_t = \frac{1}{1+b}\left\{(m_t - y_t) + \frac{b}{1+b}E_t p_{t+1} + \xi_t\right\} \tag{2}$$

Forward iteration under the assumption of rational expectations provides the following solution for the price level

$$p_t = \frac{1}{1+b}\left\{\sum_{i=0}^{\infty}\left(\frac{b}{1+b}\right)^i E_t(m_{t+i} - y_{t+i}) + \xi_t\right\} \tag{3}$$

In analogy, to the permanent income hypothesis, the current price level depends on the discounted expected future values of the money income ratio. In order to analyze the implications of this equation for the price level, we have to make an assumption with respect to the law of motion of the forcing variable $m_t - y_t$. Instead of assuming a standard non-stationary linear time series process, we assume the following Markov switching trend model for the money-income ratio $(m_t - y_t)$:

$$\begin{aligned} m_t - y_t &= n_t + z_t && (4)\\ n_t &= n_{t-1} + \gamma_0 + \gamma_1 s_t && (5)\\ s_t &= (1-p) + (p+q-1)s_{t-1} + \eta_t = a_0 + a_1 s_{t-1} + \eta_t && (6)\\ z_t &= z_{t-1} + \varepsilon_t. && (7) \end{aligned}$$

According to equation (4), the money-income ratio consists of two components: The first one is a two-state, $s_t = 0, 1$, Markov trend with different growth rates and the following transition probabilities:

$$\begin{aligned} \Pr(s_t = 0 | s_{t-1} = 0) &= p && (8)\\ \Pr(s_t = 1 | s_{t-1} = 1) &= q && (9) \end{aligned}$$

In our model, it is convenient to use the AR(1) representation of the state variable given in equation (6) with η_t representing a martingale difference sequence. Moreover, we assume that state 1 is characterized by a lower growth rate, i.e. $\gamma_0 > 0$ and $\gamma_1 < 0$. The second component of excess money is a random walk with innovation ε_t.

Using the AR(1) representation of s_t and the random walk assumption for z_t we get the following expression for the expected future money-income ratio:

$$\begin{aligned} E_t(m_{t+i} - y_{t+i}) &= n_t + \sum_{j=1}^{i}(\gamma_0 + \gamma_1 E_t s_{t+j}) + E_t z_{t+j} && (10)\\ &= n_t + z_t + i\gamma_0 + \gamma_1 \sum_{j=1}^{i}\left(a_0\left(1 + a_1 + ... + a_1^{j-1}\right) + a_1^j s_t\right) \end{aligned}$$

Inserting equation (10) in equation (3) provides, after some rearrangement, the following expression for the price level:

$$\begin{aligned} p_t &= \frac{1}{1+b}\left\{\sum_{i=0}^{\infty}\left(\frac{b}{1+b}\right)^i\left[n_t + z_t + i\gamma_0 + \sum_{j=1}^{i}\gamma_1\left(a_0\sum_{l=1}^{j}a_1^{l-1} + a_1^j s_t\right)\right] + \xi_t\right\}\\ &= n_t + z_t + \kappa_0 + \kappa_1 s_t + \frac{1}{1+b}\xi_t, && (11) \end{aligned}$$

with

$$\kappa_0 = \gamma_0 b + \gamma_1 \frac{a_0 b(1+b)}{1+b(1-a_1)}$$
$$\kappa_1 = \gamma_1 \frac{a_1 b}{1+b(1-a_1)}$$

In general, the state variable enters the solution for the price level. The corresponding coefficient κ_1 is only zero if we have a value of zero for at least one of the three coefficients: γ_1 (no Markov trend), b (no expected future values are relevant) or a_1 (unpredictable state variable). The comparison of equations (4) and (11) shows that the difference between the price level and the money-income ratio, i.e. the velocity, is given by $p_t - (m_t - y_t) = \kappa_0 + \kappa_1 s_t + \frac{1}{1+b}\xi_t$ and thus, has a state dependent intercept term. Therefore, standard cointegration tests are likely to reject the cointegration hypothesis when applied to data generated by our model. However, the no-cointegration result between money, income and prices is only brought about by expectation effects in the framework of a Markov trend model, and not by an unstable money demand.[1] Moreover, note that the random walk or stochastic trend component z_t is common to both series, p_t and $m_t - y_t$, and therefore, these components are cointegrated.

