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Differential games approach to trade with exhaustible resources

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Abstract

The paper presents a model of world economy with two countries where one of them dubbed home sells the exhaustible resource to final producers in both countries, which compete at the final goods market. The interaction between final producers is reached via the sticky price mechanics, whereas price continuously adjusts to produced final product quantities. Production technology in both countries includes the resource as an essential input plus the variety of intermediate products. We demonstrate how opening up to trade of the exhaustible resource may be beneficial for the home economy by promoting technical change and capital accumulation via increased resource rents and relative factor prices movements. This leads to the increase in social welfare due to taste for variety and fosters structural change in the home country.

Keywords: structural change, resource economics, international trade, differential games

JEL codes: F1, L16, Q37, C6, C7.

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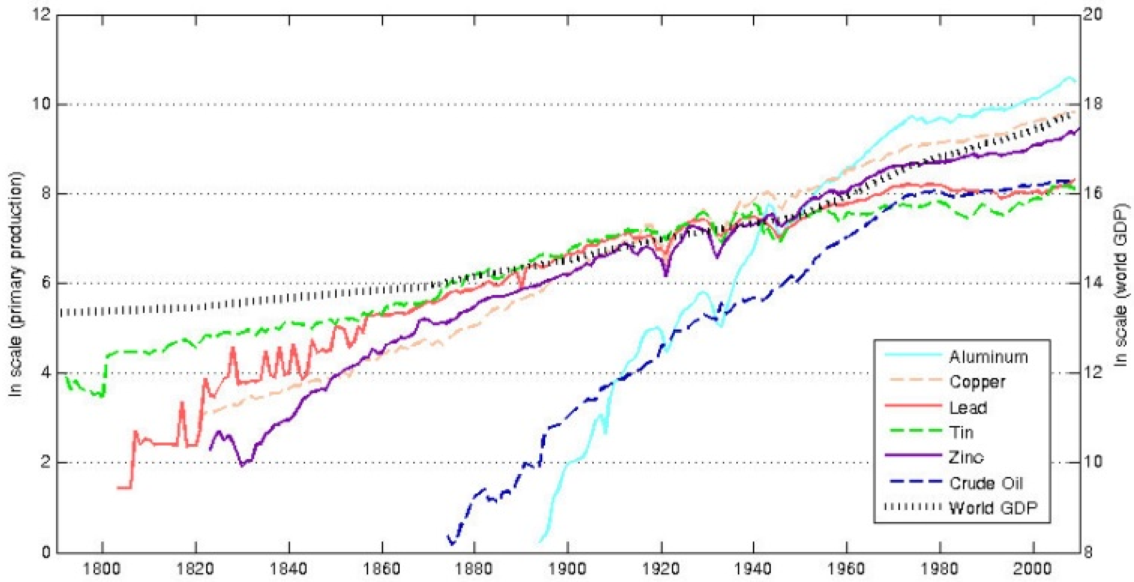
1 Introduction

The problem of exhaustible resources traded by resource abundant countries and the effects on economic growth is of long standing importance in economics. van der Ploeg (2012) points out that natural resource rents worldwide now exceed $4 trillion per annum, amounting to some 7 percent of global GDP, Stuermer (2013) showed that world primary production of non renewable resources and world GDP exhibit exponential growth since 1792 (see Figure 1) and Ruta and Venables (2012) claims that (non-renewable) resources account for 20% of world trade. Hence understanding the growth effects of natural resource revenue in an open economy cannot be overemphasised. Figure 2 illustrates an increasing trend of exports of natural resources (fuels and mining products) relative to other export merchandise providing an impetus to understand the trade-exhaustible resource dynamics.

International institutions such as the World Bank and OECD regularly promulgate advice based on the assumption that there are positive growth effects due to openness of economies. According to Srinivasan (1999) there are three sources of economic growth,(a)factor accumulation, (b)increased total factor productivity and (c)technological innovation. Trade openness can potentially enhance the growth of a country by influencing any of these three sources of growth, e.g. an open economy can obtain factors more easily from abroad, leading to factor accumulation, in cases such as foreign direct investment, countries may benefit from technological spillovers increasing TFP or the structure of the economy may change leading to the boost in technical change per se.

