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The Doping Threshold in Sport Contests*

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Abstract

We analyze the doping behavior of heterogeneous athletes in an environment of private information. In a n-player strategic game, modeled as an all-pay auction, each athlete has private information about his actual physical ability and choses the amount of performance-enhancing drugs. The use of doping substances is costly but not further regulated. The main finding of the analysis is the existence of a doping threshold. In our leading case only strong athletes dope. The level of the doping threshold is increasing in the doping costs and decreasing in the prize level. Furthermore, increasing the number of athletes affects the doping decision in two ways. More competition increases the incentives to dope for strong athletes. At the same time, we find a discouragement effect for weak athletes.

Keywords:	Auctions, Contests, Doping, Heterogeneity, Private Information.
JEL Classification:	C72, D44, D82.

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1 Introduction

Since the 1990s, the number of athletes tested positive for doping has substantially increased. Positive doping cases have been covered by the media particularly in professional cycling. The Festina affair in 1998 and the Fuentes scandal in 2006, followed by extensive legal investigations, show that many favorites and even entire cycling teams doped systematically.[1] The recently published material from the investigations of the U.S. Anti-Doping Agency against Lance Armstrong and the U.S. Postal Service Team reveal the actual dimension of doping in professional cycling (USADA, 2012). Doping is, however, by no means a new phenomenon. Written sources show that already in ancient Greece athletes used stimulants and dubious mixtures to enhance their strength and endurance (Verroken, 2005).

A closer look at doping practices reveals that athletes either dope during the training period, directly before a competition, or do both. Performance-enhancing drugs are popular because of their immediate and strong impact on individual performance. Drugs instantaneously improve performance, whereas training is time consuming and affects the performance only in the long run. Moreover, many drugs are only detectable for a short period of time. These features make doping especially attractive for athletes who wish to further enhance their performance shortly before a contest. A better understanding of athletes' incentives to take drugs is an important prerequisite to increase the efficiency of anti-doping policies.

Anti-doping agencies and the International Olympic Committee (IOC) have the objective to establish a doping-free environment for sports contests. The regulator, however, cannot directly control whether athletes take performance-enhancing drugs. Asymmetric information makes detection difficult and expensive. The current policies of anti-doping agencies are out-of-competition as well as in-competition doping controls and the sanctioning of convicted athletes. The ongoing doping cases in sports such as athletics, professional cycling or weight lifting show that despite severe sanctions and public humiliation in the case of detection, doping remains present in professional sports. Some experts even believe that doping has increased in particular sports due to the ongoing commercialization and the development of more effective drugs.

[1] Dilger et al. (2007) give a short review of the history of doping and present recent doping scandals in professional cycling and in athletics.

Asymmetric information can occur at several levels. The existing doping literature has primarily focused on asymmetric information between athletes and the regulator, leaving aside considerations about asymmetric information between athletes. Muehlheusser (2006) emphasizes the relevance of informational asymmetries in sports and recommends taking them into account when designing contests. Each athlete has private information about his actual physical ability, but can only guess the abilities of his rivals. The behavior of athletes in camouflaging and even misrepresenting their actual ability indicates that this informational advantage is important.[2] An athlete's decision to use performance-enhancing drugs will hence not only depend on regulations, but also on private information concerning his ability. So far, this issue has been neglected by the doping literature. The aim of this paper is to investigate the rationale of doping in a heterogeneous n-player game under private information.

Many researchers have studied the doping problem in the context of actual or potential anti-doping regulations. Berentsen (2002) is one of the first to analyze anti-doping regulations in a strategic two-player game.[3] Cheating and doping have recently been introduced into the theory of contests and tournaments. Their common basis is the Lazear-Rosen tournament model, extended to include a regulator who audits the athletes. Kräkel (2007), for example, analyzes the doping behavior of heterogeneous athletes who optimize the use of doping and legal inputs.[4] Another strand of the doping literature has focused on fair play norms and on peer group approval based on past doping decisions (Eber, 2008, 2011; Strulik, 2012). In both approaches an equilibrium without doping is possible; however, a reliable coordination mechanism is needed to make this equilibrium stable.[5]

Our paper is closely related to the literature that analyzes cheating in tournament models. In contrast to the random component in Lazear-Rosen tournament models, we assume that athletes' abilities are heterogeneously distributed and private information. As the interest lies in identifying how heterogeneity affects doping behavior under private information, we assume that taking performance-enhancing drugs is costly and ignore

[2]For instance, athletes play down their actual form or, conversely, conceal injuries and illnesses at press conferences.

[3]For similar contributions, see e.g., Eber and Thépot (1999), Maennig (2002), Haugen (2004). Berentsen and Lengwiler (2004) analyze doping and fraudulent accounting in an evolutionary game.

[4]Analogous is Stowe and Gilpatric (2010) who focus instead on the doping decision and different regulation regimes. Curry and Mongrain (2009) investigate the effects of the prize structure. Finally, Gilpatric (2011) analyzes how enforcement affects the effort levels.

[5]Bird and Wagner (1997) were the first who proposed decentralized mechanisms based on social norms to solve the doping problem. An example of such a mechanism is whistleblowing (see e.g., Berentsen et al., 2008).

further anti-doping regulations for the moment. The information structure of the contest is the following: In stage one, *nature* independently draws athletes' abilities from a distribution. The number of athletes and the distribution is common knowledge. The actual ability of the athlete, however, is private information. In stage two, athletes may improve their performance by taking performance-enhancing drugs. They base their doping decision on their actual ability and their beliefs about the abilities of their competitors. Finally, in stage three, the athlete with the greatest performance–the combination of ability and the chosen amount of doping–wins the prize money.

Our private information setting with heterogeneous athletes yields new insights which complement the results of existing doping literature. We analyze how the prize amount, the doping costs, the number of athletes and the distribution of abilities affect athletes' doping behavior. We show that under private information not all athletes take performance-enhancing drugs. For the majority of underlying parameter values, there exists a doping threshold. In our leading case, strong athletes dope, and athletes beneath the doping threshold have no incentive to dope. The doping behavior of an athlete depends crucially on his actual ability and the degree of competition. If the degree of competition increases, strong athletes take larger amounts of drugs. At the same time there exists a discouragement effect for athletes with low abilities. The anticipation of encountering stronger athletes in the contest discourages weak athletes from doping. For nonstandard parameterizations, three other equilibrium outcomes occur.

The paper is structured as follows: In Section 2, we describe the model and present the main results. In Section 3, we formally derive the equilibrium outcome with the doping threshold. In Section 4, we discuss the results of the doping model. First, we show how the doping threshold depends on the number of athletes and on the ratio of prize money to doping costs. Second, we analyze the influence of the distribution of abilities. Finally, we display the equilibrium results under special parameterizations. Section 5 concludes.