4 A generalized bivariate econometric model

For our empirical analysis we will use a more general model than that outlined in section 3. In particular, we will allow for autocorrelation in the error terms and lagged adjustment in observable variables. Moreover, money and income are no longer assumed to be exogenous and the real interest rate is no longer assumed to be constant. The general short run dynamics of the variables allow for variation in the real interest rate. The only (theoretically plausible) assumption is that the real rate is stationary and therefore, it does not enter the long run relation between money, real income and prices.

We use the EC-VAR framework developed by Paap and van Dijk (2003) for consumption and income. Let us define the two-dimensional vector $Y_t = [p_t, my_t]$. According to the model of section 3, we decompose this vector into three components:

$$Y_t = N_t + R_t + Z_t, \tag{12}$$

where N_t follows a bivariate Markov trend process with two-dimensional parameter vectors Γ_0 and Γ_1

$$N_t = N_{t-1} + \Gamma_0 + \Gamma_1 s_t. \tag{13}$$

The component R_t allows for a shift in the price level (see equation (11)) and is given by

$$R_t = \begin{bmatrix} \delta_1 \\ 0 \end{bmatrix} s_t \tag{14}$$

[1] A similar explanation for the failure of the expectations hypothesis of the term structure of interest rates was provided by Hamilton (1989) and Kugler (1996). In the former paper, Markov switching in the level of the short rate is considered, whereas Markov switching in the VAR for the short and the long rate is analyzed in the latter paper. In both cases, linear models lead to an erroneous rejection of the expectations hypothesis of the term structure.

Equation (13) allows for different trends in the price level and the money-income ratio, as we do not restrict Γ_0 and Γ_1 to obtain the same unconditional trend growth rates $(\Gamma_0 + \Gamma_1(1-p)/(2-p-q))$ in both series. Thus, our empirical model allows for a different unconditional trend growth in prices and the money-income ratio which may be caused by a deterministic trend in velocity. Backward iteration of equation (13) provides the following expression for the bivariate Markov trend component:

$$N_t = N_1 + (t-1)\Gamma_0 + \Gamma_1 \sum_{i=2}^{t} s_i. \tag{15}$$

Note that a constant term in the level relationship (κ_0 in equation (11)) is captured by differences in the unobserved starting values for the trend component N_1.

Finally, we assume a VAR(k) model for Z_t. As our theoretical framework of section 3 implies cointegration between the two elements Z_t, we adopt an error correction formulation of the VAR model:

$$\Delta Z_t = \Pi Z_{t-1} + \sum_{i=1}^{k-1} \Psi_i \Delta Z_{t-i} + \varepsilon_t, \tag{16}$$

where ε_t is a vector of white noise error terms. The rank of the matrix Π is a) full if Z_t is stationary, b) one if Z_t is difference stationary but cointegrated and c) zero if Z_t is difference stationary without cointegration (Johansen 1991).

To perform cointegration analysis, it is very convenient to use the parametrization of Π proposed by Kleibergen and Paap (2002), which is called the unrestricted error correction model $\mathcal{M}_{uec}$:[2]

$$\Pi' = \beta\alpha + \beta_\perp \lambda \alpha_\perp = \begin{pmatrix} \beta & \beta_\perp \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} \alpha \\ \alpha_\perp \end{pmatrix}, \tag{17}$$

where $\alpha = (\alpha_1, \alpha_2)$, $\beta = (1, -\beta_2)'$ are the vectors of the adjustment and the cointegrating coefficients, respectively. λ is a scalar and measures the deviation from the cointegration space. The orthogonal complements of the vectors α and β are denoted by $\alpha_\perp$and $\beta_\perp$. If λ is different from zero, the matrix Π is unrestricted and additionally, has full rank (the four coefficients of the matrix Π are determined by α, β_2 and λ). If λ is zero, we have cointegration and the corresponding reduced rank restriction holds. Finally, if all four coefficients of Π are zero Z_t is fully difference stationary.