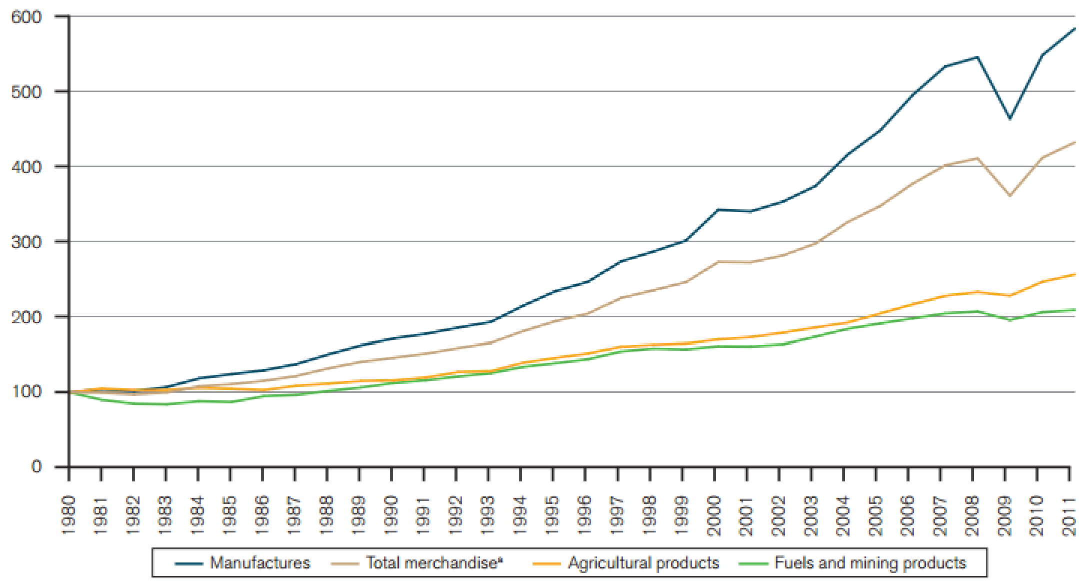
Our paper suggests that when a resource abundant country opens up to trade, demand for intermediate goods rises boosting technical change. This result is obtained from direct comparison of evolution of intermediate goods demand under both regimes (autarky and trade). We find out that there are three channels of gains for the resource abundant country; (a) variety gains in the same spirit as new trade models in which there is "love of variety" by consumers, see for example Melitz (2003); (b)technological change improvements (c)increase in rate of capital accumulation derived from non renewable resource revenue in the open economy regime. Direct comparison shows a higher autarky price relative to world equilibrium price of final goods, moreover opening of the domestic market leads to the switch in the mix of factors being used for production, as the resource price increases overtime due to exhaustibility, final good producers substitute it with intermediates in the long run boosting technical change and total factor productivity due to new varieties of intermediates goods being produced.

Figure 1: World primary production of non-renewable resources and world GDP from 1790 to 2009 in logs.



Source:Stuermer (2013)

Figure 2: Volume of world merchandise exports by major product category, 1980-2011 (index, 1980=100)



[a] Includes unspecified products.

Source: World Trade Organisation(2012)report

Not until recent, static trade models following Ricardo, like the one of Heckscher and Ohlin (1991) have fall short of studying the dynamic effects of trade opening on economic growth. Recently some dynamic arguments have been established as by Bajona and Kehoe (2010) but still the effects of exploitation of exhaustible resources in the open trade framework are missing. In support of this view, World Trade Organisation report(2010) stated that the traditional model of trade like of Heckscher and Ohlin (1991), does not directly address the problem of resource exhaustibility and the intertemporal trade-offs involved.

Therefore understanding the impact of trade on exhaustible resources requires adopting a dynamic approach that takes into account how finite resource stock varies over time. Exhaustibility of finite resources requires analysis of how the resource stock dynamic impacts production and prices and therefore the application of dynamic games framework seems quiet natural here. In light of this view and motivation, we demonstrate how opening up to trade of exhaustible resources may be beneficial to the economy which is resource abundant by promoting technical change and capital accumulation and what defines whether such an opening would be beneficial for the country.

In our study, exhaustible resource is used in production of final output like in model of Sethi (1979) with resource monopoly optimizing over the price of the resource and not its extraction rate, like in seminal paper of Hotelling (1931). We determine price evolution of the finite resource and final output when a small country opens to trade. Moreover, we explore the extent to which exhaustible resource constraints can be overcome by substitution and technological change.

There are two strands of literature studying optimality of resource extraction and effects on economic growth. The first strand is the Ramsey type growth models, like those of Stiglitz (1974), Dasgupta and Heal (1974) and Garg and Sweeney (1978). The second strand is the endogenous growth models like of Schou (1996), Aghion and Howitt (1998), Scholz and Ziemes (1999), Barbier (1999), Grimaud and Rouge (2003). Unlike in our analysis, both strands of literature do not consider impact of trade on resource price changes and extraction paths, final output price evolution or capital accumulation via trade. According to Todo (2003) developing countries usually rely on foreign direct investment(FDI) as a major source of technological development. We suggest that another channel which received less attention in models of economic growth is international trade, through which there are technological improvements from a technologically leading country to the less developed resource abundant country. We argue that capital accumulation through exhaustible resource trade has the potential to improve domestic investment and act as FDI substitute or replacement.

While most studies of endogenous growth with exhaustible resources focus on an isolated economy without trade and or FDI, a few studies related to our present work involving natural resources and trade have been carried out. A model most closely related to our work is presented by Gaitan and Roe (2012). Though they consider an infinite time horizon and trade of the exhaustible resources, they do not consider the problem within a differential game context as we have motivated above. A more recent study is by Yenokyan et al. (2014) who investigated economic growth with trade in factors of production. They concluded that trade in goods can raise growth rates of trading partners without any technology transfer or international transfer. Our study departs from their approach by considering technical change induced by intermediate goods used in final output production. Moreover we show that trade in resource may actually be harmful for less developed economy if taste for foreign product is high enough.

The rest of the paper is organised as follows, Section 2 presents the model, section 3 contains the solution (with technical steps in Appendices), in section 4 we obtain main results and propositions of the paper, in section 5 we discuss possible extensions, final section 6 concludes the paper.

2 Model

Consider the world economy with two countries, home, h and foreign, f. Home country possesses an endowment of exhaustible resource which is used in production both in home and foreign countries. The economies of both countries are otherwise symmetrical: there are representative households consuming final product being made in both countries, final goods producers, intermediate goods sectors and resource sector. Intermediate goods are produced in the Romer-fashion but we assume constant variety of available intermediaries technologies in the basic setup. We describe each sector of the home economy in turn since it is our main focus. Derivations for foreign economy are symmetric and can be easily made.