2 The Model and Main Results

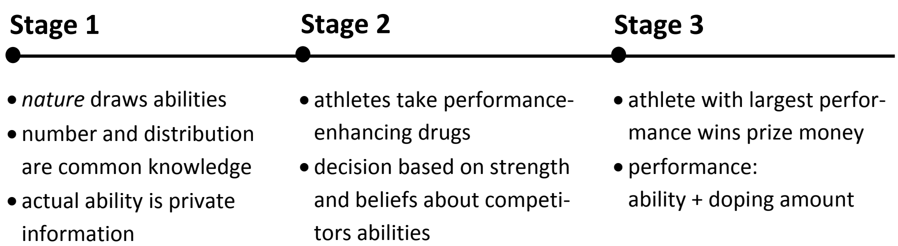
In a contest, n athletes compete against each other. The winner receives prize money v. All athletes are risk-neutral and maximize their expected payoff. Figure 1 displays the information structure of the contest. In stage one, *nature* independently draws athletes' abilities a_i, for $i = 1, ..., n$. The abilities are drawn from a given cumulative distribution function F with support $[0, 1]$. More specifically, we assume a power function distribution

$F(a) = a^{\alpha}$. This flexible functional form allows us to analyze the doping incentives for different shapes of the distribution. At the same time it ensures an explicit solution. The number of athletes and the distribution are common knowledge. The actual ability of the athlete, however, is private information.

In stage two, athletes may improve their performance by taking performance-enhancing drugs. The athlete's performance p_i is a linear combination of his ability a_i and the amount of doping d_i he chooses to take: $p_i = a_i + d_i$. We assume that athletes are free to choose an arbitrary doping amount and are not limited to the two discrete options of *doping* or *not doping*.[6] Athletes base their doping decision on their actual ability and their beliefs about the abilities of their competitors. Doping substances are not free of charge and the athlete has to pay for his doping substances before the contest begins. Since we are interested in how heterogeneity affects doping behavior under private information, we assume that taking performance-enhancing drugs is costly and ignore further anti-doping regulations for the moment.[7]

Finally, in stage three, the athlete with the greatest performance–the combination of ability and the chosen amount of doping–wins the prize money. For simplicity, the athlete with the greatest performance wins with certainty.[8] In a world without doping, the athlete with the greatest ability would win the contest. However, as athletes can choose arbitrary amounts of doping, it is, in principal, possible for a weaker athlete to beat a more talented athlete. Each athlete thus faces a trade-off between gaining a higher likelihood of winning through doping and the increased costs.

Figure 1: The information structure of the contest



[6]The model can readily be adapted to apply to a binary decision between *doping* and *not doping*. Note that this modification would not change the qualitative results.

[7]This is comparable to a situation with toothless anti-doping regulations. For example, if athletes can easily manipulate their test results.

[8]See e.g. Kovenock et al. (1996) for all-pay auctions with complete information. For all-pay auctions with private information, see among others Amann and Leininger (1996) and Feess et al. (2008).

The doping amount is the difference between the athlete's performance p_i and his ability a_i. If an athlete's performance is equal to his ability, then he does not dope and his doping costs are zero. Every athlete's doping cost function is thus a function of the difference $p_i - a_i$. The doping costs are $c(p - a_i)$, where the parameter c indicates the magnitude of marginal doping costs. The linearity of the function ensures a closed-form solution. The cost function is the same for all athletes. We denote the ratio of prize money to marginal doping costs by w ($w \equiv v/c$). From now on, we will assume that the parameter values of α, n and w satisfy $\alpha(n-1) \geq 1$ and $w\alpha(n-1) > 1$ and use the term *leading case* for these parameterizations. In Section 4.3, we will relax these assumptions and address special cases.[9]

The athlete's probability of winning depends on his chosen performance p and on the performances of the other athletes. The athlete only wins the contest if his performance $p(a_i)$ is greater than the performances of all other athletes. On the other hand, the costs accrue even if the athlete does not win the contest. Equation (1) displays the payoffs of an athlete with ability a_i who chooses the performance $p(a_i)$.

$$\Pi(a_i) = \begin{cases} v - c(p(a_i) - a_i) & \text{if} \quad p(a_i) > \max_{j \neq i} p(a_j), \\ -c(p(a_i) - a_i) & \text{if} \quad p(a_i) < \max_{j \neq i} p(a_j), \end{cases} \tag{1}$$

$$\text{nonnegativity constraint:} \qquad p(a_i) \geq a_i. \tag{2}$$

Negative doping amounts are ruled out by assumption ($d_i \geq 0$). Therefore, the nonnegativity constraint $p(a_i) \geq a_i$ has to hold for every possible a_i. Furthermore, in case of a tie between m athletes ($m \leq n$), the prize money is split up equally between the m athletes.

We are interested in athletes' optimal doping behavior. More precisely, we present the symmetric equilibrium performance function of the athletes. In equilibrium there exists a doping threshold. Athletes with ability below the threshold do not dope, whereas athletes with ability above the threshold do dope. Theorem 1 presents the equilibrium outcome of the doping model.

Theorem 1. *There exists a symmetric Nash equilibrium in pure-strategies with the doping threshold $a^\star$, for parameterizations that satisfy the conditions of the leading case.*

[9] See Appendix A.2 for an overview of all possible cases and their underlying conditions.

In the symmetric equilibrium, an athlete with ability a chooses the performance

$$p(a) = \begin{cases} a & \text{if } a < a^\star, \\ a^\star + w\left[a^{\alpha(n-1)} - a^{\star\alpha(n-1)}\right] & \text{if } a \geq a^\star. \end{cases} \tag{3}$$

The unique doping threshold $a^\star$ *is*

$$a^\star = [\alpha w(n-1)]^{\frac{1}{1-\alpha(n-1)}} \,. \tag{4}$$

Proof. See Appendix A.1.[10] □

3 The Doping Equilibrium

The actual contest is similar to an all-pay auction where every athlete has to pay for his personal doping amounts. We are interested in an equilibrium performance function which is increasing in a. If this is the case, then the probability of winning $G(a)$ is the c.d.f. of the highest order statistic $A_{(n-1:n-1)}$ of the remaining athletes.[11] Given the assumed power function distribution, the probability of having the greatest ability is $G(a) = \Pr\{A_{(n-1:n-1)} \leq a\} = F(a)^{n-1} = a^{\alpha(n-1)}$.

In order to find the symmetric Nash equilibrium in the doping contest, we apply the usual approach used in auction theory. First, we assume that in a symmetric Nash equilibrium an athlete with ability a chooses the performance $p(a)$ and then formulate the expected return for this athlete (using the highest order statistic). Every athlete can deviate from his equilibrium strategy by choosing another performance. However, it does not make sense to choose a performance lower than $p(0)$ or higher than $p(1)$. In the first instance, one would never win, and, in the second instance, one would always win, but have to pay too much. For this reason, deviations from the equilibrium performance function can be modeled as follows: An athlete with ability a who pretends to have a different ability x chooses the associated performance $p(x)$ in the contest through adjusting the doping amount.