The parametrization (17) of the matrix Π is very convenient for estimation and testing purposes as all the coefficients can be obtained by the singular value decomposition $\Pi' = USV'$ with orthonormal matrices U and V and diagonal matrix S containing the singular values of Π in decreasing order:

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}, \; V = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}, \; S = \begin{bmatrix} s_{11} & 0 \\ 0 & s_{22} \end{bmatrix} \tag{18}$$

From equation (17) we obtain the following expressions for the parameters of interest (Kleibergen and Paap, 2002, pp. 226-227) where *sig* means the sign of the argument:

$$\begin{aligned} \alpha &= u_{11} s_{11} [v_{11} \; v_{21}] \\ \beta_2 &= -u_{21}/u_{11} \\ \lambda &= sig(u_{22} v_{22}) s_{22} \end{aligned} \tag{19}$$

[2]Kleinberger and Paap (2005) show in a more recent paper that this parametrization is also useful in a classical testing framework.

5 Bayesian estimation

The estimation of model (12)–(17) is cast into a Bayesian framework and based on the sampler proposed in Paap and van Dijk (2003). To briefly outline the Bayesian setup, we write for notational convenience $Y^t = (Y_t, Y_{t-1}, \ldots Y_1)$ which gathers all observations up to time t. Conditional on initial conditions N_1, the state variable $s^T = (s_T, \ldots, s_0)$ and the model parameters θ,[3] we can factorize the likelihood

$$L(Y^T|s^T, \theta, N_1) = \prod_{t=1}^{T} f(Y_t|Y^{t-1}, \theta, s^t, N_1), \tag{20}$$

where the observation density $f(Y_t|Y^{t-1}, \theta, s^t, N_1)$ is bivariate normal

$$f(Y_t|Y^{t-1}, \theta, s^t, N_1) = \frac{1}{2\pi \det(\Sigma)^{1/2}} \exp\left\{-\frac{1}{2}\varepsilon_t'\Sigma^{-1}\varepsilon_t\right\}. \tag{21}$$

The density of s^T depends on the transition distribution given by p and q:

$$\pi(s^T) = p^{N_{00}}(1-p)^{N_{01}} q^{N_{11}}(1-q)^{N_{10}}\pi(s_0), \tag{22}$$

where N_{ij} equals the number of transitions from state i to state j and $\pi(s_0)$ represents the initial state distribution.

For the model parameters, we closely follow the prior specification of Paap and van Dijk (2003). Given the first observation in the sample, the prior distribution for the initial condition of the Markov trend, N_1, is assumed to be normal $N(Y_1, \Sigma)$. Further, we assume diffuse priors for Σ, Π, $\Psi = (\Psi_1, \ldots, \Psi_{k-1})$, and δ_1, i.e.

$$\pi(\Sigma) \propto \det(\Sigma)^{-1/2},\ \pi(\Pi) \propto 1,\ \pi(\Psi) \propto 1,\ \pi(\delta_1) \propto 1.$$

Regarding the state-specific parameters Γ_0 and Γ_1, we choose a prior which uniquely identifies the states and which restricts the unconditional growth rates $\Gamma = (\gamma_1,\ \gamma_2)'$, where $\Gamma = \Gamma_0 + \Gamma_1(1-p)/(1-p-q)$, of the Markov trends. To uniquely identify the states, we define state 1 to be the state with a lower trend growth rate in both variables, i.e. Γ_1 is restricted to be negative, $\Gamma_1 < 0$. In addition, the difference in the unconditional growth rates of the Markov trends, $\mathrm{d}\gamma = \gamma_1 - \gamma_2$, should reflect the difference in the unconditional growth rates observed in the data (see figure 3 below). Therefore, we restrict $\mathrm{d}\gamma$ to the interval $|\mathrm{d}\gamma - \mathrm{G}| \leq \mathrm{dG}$, where G is the average growth difference between the series, $\mathrm{G} = \frac{1}{T-1}\sum_{t=2}^{T} \mathrm{d}y_t$, with $\mathrm{d}y_t = \Delta p_t - \Delta(m_t - y_t)$, and dG is twice the standard deviation of the average growth difference $\mathrm{dG} = 2(T-1)^{-1/2}\mathrm{std}(\mathrm{d}y_t)$. Formally,