2.1 Households

Consumption is represented by the representative household who supplies L units of labour. As it is already standard (see for example Peretto and Connolly (2007)), we assume that the population is constant and equal in both countries of mass $L_{h,f} = 1$. We also assume that wage is equal across countries and is a numeraire such that $w = 1$. The only source of income for the households is the resource rent plus interest payments on savings. All of the income may be either consumed or saved: i.e a representative household owns the resource stock with associated revenue stream $p_R R$. Letting $r_h = r_f = r$ in the same spirit as Gaitan and Roe (2012) such that interest parity condition holds, the flow budget constraint of the representative household at home is:

$$\dot{V}_h = rV_h + p_R R + 1 - pE_h. \tag{1}$$

In the foreign country the budget constraint is the same but without resource rent:

$$\dot{V}_f = rV_f + 1 - pE_f. \tag{2}$$

The only capital in this economy are financial assets of households which are not depreciated. This assumption is not crucial and cannot influence any results of the paper except for the initial conditions on capital (assets) accumulation.

The household maximizes lifetime utility over an infinite time horizon subject to the intertemporal budget constraint and the usual No-Ponzi-game condition. We assume a constant discount rate and CES preferences preferences over goods provided by the home and foreign country. Thus the household maximizes

$$\max_{C_i} \int_0^{\infty} \mathrm{e}^{-\rho t} U(C_i) dt, \; i \in \{h, f\}; \tag{3}$$

where C_i is a consumption at country i resulting from the combination of home produced final good and foreign traded good:

$$C_i = \left[\varphi^{\frac{1}{\chi}} (C_{i,i})^{\frac{\chi-1}{\chi}} + (1-\varphi)^{\frac{1}{\chi}} (C_{-i,i})^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}} \tag{4}$$

and utility is logarithmic:

$$U(C_i) = \ln C_i \tag{5}$$

Note that χ denotes the inter temporal elasticity of substitution between home and foreign consumption goods, φ and $(1-\varphi)$ denote preferences over home and foreign goods respectively, where φ is the home bias; $\varphi \in [0, 1]$. A consumer also derives utility from consuming imported goods, the more the bias is, the less a representative household prefers imported goods.

2.2 Final goods sector

In this sector there exist home and foreign producers. They use resource being extracted in the home country and a range of intermediate inputs (country-specific) in production technology:

$$Y_i = \int_0^{N_i} Q_{ij}^{\eta} dj R_i^{\gamma},\ i \in \{h, f\}, j \in [0; N_i], \tag{6}$$
$$0 < \eta < 1,\ 0 < \gamma < 1.$$

with labour being used inelastically with $L = 1$.

The instantaneous profit function for final producer in country i is given by:

$$\pi_i = pY_i - \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - p_R R_i - 1,\ \ i \in \{h, f\}. \tag{7}$$

where last term appears due to labour costs.

We assume that the resource is traded by the home country to both home and foreign firms and the price p_R is the same for both final sector producers at home and abroad, but varieties of intermediaries are in general different and non-tradeable across countries.

The equilibrium price at time t is related to industry output by means of a linear inverse demand function:

$$p^{\infty} = (a - Y_h(t) - Y_f(t)). \tag{8}$$

In the same vein as Fershtman and Kamien (1987), we assume that the actual market price, $p(t)$, at time t is not equal to its equilibrium level $p^{\infty}(t)$, but moves towards it following a first-order adjustment process, i.e

$$\dot{p(t)} = s\left[a - p - Y_h(t) - Y_f(t)\right], \tag{9}$$

with $0 < s \leq \infty$. This implies sticky prices.

The problem of final producers thus involves instantaneous profit maximization with respect to the demand for intermediate products (capital) and intertemporal optimization w. r. t. the resource demand.

2.3 Resource sector

In our simple model, the resource sector is a monopoly, owned by the households. The behaviour is thus described by a standard resource-extraction model. The profits are given to consumers, who own this sector. In the same spirit as Dasgupta and Heal (1974), and Hartwick (1990) the firm extracts resources without any costs and offers them to the firms of the final-goods sector for a price p_R per unit, not differentiating across countries. Letting $S(t)$ denote the resource stock at time t and assuming no storage we have resource constraint

$$S_0 \geq \int_0^{\infty} R(t) dt. \tag{10}$$

The deterministic evolution of $S(t)$ is described by total extraction equal to the demand from both countries:

$$S(0) = S_0; \, \dot{S} = -R = -R_h^D - R_f^D \tag{11}$$

Since storage is not possible, the quantity supplied to the market at any time is equal to the total quantity extracted. The extracted resource is completely homogeneous across all firms, hence there is no product differentiation in the resource sector. Since we assume costless extraction, discounted profit to be maximized is given by

$$\max_{p_R} \int_o^\infty p_R R e^{-\int_0^t r(\tau)d(\tau)} dt \tag{12}$$

where r denotes the interest rate.

2.4 Intermediate producers

The intermediate goods sector consists of a variety N_i of products. We assume the range of existing intermediates to be constant, $\dot{N}_{h,f} = 0$ in both countries to simplify the analysis. Every variety is owned by a single firm, which is interpreted as R&D firm in standard dynamic case. All such monopolists have homogeneous technology of producing variety at hand. The production function for monopolist j in country i is given by

$$Q_{ij} = K_{ij}, \int_0^{N_i} Q_{ij} dj = V_i; j \in [0; N_i] \tag{13}$$

thus the total assets stock is used for production of intermediaries (capital).