The expected utility function of an athlete with ability a, who pretends to have ability x, is the product of the prize v multiplied by the winning probability of an athlete with ability x minus the cost of doping necessary in order to achieve the performance $p(x)$.

[10]The proof relies on the intermediate results of Section 3. Therefore, we recommend that readers cover Section 3 before turning to the proof.

[11]For further information on order statistics, see David and Nagaraja (2003).

His expected utility is thus: $u(a,x) = vG(x) - c(p(x) - a)$. The athlete will choose the x that maximizes his expected utility. He can only imitate performances that are equal to or greater than his ability, since the amount of doping cannot be negative. In equilibrium, the nonnegativity constraint $p(a) \geq a$ has to hold for every possible a.

The main contribution of this paper is the equilibrium performance function where the nonnegativity constraint is binding. However, in a fist step, the doping equilibrium is derived for cases where the constraint is not binding.[12] This is done in order to introduce the basic solution technique and to emphasize the importance of the nonnegativity constraint. If the nonnegativity constraint is not binding, then, the equilibrium performance function can be derived in the following way.

To obtain the optimum, we differentiate the utility function with respect to x and set it equal to zero. We obtain the FOC: $vG'(x) - cp'(x) = 0$. The optimal behavior of an athlete with ability a_i is to imitate the strategy of an athlete with ability x so that the FOC is satisfied.

In addition to the FOC, the incentive compatibility (IC) constraint has to be satisfied for every possible a in the Nash equilibrium. There must be no gain in deviating from the equilibrium strategy. The IC constraint is satisfied if $u(a,a) \geqslant u(a,x)$ for all a, x. We assume that if an athlete is indifferent between $u(a,a)$ and $u(a,x)$, he will choose the equilibrium strategy a. In a symmetric equilibrium, the optimal x corresponds to the athlete's own ability a. Therefore, we insert $x = a$ and $p(x) = p(a)$ into the FOC and obtain $vG'(a) - cp'(a) = 0$. The FOC states that the expected marginal return has to be equal to the marginal costs of increasing the winning probability. Dividing by c and solving for $p'(a)$ gives $p'(a) = vG'(a)/c$. We see that only the ratio of prize money to marginal doping costs matters. Therefore, we use w and obtain the basic equation to derive the equilibrium performance function:

$$p'(a) = wG'(a). \tag{5}$$

Our assumption of a linear doping cost function implies that an athlete with ability a_i is indifferent between his equilibrium performance $p(a_i)$ and all other $p(a)$'s for which the nonnegativity constraint of doping is not binding.[13]

[12] The nonnoegativity constraint is not binding if $\alpha < (n-1)^{-1}$ and $w > 1$.

[13] This particular circumstance has to be kept in mind when we investigate the equilibrium and for the proof of Theorem 1. If, instead, we used a doping cost function with slightly decreasing marginal costs or a quadratic doping cost function, then the implied single crossing property would guarantee a strictly separating equilibrium. For example, assume that the doping cost function is: $c(a) = c\exp(-\rho a)$, where ρ is very small. As ρ goes to zero the marginal costs go to c ($\lim_{\rho \to 0} c(a) = c$). Therefore, it seems

Integrating from 0 to a gives the performance function $p(a)$ for the cases where the nonnegativity constraint is not binding.

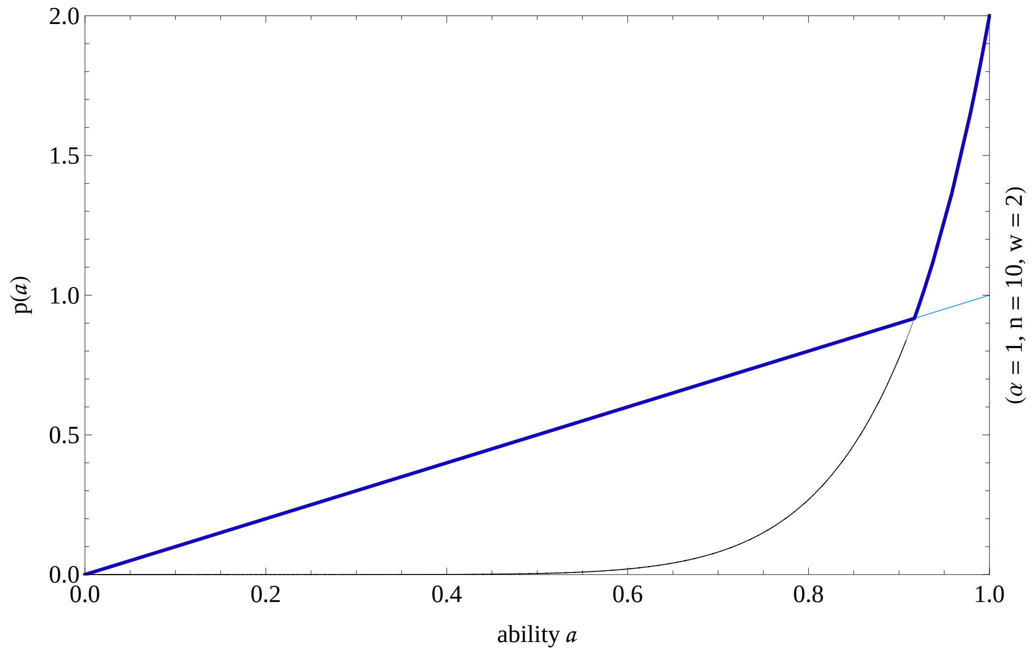
$$p(a) = wa^{\alpha(n-1)} \qquad \text{for} \quad 0 \leq a \leq 1. \tag{6}$$

The equilibrium performance function is similar to an equilibrium bid function of a standard all-pay auction.

Having specified the performance function for parameterizations where the nonnegativity constraint is not binding, we now turn to parameterizations of the leading case. Under these assumptions, the nonnegativity constraint is only binding for weak abilities. If we use performance function (6) for cases where the constraint is binding, then this would result in a performance function that lies partly below the underlying ability.

Figure 2 depicts such a situation ($\alpha = 1, n = 10, w = 2$). For athletes with an ability lower than the intersection (0.92), the performance function lies below the underlying ability. Hence, the nonnegativity constraint is violated. One could argue that augmenting the performance function of the lower section of the curve to the 45-degree line

Figure 2: Optimization that neglects the nonnegativity constraint



plausible that the equilibrium outcome in the limit is similar to the outcome of a linear doping cost function.

(to their corresponding ability) would solve the problem. However, we will show below, that this is not a Nash equilibrium.