$$\pi(\Gamma_0, \Gamma_1) \propto \begin{cases} 1 & \text{if } (\Gamma_0, \Gamma_1) \in \{\Gamma_0 \in \Re^2, \Gamma_1 \in \Re^2 |\ \Gamma_1 < 0,\ |\mathrm{d}\gamma - \mathrm{G}| \leq \mathrm{dG}\} \\ 0 & \text{otherwise} \end{cases}.$$

Finally, we assume a uniform prior for p and q, $\pi(p) = U(0,1)$ and $\pi(q) = U(0,1)$.

To obtain draws from the joint posterior distribution $\pi(\theta, s^T, N_1|Y^T)$ we iteratively sample from the conditional posterior distributions $\pi(s^T|\theta, N_1, Y^T)$, and $\pi(\theta, N_1|s^T, Y^T)$. The first step is based on the single-move sampler derived in Albert and Chib (1993). To

[3] i.e. $\theta = (\Gamma_0, \Gamma_1, \Pi, \Psi, \delta_1, \Sigma, p, q)$, with $\Psi = (\Psi_1, \ldots, \Psi_{k-1})$.

sample from $\pi(\theta, N_1|s^T, Y^T)$, additional blocking is useful to derive conditional distributions from which sampling is standard. Basically, all parameters except Σ, p and q can be sampled from normal or matrix normal distributions. Σ is sampled from an inverted Wishart distribution, and p and q are sampled from independent beta distributions (see Paap and van Dijk, 2003).

The cointegration analysis is based on the decomposition given in (17). If the variables are non-stationary but not cointegrated, then $\Pi = 0$. If the variables are cointegrated, the reduced rank restriction applies and $\lambda = 0$. Thus, the Bayes factor to evaluate the model of no cointegration, $\mathcal{M}_{\Pi=0}$ against the unrestricted error correction model $\mathcal{M}_{uec}$ amounts to evaluate the Bayes factor for $\Pi = 0$ in $\mathcal{M}_{uec}$. Correspondingly, the Bayes factor to evaluate the model of cointegration, $\mathcal{M}_{\lambda=0}$ against the unrestricted model $\mathcal{M}_{uec}$, is in fact the Bayes factor for $\lambda = 0$ in $\mathcal{M}_{uec}$. Both Bayes factors may be estimated using the Savage-Dickey density ratio (Dickey, 1971), which equals the ratio of the marginal posterior density to the marginal prior density of Π and λ, respectively, evaluated at $\Pi = 0$ and $\lambda = 0$:

$$\begin{aligned} \mathrm{BF}(\mathcal{M}_{\Pi=0}|\mathcal{M}_{uec}) &= \frac{\pi(\Pi|Y^T)|_{\Pi=0}}{\pi(\Pi)|_{\Pi=0}} \\ \mathrm{BF}(\mathcal{M}_{\lambda=0}|\mathcal{M}_{uec}) &= \frac{\pi(\lambda|Y^T)|_{\lambda=0}}{\pi(\lambda)|_{\lambda=0}} \end{aligned} \tag{23}$$

The marginal posterior density of Π can be inferred directly from the Gibbs output by averaging over the full conditional posterior distributions evaluated at zero, while the posterior distribution of λ may be estimated using a kernel estimator. The prior height, which are not defined given that we use diffuse priors, is approximated by the penalty function $(2\pi)^{-\frac{1}{2}(2-r)^2}$, (see Kleibergen and Paap, 2002 for more details).

If evidence for cointegration is present, we can sample β_2 and α adding a Metropolis-Hastings step into the Gibbs sampler, where the acceptance rate at iteration (m) for the values $(\alpha^{(m)},\ \beta_2^{(m)})$ obtained by the transformation (19), depends on the value $\omega(\cdot)$ which is given by the ratio of the posterior of the cointegration model and the posterior of the unrestricted error correction model. Thus, at each iteration, $(\alpha^{(m)},\ \beta_2^{(m)})$ are accepted with probability $\tau = \min\left(\frac{\omega(\Sigma^{(m)},\alpha^{(m)},\beta_2^{(m)},\lambda^{(m)})}{\omega(\Sigma^{(m-1)},\alpha^{(m-1)},\beta_2^{(m-1)},\lambda^{(m-1)})}, 1\right)$, see Paap and van Dijk (2003) and Kleibergen and Paap (2002) for more details.