In each country, each firm j in the intermediate good sector maximizes the present discounted value of profits flow:

$$V_{ij} = \max_{p_{Q_{ij}}} \int_t^\infty e^{-r(s)} [p_{Q_{ij}} Q_{ij} - r_i K_{ij}] ds \tag{14}$$

and establishes the equilibrium price for each individual intermediate product given the demand from the final producers sector.

3 Solution

3.1 Households

The maximization problem of the household, (3) subject to (1) or (2) yields demand for homogeneous final product. Households solve the problem in two steps,

1. **They decide how to allocate expenditures between imports and domestically produced goods.**

   Let E_i be the aggregate consumption expenditure in country $i = h, f$. The instantaneous expenditure constraint is given be

$$E_h = p_h C_{hh} + p_f C_{fh} \tag{15}$$

$$E_f = p_f C_{ff} + p_h C_{hf} \tag{16}$$

   No trade friction implies that the law of one price holds across countries,

$$p_h = p_f = p \tag{17}$$

   thus prices of final goods sold at home and foreign countries are equal.

2. **Agents choose the time profile of expenditures by maximising present value utility.**

$$\max_C \int_0^\infty \mathrm{e}^{-\rho t} \ln \left[\frac{E_i}{\varphi^\chi p^{1-\chi} + (1-\varphi)^\chi p^{1-\chi}} \right] dt; \tag{18}$$

   subject to (1) for the home country and to (2) for the foreign country.

   Optimality conditions give the standard Euler equation in which at each moment in time the rate of growth of expenditures on consumption is equal to the difference between the instantaneous interest rate and the rate of time preference.

$$\frac{\dot{E}_i}{E_i} = r_i - \rho,\ i \in \{h, f\}. \tag{19}$$

   Assuming equal interest rates across countries, $r_h = r_f$ yields equal growth rates of expenditures.

3.2 Final producers

Final producers are facing the consumers demand, C_h, C_f and optimize over demand for resource, R_i^D and demand for intermediate products, Q_{ij}^D. The total resource demand is dynamic and has to be chosen by both firms, but demand for intermediaries is obtained via maximization of the instantaneous profit functions.

We solve this problem in two steps.

1. **Demand for intermediate products.**

   Perfect competition and profit maximization implies that marginal products are equalised to factor prices. The demand for the jth variety of intermediate input in country i can be written in the well known Dixit-Stiglitz form:

$$Q_{ij}^D : \eta Q_{ij}^{\eta-1} R_i^\gamma - p_{Q_{ij}} = 0; \tag{20}$$

$$Q_{ij}^D = \left(\frac{\eta}{p_{Q_{ij}}} R_i^\gamma\right)^{\frac{1}{1-\eta}} = \eta Y_i \frac{p_{ij}^\epsilon}{\int_0^{N_i} p_{ik,k\neq j}^{1-\epsilon} dk} \tag{21}$$

Note that $\epsilon = \frac{1}{1-\eta}$ is the elasticity of demand with respect to intermediary's price which depends on the elasticity of substitution between varieties, η. The demand (21) is the downward sloping for each intermediate input j and is known to the monopolistic intermediate producer.

2. **Demand for the exhaustible resource.**

   The only dynamic choice variable for both foreign and home final producers is the resource demand. The dynamic problem for both firms is thus:

$$\max_{R_i^D} \int_0^\infty e^{-rt} \left(pY_i - \int_0^N p_{Q_{ij}} Q_{ij} di - p_R R_i^D - 1\right) dt, \tag{22}$$

s.t.

$$\dot{p} = s\left[a - p - Y_h - Y_f\right] \tag{23}$$

   where Q_{ij} is given by (21) and output is given by (6) for the home firm and the same for the foreign firm.

Using the open-loop equilibrium concept we derive resource demand through application of the Maximum Principle for both countries. Details can be found in the Appendix B. The optimal demand for the resource is

$$R_i^D = \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}} \left(\frac{(p-\lambda_i)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_i^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \tag{24}$$

a function of final product's price p and the shadow costs of production λ_i. Provided the resource demand is a differentiable function, we can reformulate the problem to get rid of shadow costs in the system. For that we use F.O.C. (B.6) to express λ_i through resource demand and price and then make use of (B.11) to get the differential equation on resource demand as a function of resource price, giving (B.14) in Appendix B.

$$\dot{R}_i = \left(\frac{\dot{p_R}}{p_R} - (1+r)\right) \frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(a - Y_{-i} - (2+r)p\right) \frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}. \tag{25}$$

We can deduce that an increase in the resource price p_R leads to a decline in the resource demand. Increasing varieties of intermediate inputs N would imply that a monopolist charges a lower price p_Q which gives rise to an increase in resource price. The net effect is a decrease in non-renewable resource demand. The interpretation is that capital accumulation takes place via expansion of varieties, presence of imported goods leads to improvement in existing goods through learning by doing effect. Consequently as resources get depleted the final good producers replace them with intermediate varieties.

As the substitution parameter η increases(more intermediates inputs substituting resource inputs) p_R rises hence less resources are demanded by final good producers confirming the above assertion. Using data from 1792 to 2009, Stuermer (2013) illustrated that prices for exhausted resources are roughly trendless (see figure3 in the appendix E), therefore the growth rates of all prices of the exhaustible resources covered in the study(five major base metals and crude oil) were not significantly different from zero. However, the analysis in this present study within a dynamic setting shows rich implications of exhaustible resource prices on input switching effect (substitution) in final good production.