In the following, we will derive the equilibrium results of Theorem 1. To ensure that an athlete's performance is equal to or greater than his ability, we introduce the reserve ability approach.[14] This approach to find the equilibrium subject to the nonnegativity constraint is non-standard in auction literature. Therefore, we derive the equilibrium with the doping threshold step by step and, if necessary, provide further explanations.

The **reserve ability approach** is a three-step procedure to find the equilibrium outcome. First, assign an arbitrary reserve ability and assume that athletes with abilities below the reserve ability choose their ability as performance. Second, derive the equilibrium doping behavior of athletes with abilities above the reserve ability. And finally, determine the proper reserve ability such that athletes with abilities below it behave optimally. The approach can only be applied if the constraint is binding for a closed interval that includes the lowest ability and the c.d.f. is continuous.

The introduction of the reserve ability approach ensures that the performance of weak athletes is equal to their ability, such that $p(a)$ is no longer smaller than a. In contrast to an auction with a reserve price, weaker athletes can still win, since their performance is the sum of the chosen doping quantity and their ability. The reserve ability is non-effective under parameterizations where the constraint is not binding and performance outstrips ability over the whole support.

In order to obtain the performance function $p(a)$ of an athlete with ability a, we integrate Equation (5) from an arbitrary reserve ability a^r with respect to a.

$$\int_{a^r}^{a} p'(z)dz = w \int_{a^r}^{a} G'(z)dz = w[F(a)^{n-1} - F(a^r)^{n-1}].$$

We solve the integral on the left-hand side of the equation above and take the obtained term $p(a^r)$ over to the right-hand side. Since the performance function at position a^r must be equal to a^r, we can replace $p(a^r)$ by a^r. Finally, we substitute the power distribution function. We thus obtain the performance function of athletes, given the underlying distribution of abilities and the reserve ability a^r.

$$p(a, a^r) = a^r + w\left[F(a)^{n-1} - F(a^r)^{n-1}\right] = a^r + w[a^{\alpha(n-1)} - a^{r^{\alpha(n-1)}}] \text{ for } a^r \leq a \leq 1. \quad (7)$$

[14] The term *reserve ability* comes from the concept of the *reserve price*. The reserve price is used in auction theory in order to close out bid valuations that are too low. The bids have to at least meet the reserve price, which prevents bidders with valuations lower than r from placing a bid. For a good overview on auctions and the reserve price see Krishna (2002).

The proper reserve ability satisfies two conditions. First, the reserve ability a^r has to be chosen such that the performance function $p(a, a^r)$ of athletes with ability above the reserve ability does not sink below the 45-degree line ($p(a, a^r) \geqslant a$ for $a \in [a^r, 1]$). Second, a^r must not be chosen too large such that athletes with abilities below the reserve ability have no incentives to deviate. For our continuous power distribution function, these two conditions are satisfied if $p(a, a^r)$ has the same slope as the 45-degree line at position $a = a^r$. Hence, the slope of the performance function is equal to 1 at the proper reserve ability, which we denote the doping threshold. The reason for this smoothness condition can be seen in the athletes' optimization problem. In equilibrium, every athlete chooses his performance such that weaker athletes have no interest in imitating that performance. Therefore, the equilibrium performance function does not have a kink at the doping threshold.[15]

To determine the proper reserve ability a^r, we set the derivative of the performance function (7) equal to one. By solving the equation for a, we obtain Equation (4) which is the doping threshold. The doping threshold $a^\star$ defines the ability level where the athlete is indifferent between doping and not doping. Athletes with an ability beneath the threshold do not dope, and athletes with an ability above this value take performance-enhancing drugs. The doping threshold depends on the distribution of abilities, the number of competing athletes, and the ratio of prize money to marginal doping costs. In Proposition 1, we show that under the parameterization of the leading case the doping threshold exists.

Proposition 1. *If $\alpha(n-1) > 1$ and $\alpha(n-1)w > 1$, then there exists a unique doping threshold $a^\star \in (0, 1)$, and strong athletes dope, and weak athletes abstain from doping.*

Proof. The derivative of Equation (7) is continuous and strictly increasing if $\alpha(n-1) > 1$. Given that $a^r = 0$, the derivative $p_a(0, 0)$ is zero. If the derivative $p_a(1, 0) = \alpha(n-1)w > 1$, then it follows that a unique solution of the doping threshold $a^\star$ must exist over the support $\in [0, 1]$. □

The next step is to derive the performance function and the doping amount of the athletes. The athlete's performance function is obtained by inserting Equation (4) in Equation (7). The result is Equation (3), which is the equilibrium behavior of the athletes.

Besides the performance function, the doping behavior of the athletes is of interest. As the performance function $p(a)$ is the sum of ability a and the doping quantity d,

[15] For the mathematical proof see Appendix A.1.

the doping function can be simply derived from the performance function. The doping quantity is an athlete's performance minus his ability. Having determined the doping threshold, the performance function and the doping quantity, we can now describe the equilibrium behavior of athletes in the doping model. Section 4 will discuss the outcomes of the doping model in more detail. Using comparative statics and figures, we will show how different values of α, n and w influence the doping threshold, the performance function and the doping function.

4 Discussion

In our model, athletes usually choose an amount of doping such that the marginal costs of doping are equal to the marginal expected increase in prize revenue. Depending on the distribution of abilities, however, it is possible that the marginal return to be gained by doping is smaller than the marginal costs of doping. In the leading case, weak athletes abstain from doping, because it would cost more to imitate the performance of a slightly stronger athlete than the extra return from having a higher probability of winning. Thus, we identify a discouragement effect for weak athletes, similar to the discouragement effect in Lazear-Rosen tournament models with head-starts or handicaps (see e.g., Weigelt et al. (1989) and Schotter and Weigelt (1992)). Crucial for an athlete's doping decision is his actual ability and the shape of the winning probability function G. The function becomes more convex, the more competitors there are. This is the reason that a threshold value exists for the majority of parameterizations, below which it is optimal to abstain from doping.