6 Results

6.1 The data

We use quarterly data covering the period from 1975 to 2003. They stem from the area wide model database provided by the ECB, except for M3.[4] All series are seasonally adjusted and taken in logs. The price level is measured by the log of the harmonized index of consumer prices (p) and the money-income ratio is measured by the log ratio of M3 over GDP, $(m-y)$. The data cover most of the post Bretton-Woods period and include valuable information about prices and money, in particular a period of high variation in the data during the second half of the seventies.

[4] We kindly thank Boris Hofmann who provided us with the time series.

Table 1: Posterior distribution of Π and λ. Logarithm of the Bayes factors and posterior model probabilities.

Posterior distribution

$$\Pi = \begin{bmatrix} -0.06 & 0.08 \\ (-0.11 \ -0.00) & (0.00\ 0.16) \\ 0.06 & -0.09 \\ (-0.04\ 0.15) & (-0.20\ 0.04) \end{bmatrix} \quad \lambda = \begin{bmatrix} -0.004 \\ (-0.01\ 0.00) \end{bmatrix}$$

Cointegration analysis		
model	log BF$(\cdot\|\mathcal{M}_{uec})$	$P(\cdot\|Y^T)$
$\mathcal{M}_{\Pi=0}$	-12.00	0.00
$\mathcal{M}_{\lambda=0}$	4.94	0.99
$\mathcal{M}_{uec}$	0.00	0.01

Figure 3 depicts the level series. The price level is the bold solid line and the money-income ratio is the bold dashed line. For graphical convenience, the price level is normalized to the initial level of the money-income ratio. We can observe that until the end of 1986, both series grew at the same average rate, namely 1.9% on a quarterly basis. Afterwards, the growth rate of the money-income ratio averaged 1% on a quarterly basis, and exceeded the inflation rate which amounted to a quarterly average rate of 0.65% (see also figure 4 for the decrease in both growth rates). The average difference in the quarterly growth rates over the whole observation period is 0.24%, which reflects the decline in velocity over time. As already mentioned in section 5, this stylized fact is also used to design the prior distribution of the Markov switching trend parameters.

6.2 Posterior distributions

The posterior estimates are obtained by iterating 30,000 times over the sampler. The first 18,000 draws are discarded to remove dependence from initial conditions. The posterior inference is then based on every 4th draw of the remaining 12,000 draws. A preliminary analysis revealed that the inclusion of 4 lagged values of ΔZ_t is appropriate to remove autocorrelation in the residuals.

The cointegration analysis is based on the estimate of the unrestricted error correction model. The posterior distribution of Π and λ, obtained from the decomposition (17)-(19), are given in table 1. The logarithm of the Bayes factors $\text{BF}(\mathcal{M}_{\Pi=0}|\mathcal{M}_{uec})$ and $\text{BF}(\mathcal{M}_{\lambda=0}|\mathcal{M}_{uec})$ are given in the bottom panel. While the Bayes factor $\text{BF}(\mathcal{M}_{\Pi=0}|\mathcal{M}_{uec})$ clearly rejects a model of no-cointegration, the Bayes factor $\text{BF}(\mathcal{M}_{\lambda=0}|\mathcal{M}_{uec})$ favors a specification with reduced rank, i.e. cointegration. Alternatively, in terms of model averaging, the model with one cointegration vector has the highest posterior probability. Given this evidence, we estimate the model with one cointegration vector, sampling the cointegration and the error correction parameters by Metropolis-Hastings. The acceptance rate proved to be relatively high (49%).