3.3 Resource extraction

Profit maximization problem in the resource sector becomes :

$$\max_{p_R} \int_o^\infty p_R(t)R(t)e^{-\int_0^t r(\tau)d\tau}dt, \tag{26}$$

$s.t.$

$$\int_o^\infty R(t) \leq S(0) \tag{27}$$

$$\dot{S} = -R \tag{28}$$

with $R = R_h^D + R_f^D$. The application of Maximum Principle (details in Appendix C) yields constant price growth rate to ensure infinite time of total extraction: Thus the growth of resource price is proportional to the interest rate:

$$\frac{\dot{p}_R}{p_R} = r\frac{\eta + \gamma - 1}{\gamma}. \tag{29}$$

This is consistent with Hotelling (1931) rule that the optimal extraction path of an exhaustible resource is one along which the exhaustible resource price increases at the rate of interest times the ratio of total productivity and intermediaries (which are substitutes to the resource). This is a no-arbitrage condition equalizing the returns between the exhaustible resource and other assets.

3.4 Intermediate goods producers

The producer maximizes profit (we assume $\xi = 1$) function:

$$\max_{p_{Q_{ij}}} \pi_{Q_{ij}}(Q_{ij}) = p_Q Q_{ij} - rK_{ij} \tag{30}$$

$$s.t.$$

$$Q_{ij} = K_{ij}; j \in [0 : N_i]. \tag{31}$$

Since every intermediate producer is a monopolist in his/her own product, the firm optimizes over the price with the demand given by (21). From this the price for every intermediary may be defined and turns to be equal across intermediaries (since there is no technical change and homogeneous technologies for all of them).

$$r = \eta^2 \left(\frac{R_i^\gamma}{Q^{1-\eta}} \right) \tag{32}$$

Substituting for Q_D we get the optimal pricing rule of a constant mark up over the marginal cost (see Appendix D)

$$p_{Qij} = p_{Qi} = \frac{r}{\eta} \tag{33}$$

Due to product symmetry, prices in all sectors are identical, The only parameters affecting prices are costs r and elasticity of demand η. We assume interest parity such that $r_i = r_{-i}$, which implies the equal growth rates given by the Euler equation (19). Low values of η allow monopolist to charge a higher mark up and earn higher revenues and profits. η also captures intermediate input share in output, low values of η presume less demand for intermediate inputs.

We can substitute for p_Q and rewrite Q^D as:

$$Q_{ij}^D = \left(\frac{\eta}{p_Q} R_i^\gamma \right)^{\frac{1}{1-\eta}} = \left(\frac{\eta^2}{r} R_i^\gamma \right)^{\frac{1}{1-\eta}} = \eta^{2\epsilon} r^{-\epsilon} R_i^{\epsilon\gamma}, \tag{34}$$

where $\epsilon = \frac{1}{1-\eta}$. Now substituting for the pricing rule in the profit function we get

$$\begin{aligned} \pi_i &= \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - rK_i = \eta^{2\epsilon-1} r^{1-\epsilon} R_i^{\epsilon\gamma} - \eta^{2\epsilon-} r^{1-\epsilon} R_i^{\epsilon\gamma} \\ &= (1-\eta) \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj = \frac{1-\eta}{\eta} r \int_0^{N_i} Q_{ij} dj \end{aligned} \tag{35}$$

That is, monopolist profits at every instance is $(1-\eta)$ share of revenue. Thus with $\eta > 1$ the R&D sector is making losses and economy collapses (since no growth of varieties is assumed).

3.5 Equilibrium, market clearing

consists of:

1. Final goods market clearing $C_h + C_f = Y_h + Y_f$;
2. Capital market clearing for every country $K_i = \int_0^{N_i} Q_{ij} dj$;
3. Intermediaries market clearing for every country $Q_{ij} = Q_{ij}^D$;
4. Resource market clearing, $R = R_h^D + R_f^D$.

Now observe that the resource price growth rate is constant, (29). Thus the resource demand dynamics (25) are transformed into

$$\forall i \in \{h; f\} : \dot{R}_i = \left(r\frac{\gamma + \eta - 1}{\gamma} - (1 + r) \right) R_i -$$
$$\frac{1}{p_R} \left(a - N_{-i} \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_{-i}^{\frac{\gamma}{1-\eta}} - (2 + r)p \right) \frac{\gamma}{\gamma + \eta - 1} N_i \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_i^{\frac{\gamma}{1-\eta}} \tag{36}$$

and substituting for output and resource price solution (which is a straightforward monotonic solution for the ODE (29)) one gets the triple of equations, which describes the evolution of the world economy:

$$\dot{R}_h = \left(r\frac{\gamma + \eta - 1}{\gamma} - (1 + r) \right) R_h -$$
$$\frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a - N_f \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_f^{\frac{\gamma}{1-\eta}} - (2 + r)p \right) \frac{\gamma}{\gamma + \eta - 1} N_h \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}}; \tag{37}$$
$$\dot{R}_f = \left(r\frac{\gamma + \eta - 1}{\gamma} - (1 + r) \right) R_f -$$
$$\frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a - N_h \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}} - (2 + r)p \right) \frac{\gamma}{\gamma + \eta - 1} N_f \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} R_f^{\frac{\gamma}{1-\eta}}; \tag{38}$$
$$\dot{p} = \left(a - p - R_h^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} N_h - R_f^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r} \right)^{\frac{\eta}{1-\eta}} N_f \right). \tag{39}$$

where p_0 denotes the initial price for the resource.

The evolution of the world economy with two countries and open trade for exhaustible resource and final product is thus given by the triple (37), (38), (39).