4.1 The Number of Athletes and the Costs of Doping

In discussing the results, we first investigate the effects of n and w. For this, we will assume that $\alpha = 1$. The abilities are uniformly distributed between 0 and 1. First, we will consider the doping threshold. With a uniform distribution of abilities, Equation (4) becomes

$$a^{\star}_{(\alpha=1,n,w)} = [w(n-1)]^{\frac{1}{2-n}} . \quad (8)$$

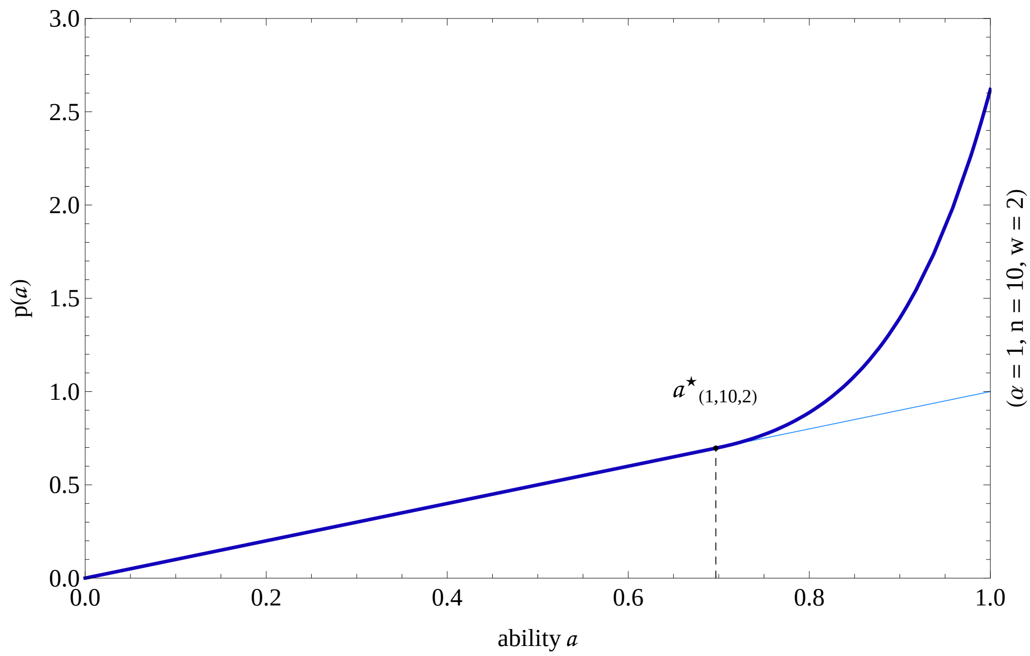
The number of competitors is decisive for the size of the doping threshold. If only two athletes compete, then the function of the doping threshold does not exist. In a two-player contest, the performance function consequently has the following appearance: $p(a) = wa$ for $w > 1$. The doping quantity, thus, increases linearly in line with ability.

If $w \leq 1$, the two athletes do not dope, because doping is too expensive. Hence, either both athletes dope or neither dopes.

In contests with more than two athletes, a doping threshold does exist. An athlete is indifferent between doping and not doping if his ability is equal to the threshold. The more athletes participate, the higher the value of the doping threshold. A higher number of participants makes it increasingly unattractive for weak athletes to dope. Furthermore, the ratio of prize money to marginal doping costs w determines the level of the doping threshold. If w is large, then the participants can win a large amount of prize money and the doping costs are relatively low. If w becomes smaller, that is, the relative doping costs increase, then the doping threshold rises.

Figure 3 shows the performance function for a contest between 10 athletes and a ratio of prize money to marginal doping costs of 2. The performance function (thick line) represents an athlete's ability up to the threshold. Beyond this point, the athlete's performance is greater than his ability. The doping threshold lies at approximately 0.7. An athlete's doping amount is the difference between his performance and his ability. Up to the doping threshold, the doping amount is equal to zero. Doping does not pay in this array, and athletes' true abilities determine the outcome. For abilities above

Figure 3: Performance function $p(a)$



the doping threshold, the doping function has a positive value. In other words, athletes above this threshold will dope. The more talented such an athlete is, the more he will dope, so that the athlete with the greatest ability will dope the most.

Under standard parameterizations, an athlete will have difficulties winning without resorting to doping. In order to illustrate this, we use the example of an athlete with ability $a_{\text{fair}} = 1$. This athlete always wins in a world where no athlete resorts to doping. However, if he decides not to dope, his chances of winning falls dramatically. In order to calculate his chance of winning in this contest, we need ability a°, for which the performance function $\mathrm{p}(a^{\circ})$ is equal to 1.[16] Our best athlete, who is also honest, is only able to win against athletes whose $a < a^{\circ}$. His likelihood of winning corresponds to the highest order statistic of a°. $\mathrm{G}(a_{\text{fair}} = 1) = a^{\circ\, n-1} = 0.19$. In a doping environment, his chance of winning thus falls from 100 percent to just 20 percent. This low winning probability is due to the fact that he does not simply have to compete against one competitor whose ability should be smaller than his ($a_i < a^{\circ}$), but has to win against all nine competitors.

Commercialization has caused prize money in certain sports to surge and has allowed successful athletes to skim additional cash from private companies. Nowadays, it is quite common, that companies employ the images of successful athletes in corporate sponsoring events to position a brand or to ameliorate their images. Thus, the ratio w seems to have increased rather than decreased over the last two decades. In our model, a larger w leads to a smaller doping threshold and raises the amount of doping substances used. The implications of our model are supported by the observation that there are more doping cases in popular sports than in sports where the prize amount is lower, or where doping offers only a small competitive advantage.

4.2 The Distribution of Abilities

The nature of the distribution affects the doping threshold and the equilibrium amount of doping substances used. The power distribution with an arbitrary α illustrates different distributions of abilities. For $\alpha < 1$, the density of the ability distribution is the highest for small a's, while on the other hand for $\alpha > 1$, there are relatively more strong than weak athletes. We accentuate the importance of the distribution of abilities by contrasting the outcome of a contest of numerous strong athletes with the outcome of a contest of numerous weak athletes.

[16] We set the performance function equal to 1 and then solve for a. For the contest with $n = 10$ and $w = 2$ we receive: $a^{\circ} = 0.832$.

Figure 4 compares two different distributions of abilities. In both cases, 10 athletes compete against each other, and the ratio of prize money to marginal doping costs is equal to 2. The dashed line is the density of athletes' abilities. In Graph (a), the density of the power function distribution has a parameter value of $\alpha = 0.3$, and in Graph (b) it has one of $\alpha = 3$. The thin line is the performance of the athletes when they do not dope. The athlete's behavior–when doping takes place–is illustrated by the performance function.