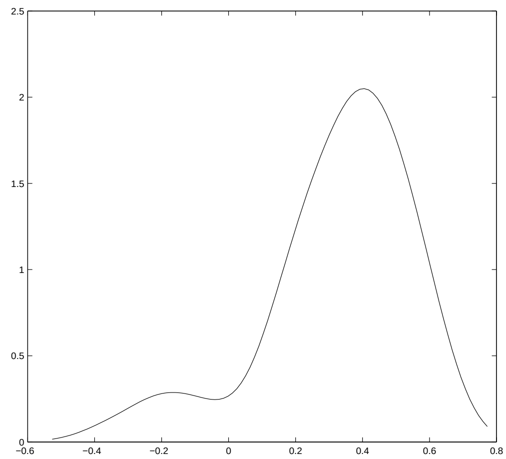
The posterior distribution of the parameters is displayed in table 2. We show the mean and the shortest interval covering 95% of the posterior density. With some exceptions,

Table 2: Posterior distribution of parameters. The 95% HPD interval is the shortest interval covering 95% of the posterior density

	price		money-income ratio	
	mean	95% HPD	mean	95% HPD
N_1	339.65	(339.19 340.14)	-1116.60	(-1117.69 -1115.42)
Γ_0	1.48	(0.92 2.24)	1.92	(1.43 2.63)
Γ_1	-0.21	(-0.41 -0.00)	-0.75	(-1.06 -0.45)
δ_1	0.33	(-0.21 0.68)	0	(–)
β	1	(–)	-1.34	(-1.76 -0.99)
α	-0.06	(-0.11 0.01)	0.06	(-0.03 0.14)
$\Psi_{1,1\cdot}$	0.36	(0.03 0.72)	0.19	(-0.30 0.78)
$\Psi_{1,2\cdot}$	-0.17	(-0.29 -0.07)	0.25	(0.02 0.47)
$\Psi_{2,1\cdot}$	-0.05	(-0.37 0.20)	-0.11	(-0.70 0.46)
$\Psi_{2,2\cdot}$	-0.01	(-0.12 0.11)	0.12	(-0.13 0.38)
$\Psi_{3,1\cdot}$	0.33	(0.09 0.60)	0.30	(-0.19 0.81)
$\Psi_{3,2\cdot}$	-0.04	(-0.15 0.07)	0.09	(-0.16 0.32)
$\Psi_{4,1\cdot}$	0.44	(0.15 0.74)	0.07	(-0.50 0.63)
$\Psi_{4,2\cdot}$	-0.07	(-0.17 0.04)	0.06	(-0.16 0.28)

$$\Sigma = \begin{bmatrix} 0.06 & -0.01 \\ (0.04\ 0.08) & (-0.04\ 0.03) \\ -0.01 & 0.37 \\ (-0.04\ 0.03) & (0.24\ 0.48) \end{bmatrix} \qquad \begin{array}{ll} p = & 0.84 \\ & (0.69\ 0.97) \\ q = & 0.82 \\ & (0.67\ 0.96) \end{array}$$

Figure 2: Marginal posterior distribution of δ_1.



the parameters of interest are significant. The presence of two states is significant, in particular for the money-income ratio we estimate two significantly different growth states. The 95% interval of the shift parameter δ_1 includes zero. This is due to the fact that the estimated posterior distribution is bimodal, whereby most of the posterior mass is clearly concentrated around the positive mode at 0.41 (see figure 2). This estimate indicates that the model specification potentially should allow for a third state to capture the shifting relationship between prices and the money-income ratio. This will be explored in further work. Concerning the dynamic part of the model, i.e. the coefficients $\Psi_1, \ldots, \Psi_4$, the inclusion of four lags is mainly due to the inflation dynamics. All coefficients on lags higher than one are insignificant except for the own third and fourth lag of the inflation rate. Given this over-parametrization, the significance of the Markov switching part of the model additionally gains in strength.[5] Finally, the estimates of the transition probabilities p and q reveal that the two states are persistent, both states on average last for around six quarters.