It is clear that under autarky the resource demand from the foreign country is zero and the system is reduced to:

$$\dot{R}_h^A = \left(r\frac{\gamma+\eta-1}{\gamma} - (1+r) \right) R_h - \frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a-(2+r)p\right) \frac{\gamma}{\gamma+\eta-1} N_h \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}}; \tag{40}$$

$$\dot{p}^A = \left(a - p - R_h^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_h \right). \tag{41}$$

where superscript A denotes autarky regime. If the trade for resource is absent as well as for final product the dynamics of the home economy is characterized by the couple (40), (41).

4 Results

We analyze here the comparative dynamical behaviour of the system under open trade of the resource and under autarky, when the resource is traded only within country as well as the final product. It is relatively straightforward to see from the price dynamics, (39), (41) that opening the national markets may either decrease or increase prices of the final product and subsequently change the total demand for the resource. This in turn characterises the usage of intermediaries (capital and technology).

We start by defining the steady states of both systems.

Proposition 1 (Steady states existence).
Steady states for both systems (39), (37), (38) *and* (41), (40) *exist only if*

$$\gamma + \eta > 1 \tag{42}$$

otherwise resource demand has increasing growth rates until the full exhaustion of the resource.

Indeed observe that only under this condition the exponent in resource demand equations have negative power yielding contraction in time. Non-autonomous ODE systems may have steady states only if their non-autonomous part is a contraction. Hence the result.

This means prices and resource demand may be stabilized in the system if economy (both domestic and abroad) has high enough overall productivity. In this case the accumulation of capital can substitute for resource in production and thus demand for resource will stabilize at some level as well as prices. If instead, productivity is relatively low, there is not enough additional savings to replace resources in production and rising demand for goods (because of higher income of households) would lead to resource demand increase and thus price increases up to the point when the resource is fully exhausted. At this time no production will take place, since we assumed Cobb-Douglas production technology.

4.1 Autarky

The steady state for the autarky is defined by stable price for final product and stable demand for the resource.

The implicitly defined steady state level of resource demand under autarky results from substituting (44) into (43) and observing that the demand is stabilized only when

1. Economy is productive, e. g. $\gamma + \eta > 1$;
2. The fluctuations of demand stop, e. g. $\mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \to 1$.

If the total factor productivity is lower than 1, $\gamma + \eta < 1$ demand cannot be stabilized., since the non-autonomous part does not vanish at infinity, $\mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \to \infty$ and the system explodes. If conditions above hold, there exist three different steady state levels of resource demand: one of them trivially sets demand to zero, another one is close to zero and the last one is non-trivially set at some positive level. These follow from the algebraic properties of the polynomial (43), which has up to 3 real roots.

$$\left(r\frac{\gamma+\eta-1}{\gamma} - (1+r)\right) R_h^A - \frac{1}{p_0}\left(a - (2+r)p^A\right)\frac{\gamma}{\gamma+\eta-1} N_h^A \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} (R_h^A)^{\frac{\gamma}{1-\eta}} = 0, \tag{43}$$

$$\bar{p}^A = a - (R_h^A)^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_h^A. \tag{44}$$

4.2 Opened trade

The steady state for the system under trade is defined similarly from the system (37),(38),(39) under the condition of productive economy and thus vanishing exponent in resource demand equations:

$$\left(r\frac{\gamma+\eta-1}{\gamma}-(1+r)\right)R_h^O-$$
$$\frac{1}{p_0}\left(a-N_f\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_f^O)^{\frac{\gamma}{1-\eta}}-(2+r)p^O\right)\frac{\gamma}{\gamma+\eta-1}N_h^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_h^O)^{\frac{\gamma}{1-\eta}}=0 \quad (45)$$
$$\left(r\frac{\gamma+\eta-1}{\gamma}-(1+r)\right)R_f^O-$$
$$\frac{1}{p_0}\left(a-N_h^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_h^O)^{\frac{\gamma}{1-\eta}}-(2+r)p^O\right)\frac{\gamma}{\gamma+\eta-1}N_f^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_f^O)^{\frac{\gamma}{1-\eta}}=0 \quad (46)$$
$$\bar{p}^O=a-(R_h^O)^{\frac{\gamma}{1-\eta}}\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}N_h^O-(R_f^O)^{\frac{\gamma}{1-\eta}}\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}N_f^O. \quad (47)$$

with superscript O denoting the open trade regime.

As it can be seen, the same is true for the opened system. The difference comes from the boost in resource demand and lower price for the final product experienced by home final producers. At the same time under the open trade revenues of resource monopoly are growing, increasing the households income. This income is then used for faster capital accumulation and higher production of existing varieties.

For simplicity of further analysis we assume that varieties which are produced at home and abroad are fixed and are not changed as a result of opening trade, that is $N_h^A = N_h^O = N_h$, $N_f^A = N_f^O = N_f$ thus only the level of production of each variety , Q_{ij} may change. We start with the direct comparison of two steady states for the home economy.

Compare first steady state price levels in the opened and closed economy. The difference is given by

$$\bar{p}^A-\bar{p}^O=\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}\left((R_f^O)^{\frac{\gamma}{1-\eta}}N_f+(R_h^O)^{\frac{\gamma}{1-\eta}}N_h-(R_h^A)^{\frac{\gamma}{1-\eta}}N_h\right) \quad (48)$$

which is the difference between home output under open trade and autarky plus foreign output. This difference can be negative (thus final product price increases) only if the home output fall more due to foreign competition then the total foreign output. This could be true only if the home economy is relatively large comparing to the rest of the world (foreign) and representative households preferences are biased towards foreign final product. In this case the resulting reduction of home output is so strong, that it is not compensated by foreign output and final price increases instead of decreasing due to increased competition.