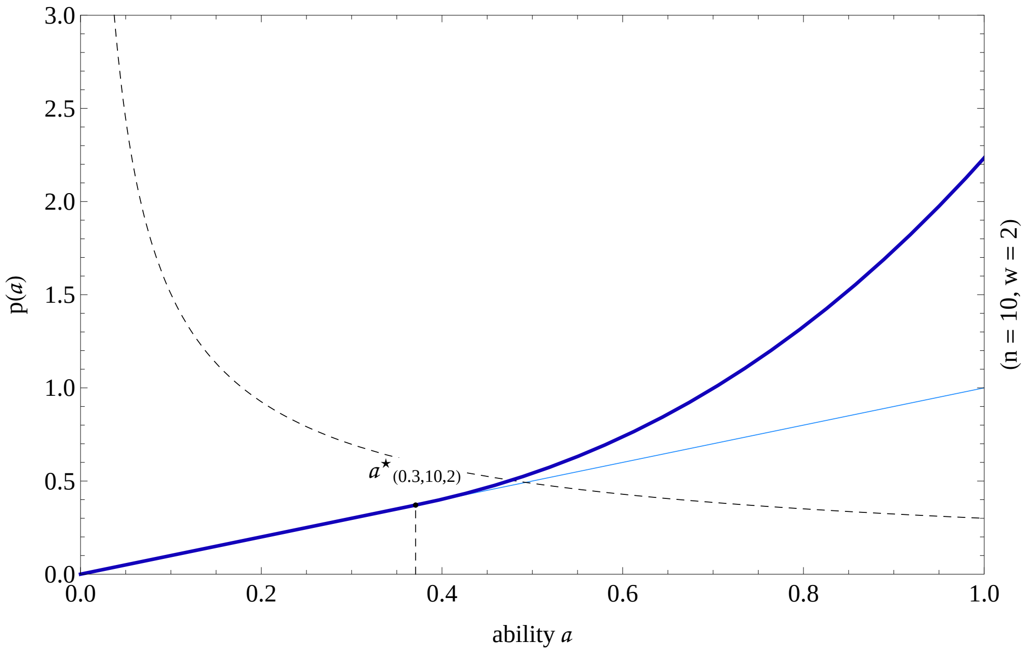
Graph (a) illustrates the outcome of a contest with numerous weak athletes. Here, the doping threshold is very low. Since there are only a few strong athletes, even relatively weak athletes make use of doping. The explanation for this result is that the probability of coming up against a stronger athlete in the contest is relatively small. With a low probability of strong athletes, the performance function increases gradually.

Graph (b) illustrates the outcome of a contest with numerous strong athletes. Here, doping behavior is quite different in comparison to the first case. The doping threshold is higher. This is because a weak athlete can expect to come up against a stronger competitor, given the higher probability of strong athletes. The performance function increases much more sharply than it did in Graph (a). The reason is that the degree of competition is more intense among strong athletes. This leads the strongest athlete to take more doping substances than he would have in the first case. In Graph (a), the performance of the strongest athlete is 2.2, while it is above 2.6 in Graph (b). Hence, although the doping threshold is further to the right, the actual performance of an athlete with $a = 1$ is greater than in a contest with numerous weak athletes.

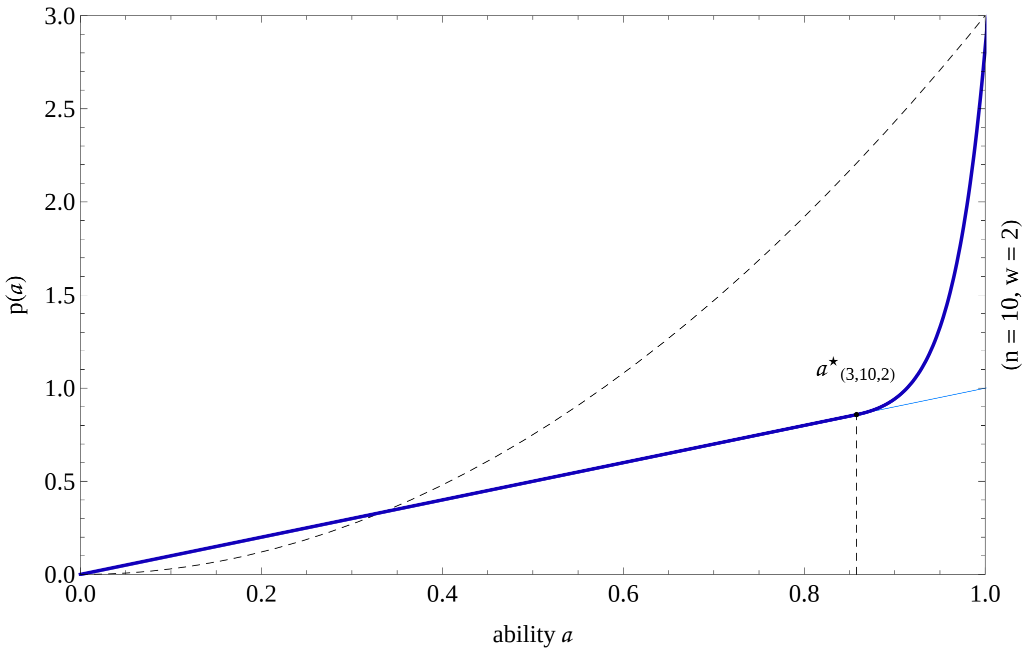
This section shows that in addition to the number of athletes and the ratio of prize money to marginal doping costs, the distribution of abilities also plays a significant role. An athlete's strength relative to his opponents and, as in Dilger and Tolsdorf (2010), the competitive pressure which weighs on an athlete from competitors with similar abilities are crucial for the doping decision. However, note that a high doping threshold does not always imply that strong athletes dope less than in circumstances with a lower doping threshold. In summary, competitive pressure and the distribution of abilities play decisive roles in an athlete's choice of the optimal doping quantity.

Figure 4: Comparison of different distributions

(a) Many weak contestants: $\alpha = 0.3$



(b) Many strong contestants: $\alpha = 3$



4.3 Special Cases

All parameter values for α, n and w that meet the assumptions of the leading case lead to an equilibrium outcome where weak athletes abstain from doping. Relaxing the conditions of the distribution of abilities and allowing for extreme values of the ratio of prize money to doping costs makes three other outcomes possible. The leading case arises in the majority of underlying parameter values. The other three cases are special cases that emerge only under exceptional circumstances: if doping costs are very high or low, or if the distribution of abilities is extremely skewed to the left.

For the prevailing equilibrium outcome, the derivative of Equation (7) with respect to a plays a central role. The derivative is the expected marginal gain divided by the marginal costs of doping. The second derivative shows that the first derivative is increasing if $\alpha(n-1) > 1$ and decreasing if $\alpha(n-1) < 1$ for every $a \in (0, 1]$. This critical value is the key to distinguish between the different doping outcomes. The underlying parameter values of α, n and w determine which outcome arises. Three additional outcomes are possible. In the first case, no athlete dopes. In the second case, only weak athletes dope. And in the third case, everybody dopes.[17]

The outcome that nobody dopes results if the ratio of prize money to marginal doping costs is low in comparison to the product of the shape parameter and the number of rivals. Nobody dopes because the marginal doping costs are greater than the marginal increase in the expected return. Hence, everybody would lower his expected utility by doping.