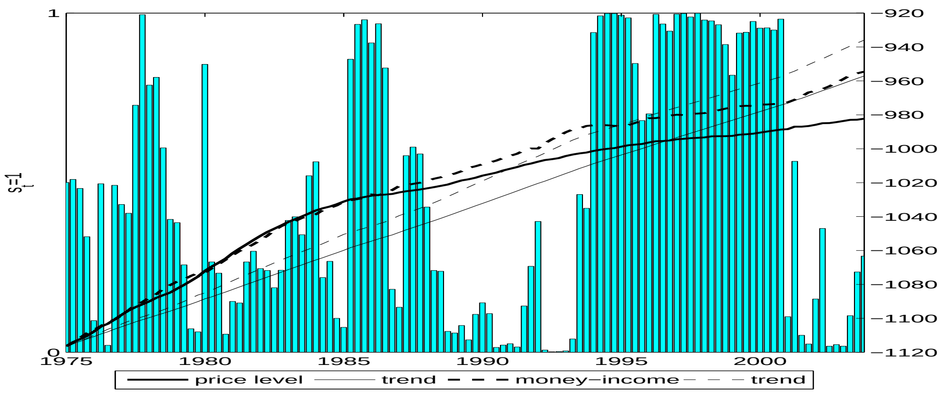
6.3 Discussion

Given the state identifying restriction with which we truncate the prior distribution, the posterior estimates of the Markov switching parameters relate state 0 to periods with higher trend growth. The price level trend grows at a lower level than the trend in the money-income ratio during state 0, 1.48% versus 1.92%, respectively. On the other hand, during state 1, both trends grow at nearly the same rate, 1.27% and 1.17% for the price level and the money-income ratio, respectively. If we compute the unconditional growth rates of the Markov switching trends, $\Gamma_0 + \Gamma_1 \frac{1-p}{2-p-q}$, the price level trend on average grows at 1.38% and the money-income ratio at 1.57% over the observation period. This amounts to a difference in the growth rates of 0.19% which lies within the 95% interval around

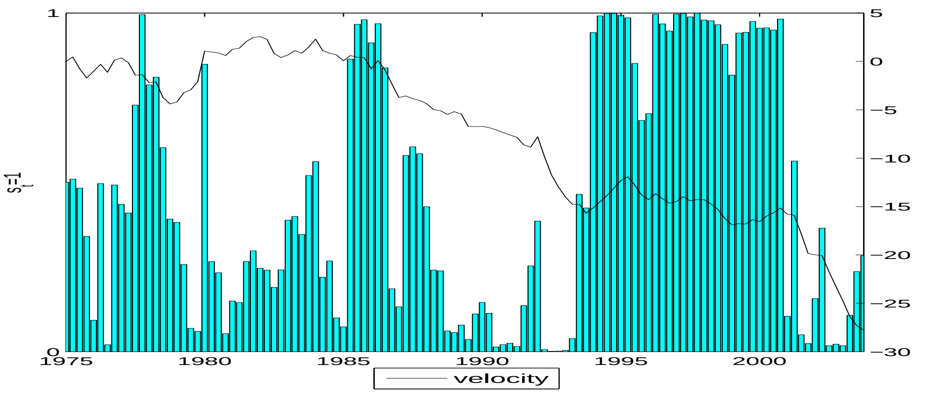
[5]It is known that dynamic (linear) features of data are spuriously captured by a Markov switching specification if the number of included autoregressive lags is too small.

Figure 3: (a) Price level (solid line) and money-income ratio (dashed line), (b) velocity $(p_t - (m_t - y_t))$. The posterior probabilities, $P(S_t = 1|Y^T)$ and the mean estimate of the Markov switching trend components are estimated by averaging over the MCMC draws. The initial values of the time series are normalized to the initial level of the money-income ratio.