Proposition 2 (Steady states prices comparison).
The steady state price decreases after opening up trade, $\bar{p}^A-\bar{p}^O>0$ if:

1. *The home final output at least does not decrease under trade,*

$$(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}} \geq 0 \tag{49}$$

2. *The home output decreases to a lesser extent than the foreign output increases:*

$$(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}} < 0,\ N_h|(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}}| < (R_f^O)^{\frac{\gamma}{1-\eta}} N_f \tag{50}$$

otherwise the final product price increases, $\bar{p}^A - \bar{p}^O < 0$

This is a straightforward conclusion which does not need formal proof. Since our main goal is to study how the opening trade would affect the developing country, we assume $N_f/N_h > 1$ meaning the rest of the world is more developed. This increases the likelihood of final price to decrease as a result of opening trade. From this observation we deduce a corollary:

Corollary 1. *The higher is the technological distance between home and foreign country,* N_f/N_h *the more final price reduction have to be expected from opening trade.*

Now we analyze the changes in the resource rent, which governs the capital accumulation at home.

Proposition 3 (**Resource rent**).

1. *The price of the resource does not change from opening up the trade as long as* $r_f = r_h$ *and production technologies are the same,* $\eta_f = \eta_h, \gamma_h = \gamma_h$*:*

$$p_R^O = p_R^A \tag{51}$$

2. *The resource rent under open trade (as long as resource is available) is higher than under autarky by the factor of*

$$p_R R^O - p_R R^A = p_R \left(R_f^O + (R_h^O - R_h^A)\right) > 0. \tag{52}$$

Proof is made by:

1. Checking the resource price movement equation which is independent of country-specific parameters, (29);

2. Done by checking resource demand (C.1) with zero and non-zero foreign demand and observing from proposition 1 that home demand changes is negative.

Thus the resource rent grows from opening trade as long as the final price drops (the home country is technologically significantly backward) and technology is not super-productive $\eta < 1$. Resource rent may also grow if the final price grows but technology is very productive, since the relationship in Proposition 1 is reversed.

Observe that the quantity $(R_h)^{\frac{\gamma}{1-\eta}}$ governs the home demand for each of the intermediaries expressed in terms of resource demand. As long as $\eta < 1$ intermediaries are complements with the resource and increase in resource demand leads to the increase in intermediaries usage too. However if $\eta > 1$ which means more productive technologies, the increase in resource demand will actually decrease intermediaries usage, making them substitutes. This of course holds only as long as prices both for intermediaries and the resource have fixed schedules, that is, $r_h = r_f$. Otherwise the home economy would adapt its interest rate to the world one, making intermediaries cheaper.

From the (21) it follows, that under open trade the demand for intermediate products as a substitute for resource is higher, since the resource demand is lower. This result is in tandem with a widely held view in literature that introducing new intermediate goods leads to an increase in total factor productivity(TFP) and causes growth. Thus the opening of the domestic market leads to the switch in the mix of factors being used for production, boosting technical change. This last is reflected in higher levels of Q_{hj}, since the price ratio between the resource and intermediaries becomes more favourable for the latter.

Proposition 4 (**Structural change under open trade**).

When the resource abundant country opens the trade, then:
For $\eta < 1$*:*

1. *If* $r_h > r_f$ *the demand for intermediate products increases, boosting technical change. through cheaper capital;*

2. *If* $r_h < r_f$ *the opposite effect occurs, making intermediaries relatively more expensive and the economy more resource oriented;*

3. *If* $r_h = r_f$ *intermediaries usage may decrease at home, following the resource demand.*

It is interesting to note that it is not always good for technical change in the home country to open up trade. The effect would depend on the relative price of capital at home and abroad and on the productivity of technology. If capital at home is scarce which is often the case for developing countries, structural change may be ignited by the change in relative price of technology and resource based factors. However if the technology is more productive, the effect would be reversed, since additional revenue will increase resource expenditures and not the intermediaries demand. However this can be the case only for $\eta > 1$, which means collapse of the home economy without technical change. Thus we restrict ourselves only to the case of $\eta < 1$.

This may be demonstrated by direct comparison of evolution of intermediate demand (21) under both regimes. The ordering would follow for the case of $R_h^A > R_h^O$ in the steady state.

Proposition 5 (**Hartwick's rule in the economy under open trade**).

Due to Proposition 1 Hartwick's rule does not always hold in the opened economy: not all resource rents are re-invested into the capital accumulation.

Proof follows from the fact that consumption expenditures may increase after opening trade, $E_h^O \geq E_h^A$, both under decrease and increase of final price, depending on the taste for foreign goods , $1 - \phi$ parameter. Then it is possible to find such a configuration of parameters space, $\phi, \rho, \eta, \gamma, r$ that simultaneously

$$E_h^O > E_h^A,\, p_R R^O \leq p_R R^A \tag{53}$$

hold. This is sufficient for Hartwick's rule to fail.

Observe that we do not discuss here how *generic* is the situation of Hartwick's rule breakage. This would require much more detailed simulations and parametric analysis. However some raw estimations point to the key role of elasticity ϕ. The more home consumers like foreign products, the more likely expenditures will increase higher, than resource revenues. This is indeed the case for many developing countries: taste for foreign products is high, and demand for resource is low since the rest of the world is more technologically advanced. Thus we claim that according to our analysis, the open trade may benefit home country only if it is not too much backwards and there is some support of home produced products.