Proposition 2. *If $\alpha(n-1) \geq 1$ and $\alpha(n-1)w \leq 1$, then no doping threshold $a^\star \in [0, 1]$ exists and nobody dopes.*

Proof. The derivative of Equation (7) with respect to a at $a^r = a$ is continuous and strictly increasing in a if $\alpha(n-1) > 1$. The derivative at $a = 0$ is zero. If the derivative $p_1'(1, 1) = \alpha(n-1)w < 1$, then it follows that no doping threshold $a^\star$ exists over the support $[0, 1]$. □

The two other outcomes arise when the distribution of abilities is highly skewed to the left. In case (ii) only weak athletes dope, and in case (iii) everybody dopes. We investigate equilibrium outcomes for distributions of abilities that satisfy the inequality $\alpha(n-1) < 1$. Then, the derivative of Equation (7) is decreasing. Furthermore, $p_1(0, a^r)$

[17] Appendix A.2 presents the conditions, the parameter values have to meet for each of the three special cases.

is not defined under this condition. It is the case that $\lim_{a \to 0} p_1(a, a^r) = \infty$. Therefore, the nonnegativity constraint is not binding for small a's. Depending on the underlying parameter values, it is possible that the constraint is not binding over the whole support. In such a case, the equilibrium performance function is Equation (6). Generalizing the performance function in such a way that it displays the equilibrium outcomes of case (ii) and (iii), gives

$$p(a) = \max\{wa^{\alpha(n-1)}, a\} \qquad \text{for} \quad 0 \leq a \leq 1. \tag{9}$$

The doping quantity is the difference between an athlete's performance and his ability. In the equilibrium outcome, only weak athletes take performance-enhancing drugs. The formula of the doping threshold in case (ii) differs from the leading case. The doping threshold in case (ii) is given as follows:

$$a^{\star}_{\alpha,n,w} = w^{\frac{1}{1-\alpha(n-1)}}. \tag{10}$$

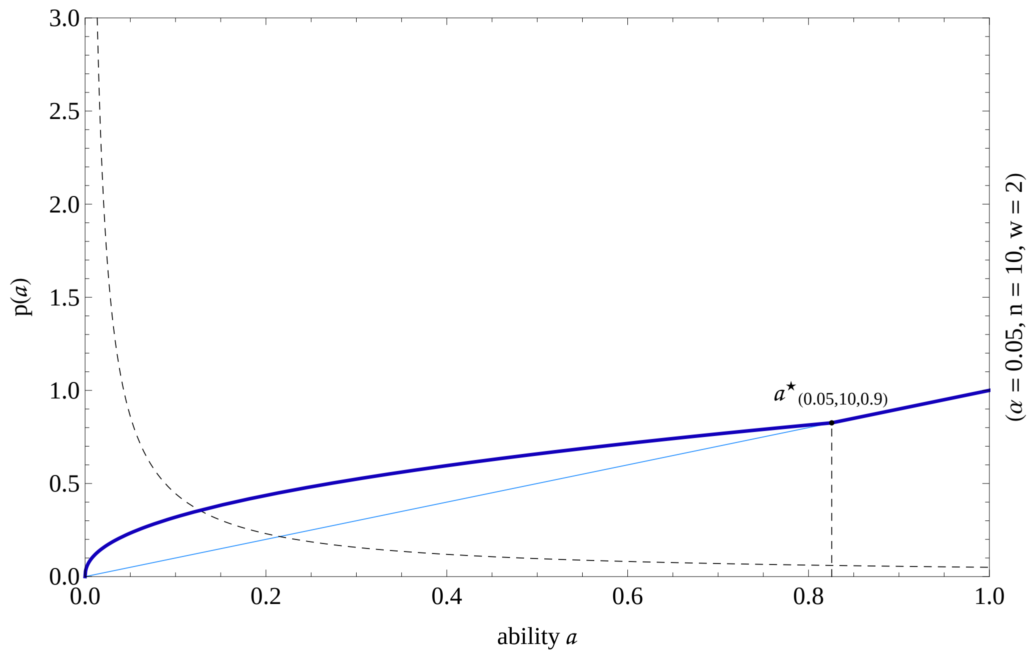
Proposition 3 presents the outcome of case (ii) and (iii) and the underlying conditions.

Proposition 3. *If $\alpha(n-1) < 1$, then two doping outcomes are possible, depending on the underlying value of w. In the first outcome (ii), only weak athletes dope. If $w \leq 1$, then there exists a unique doping threshold $a^{\star} \in [0, 1]$ and only weak athletes have an incentive to dope.*
In the second outcome (iii), everybody dopes. If $w > 1$, then no doping threshold exists over the support $[0, 1]$, and every athlete has an incentive to dope.

Proof. If $\alpha(n-1) < 1$, then the derivative of Equation (7) with respect to a for $a^r = 0$ is strictly decreasing in a. Furthermore, the derivative $p_1(0, a^r)$ is not defined. It can be shown that $\lim_{a \to 0} p_1(a, a^r) = \infty$. First, consider special case (ii). The performance function $p(a, 0)$ at $a = 1$, not considering the doping nonnegativity constraint, would be $p(1, 0) = w1^{\alpha(n-1)} = w \leq 1$. Therefore, there exists a unique intersection point with the 45-degree line. Hence, there exists a doping threshold $a^{\star}$ over the support $\in [0, 1]$.
Using the same reasoning for special case (iii), no doping threshold exists over the support $[0, 1]$ if $w > 1$. □

Figure 5 displays the performance function $p(a)$ in the special case (ii). The dashed line is the density distribution of abilities. The distribution is very skewed to the left, which implies that mostly weak athletes compete in the contest. Ten athletes participate in the contest, and the ratio of prize money to marginal doping costs is assumed to be 2. The equilibrium outcome is that weak athletes dope. The doping amount starts to decrease after a certain ability level and is zero for abilities above the doping threshold. The reason for this outcome is that the degree of competition is greatest for weak abilities. Strong athletes do not dope, because the probability of encountering a stronger athlete and the ratio of prize money to marginal doping costs are so small that it is not optimal to increase their performance over their ability level under private information.

Figure 5: Weak athletes dope, special case (ii)



At the doping threshold, the performance function has a kink. Having argued in the leading case that there can be no kink at the doping threshold, in the special case (ii) this is different. Given that all other athletes play the symmetric equilibrium of the performance function in Equation (9), it can be shown that an athlete with an ability a_i below the doping threshold is indifferent to imitating the performance of an athlete with an ability $a_j < a^\star$. If he would instead imitate the performance of an athlete with an ability $a_j > a^\star$, his utility decreases. For athletes with an ability above the doping threshold the nonnegativity constraint is binding. This implies that the marginal return of increasing the performance is lower than the marginal costs of doping.

5 Conclusion

We study the doping behavior in an environment of heterogeneous agents and private information. Our setting yields new insights which complement existing results of the doping literature. For the majority of underlying parameter values a doping threshold exists. In our leading case, weak athletes will abstain from doping even without doping controls. Athletes with abilities above the doping threshold resort to doping substances. The doping behavior of athletes and the level of the doping threshold are sensitive to the underlying parameterization. Three other equilibrium outcomes occur when we investigate nonstandard parameterizations. In this paper, we restrict our attention to doping in sport contests. However, our private information setting may also be of interest in areas such as promotion tournaments or public procurement.