(a)



(b)

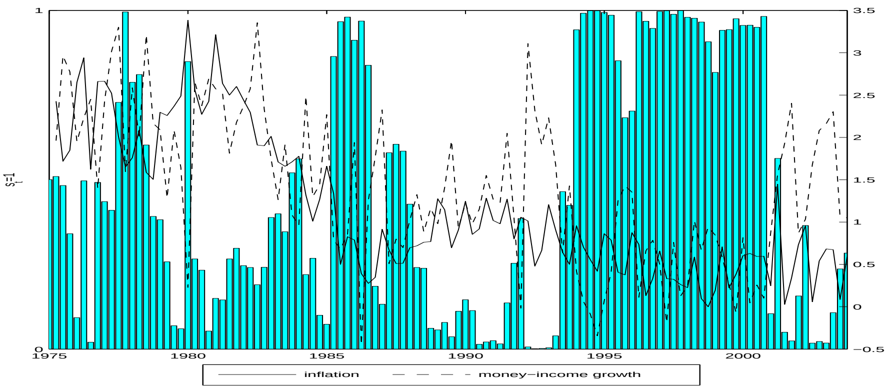


the empirical mean growth difference of 0.24% (0.09%-0.39%). As already mentioned, most of the posterior mass for δ_1 is concentrated around the positive mode at 0.41. This reflects the empirical fact that the price level trend grows at a lower rate than the trend in the money-income ratio in state 0. If the series are not drifting apart unboundedly, i.e. in order to "catch up" the trend in the money-income ratio, a positive shift in the price level is needed when switching to state 1, in which both trends grow at equal rates. This effect dominates the negative shift which would be observed if both series would follow the same Markov trend (see equation 11).

The posterior estimate of the identified cointegration vector shows that the vector $[1 -1]$ marginally is contained in the cointegration space. Note that, although both series display a symmetric mean adjustment to lagged errors, namely 0.06 in absolute values, the money-income ratio seems to be nearly weakly exogenous.

Given the numerical results, we now turn to the estimate of the posterior state probabilities. Figure 3, panel (a), depicts the price level and the money-income ratio in bold

Figure 4: Posterior probabilities, $P(S_t = 1|Y^T)$, the inflation rate (solid line) and the growth rate of the money-income ratio.



solid and dashed lines, respectively, and the normal faced lines represent mean estimate of the trend component of each series. We observe that the Markov trend basically captures the average long run trend in both series. The graph also plots the posterior probability of state 1, the state which is characterized by nearly equal growth rates in both series. This is particularly observable in the second half of the 1990s, a period of disinflation in the forerun to monetary union. Figure 3, panel(b) displays velocity $(p_t - (m_t - y_t))$ against the posterior state probabilities and gives another interpretation of the states. Periods of trend growth differences (state 0) mean periods of decreasing velocity while equal trend growth periods mean constant velocity. This is particularly the case since 1985.

It is worth mentioning, that state 1 does not capture the growth slowdown that has taken place in both series after 1986. In figure 4, we display the inflation rate and the money-income ratio growth rate along with the posterior probabilities of state 1. We can observe that state 0 relates to periods where a higher growth in the money-income ratio did not materialize in higher inflation.

7 Conclusion

This paper analyzes the recently documented instability of money demand in the euro area in the framework of a Markov switching trend model. First, we consider a standard flexible price model with stable money demand, rational expectations and exogenous output and money. The difference of log money and log income (the money-income ratio) is assumed to follow a Markov trend with two states, high and low growth, respectively. This approach is based on the corresponding analysis of the implications of a Markov switching income trend on the cointegrating relationship between consumption and income within the framework of the permanent income hypothesis (Hall, Psaradakis and Sola, 1997). This framework, which implies an influence of the "permanent" money-income ratio (current and discounted expected future values) on prices, leads to a cointegrating relationship between (log) prices and the (log of the) money-income ratio with a switching intercept term. Of course, this likely leads to a rejection of cointegration by standard tests and to the erroneous conclusion of an unstable money demand. However, the no-cointegration result between money, income and prices is only brought about by expectation effects in the framework of a Markov trend model and not by an instability in the money demand. Second, a more general model allowing for endogeneity and more general dynamics is estimated for euro area data from 1975-2003 by applying Bayesian methods used in Paap and van Dijk (2003) for the permanent income model. This exercise provides support for our model and the stability of the demand for M3 in the euro area. Two rather persistent states of different trend growth in the money-income ratio, lasting both on average for 6 quarters, are found, and evidence for cointegration between the stochastic trend components of prices and the money-income is rather strong in this model. However, the posterior distribution of the level shift parameter for prices indicates that a third state could be relevant. A corresponding extension of our model will be considered in future research.

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