Our findings can be summarized as follows. First, an athlete's doping decision depends on the ratio of prize money to marginal doping costs and not on absolute values. In the doping model, a higher ratio decreases the doping threshold and more athletes dope. Thus, the ongoing commercialization and new discoveries of the pharmaceutical industry have increased the incentives to resort to doping. Second, increasing the number of athletes affects the doping decision in two ways. The increased competition forces strong athletes to take larger amounts of drugs. At the same time a discouragement effect exists for weak athletes. The anticipation of facing a higher probability of encountering stronger athletes discourages weak athletes from doping. Finally, our comparison of outcomes between a contest with many weak athletes and a contest with many strong athletes shows that competitive pressure and the distribution of abilities play decisive roles in athletes' doping behavior.

The results of our model would be even more convincing if we could test our findings empirically. Unfortunately, hardly any empirical studies about doping exist, since doping is not directly observable. However, there are indications that support our findings. Empirical evidence shows that there are more positive doping cases in commercial sports with high prize amounts and where athletes receive substantial payments from sponsorships. On the other hand, doping is only rarely detected in technical sports such as tennis, where the use of performance-enhancing drugs helps only marginally.

A promising extension of our model would be the inclusion of a regulator who checks the pool of athletes. But, such a model that combines our private information setting with the asymmetric information problem between athletes and the regulator would be demanding. This is because we would have to incorporate disqualifications into our model, which makes closed-form solutions impossible.

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A Appendix

A.1 Proof of Theorem 1

The equilibrium performance function (Equation (3)) with the doping threshold (Equation (4)) is a Nash equilibrium if no athlete with ability a_i for all $a_i \in [0, 1]$ is better off by unilaterally deviating from the equilibrium strategy. Note that the performance function is strictly increasing in a. It follows that, if an athlete imitates the strategy of an athlete with ability a_j, his winning probability is $G(a_j)$. By using the term imitating, we mean that the athlete choses the performance $p(a_j)$ of an athlete with ability a_j. Note that an athlete with ability a_i can only imitate athletes with a performance greater than his ability ($p(a_j) \geq a_i$). Thus, we can limit the verification on the range $p(a_j) \geq a_i$, for all $a_i \in [0, 1]$.

Before starting with the proof, we ask the question: How large would the doping costs have to be for an athlete with ability a_i to win with probability $G(a_j)$, such that he would be indifferent to his equilibrium strategy? An athlete is indifferent if the difference of the winning probability times prize money is equal to the difference of doping costs. We denote the performance level where the athlete would be indifferent by $\hat{p}_{a_i}(G(a_j))$. The performance level satisfies the following equation:

$$vG(a_j) - c(\hat{p}_{a_i}(G(a_j)) - a_i) = vG(a_i) - c(p(a_i) - a_i). \qquad (11)$$

Generalizing Equation (11) for an arbitrary $a \geq a_i$ gives an indifference function.

Definition 1. The indifference function $\hat{p}_{a_i}(a)$ indicates the performance level at which an athlete with ability a_i is indifferent to his equilibrium strategy $p(a_i)$ if he were to win with the probability of an athlete with ability a. Function $\hat{p}_{a_i}(a)$ is defined in the range $a \in [a_i, 1]$. More formally the function is

$$\hat{p}_{a_i}(a) = p(a_i) + w[G(a) - G(a_i)] = p(a_i) + w[a^{\alpha(n-1)} - a_i^{\alpha(n-1)}] \text{ for } a \geq a_i. \qquad (12)$$

Comparing the functional form of Equation (12) with Equation (7) shows that the only difference is the threshold value. For abilities $a_i > a^\star$, the indifference function is even identical to the equilibrium performance function within the range of $a \in [a^\star, 1]$. This is a direct result of the assumed linear doping cost function. The indifference function illustrates whether an athlete would have an interest in deviating from the equilibrium. If the performance function $p(a)$ runs below (above) $\hat{p}_{a_i}(a)$, then the athlete is better off (worse off) by deviating. The proof of Theorem 1 consists of two steps. First, we prove that the performance function is a Nash equilibrium ((i), (ii)). Second, we prove that the doping threshold $a^\star$ is unique.

Proof. For athletes with ability a_i above the doping threshold, the indifference function is identical to the performance function. (i) Hence, for all $a_i \in [a^\star, 1]$ no athlete can be better off by imitating another performance ($p(a_j) \geq a_i$).

For athletes with abilities $a_i < a^\star$ below the doping threshold, the indifference function is not identical to the performance function. Note that the derivative with respect to a of $\hat{p}_{a_i}(a)$ is equal to the derivative of Equation (7). For parameterizations of the leading case, the derivative $\hat{p}_{a_i}{}'(a)$ at $a = a_i$ is strictly increasing in a_i. Remember that $\hat{p}_{a_i}(a)$ for $a_i = a^\star$ is identical to the performance function. Therefore, the derivative of $\hat{p}_{a_i}(a)$ at $a = a^\star$ is equal to 1. (ii) This implies that $\hat{p}_{a_i}(a)$ runs below the performance function $p(a)$ in the range of $t \in [a_i, 1]$ for all $a_i \in [0, a^\star)$. Hence, an athlete with ability a_i is worse off by imitating the strategy of an ability $a > a_i$.

Finally, we prove that the doping threshold $a^\star$ is unique. Suppose that the optimal threshold $\widetilde{a^\star}$ is smaller than $a^\star$. Then, the performance function would violate the nonnegativity constraint, since the derivative of the performance function is smaller than 1 for all $\widetilde{a^\star} \in [0, a^\star)$. Now suppose that the optimal threshold $\widetilde{a^\star}$ is greater than $a^\star$. Then, $\hat{p}_{a_i}(a)$ of an athlete with ability a_i which is slightly smaller than $\widetilde{a^\star}$ would run above the performance function, since the derivative of $\hat{p}_{a_i}(a_i)$ for $a_i \in (a^\star, 1]$ is greater than 1. Hence, the athlete with ability $\widetilde{a^\star} - \epsilon$ would be better off by deviating from his equilibrium strategy. It follows that the doping threshold $a^\star$ is optimally chosen, and therefore the performance function is a Nash equilibrium. □

A.2 The Four Doping Outcomes

The following description presents the different doping outcomes. Moreover, the conditions on the parameterization values of α, n and w that lead to the four outcomes are displayed.

Table 1: The doping outcomes

(A) Leading Case	**(B) Special Cases**
	(i) Nobody dopes
	$\alpha \geq (n-1)^{-1} \cap w\alpha(n-1) \leq 1.$
Strong athletes dope	**(ii) Weak athletes dope**
$\alpha \geq (n-1)^{-1} \cap w\alpha(n-1) > 1.$	$\alpha < (n-1)^{-1} \cap w \leq 1.$
	(iii) Everybody dopes
	$\alpha < (n-1)^{-1} \cap w > 1.$