5 Extensions

The model allows for immediate extension of results on arbitrary n of resource-importing countries. The extension of the number of exporting countries is non-trivial, since if one allows for strategic interactions between them, the cartel collision behaviour may be observed.

Some more elaborated extensions may be considered also. First, it is interesting to study the effects of price discrimination for the resource monopoly, as it is frequently the case: home producers enjoy lower prices than foreign ones. In our setup such a discrimination may indeed boost home country revenues and thus capital accumulation, but the decrease in foreign output would drive prices for the final product higher and decrease the utility of households. Thus the price discrimination should be treated with caution while might be implemented if the difference in home and foreign technologies, represented by the level of available varieties is big enough.

Second, the process of technical change need not to be restricted to the growth in the level of usage of existing intermediaries. One could also assume dynamic character of N_h, N_f in the spirit of Peretto and Connolly (2007), Belyakov et al. (2011) or Bondarev (2012). However this extension should require rather challenging analytical elaboration.

Third, the limited capacity of the resource seems to play no role in the current model, because the time of extraction is not optimized upon by the resource monopolist. However this time of full exploitation should play an important role in the decision of how much resource to export to foreign country and at which price. Moreover the market structure of the resource should be paid more attention. The existence of oligopolistic structure is common for resource abundant economies and is a matter of ongoing debates. This structure may be modelled by the means of usual dynamic oligopoly with the application of the same Differential games apparatus.

6 Conclusion

In this paper we have studied the consequences of opening up to trade in a resource abundant economy,with particular attention on technical change and capital accumulation. Low income resource abundant countries tend to favour foreign direct investment(FDI) as a driver of technological advancement, e.g through technological spillovers. These countries even engage in fiscal policy incentive based competition to attract FDI. In this paper we posit that technological improvements may take place via trade with developed countries which we term technological leaders, the main process being learning by doing on the less technologically advanced resource abundant countries. This approach is different from endogenous growth models that study an isolated country and do not take into account trade relations.

We postulate three channels of gains for the resource abundant country:

1. The variety gains: Variety gains have been investigated extensively in new-trade models of monopolistic competition annd models measuring gains from trade and FDI, see for example(Melitz (2003), Arkolakis et al. (2012)). Since consumers have access to both domestic and new import (foreign) varieties, world equilibrium price goes down, and this was shown in our model to be lower than autarky price. In a similar frame, Melitz and Ottaviano (2008) contended that , larger markets exhibit tougher competition resulting in lower average mark-ups and higher aggregate productivity.

2. Technological change: technological improvements spread from the technological leader via learning effects, when goods produced with better and advanced methods, processes and technologies are traded to the technological "laggers". Every time intermediate firms introduce intermediate goods used in producing final goods this increases total factor productivity and economic growth.

3. Capital accumulation: we argue that larger initial exhaustible resource stocks leads to increased capital stock in the long run for the resource abundant country since they receive trade proceeds of the exhaustible resources to the foreign countries. Larger revenues may in turn be used as FDI replacement to stimulate consumption, this is in the same spirit as Asheim (1986) who asserted that resource-rich economy can allow itself to use revenues arising from resource exports to finance additional consumption.

However there are situations when Hartwick's rule fails and thus opening trade will be harmful for the home economy. The more is foreign bias in consumer tastes, the more likely this can happen.

The policy implications and recommendations for the developing country are:

1. To boost the technological transformation of the home industry it might be more beneficial to trade the resource rather than seek FDI and implement fiscal policies. As in Chalk (1998) and Gelb (1988) governments with non-renewable resources could spend on investment goods. As this was often the case in oil producing countries, such revenues increased future outputs, generated non-oil revenue and reduced future deficits. IMF (2013) report on natural resource wealth for sub-Saharan Africa pointed out that the fiscal framework to be employed should be clear on the role of resource revenue, recognizing its exhaustibility and volatility, and institutional rules and structures that promote fiscal sustainability;

2. The opening up to trade may boost home direct investments through faster capital accumulation and will benefit consumers without harming domestic industry. The higher is the current resource stock, the more beneficial such a policy may be.

3. Investing in R&D in extraction technology might offset the depletion of todays resources. Stuermer and Schwerhoff (2013) found out in their study that resource stock may be increased through R&D investment in extraction technology, they argue that even if non-renewable resource use and production increase exponentially, resource prices might stay constant in the long term. In the same spirit, WTO (2010) reported that allowing for technological change in the extractive sector can effectively increase the supply of resources by contributing to new discoveries and allowing extraction of stocks that could not be reached before.

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Appendices

A Derivation of the Euler Equation

First derive the Indirect Utility

Lets rewrite the utility $U(C)$ as

$$U(C_{i,i}, C_{-i,i}) = \ln\left(\varphi C_{ii}^{\rho} + (1-\varphi)C_{-i,i}^{\rho}\right)^{\frac{1}{\rho}}, \; i \in \{h, f\} \tag{A.1}$$

The consumers' budget constrained choice problem has the Langragian

$$\mathcal{L} = U(C_{ii}, C_{-ii}) - \lambda\left(E_i - pC_{ii} - pC_{-ii}\right) \tag{A.2}$$

The demand functions then satisfy the familiar tangency condition:

$$\frac{MRS_{ii}}{MRS_{-ii}} = \frac{\varphi C_{ii}^{\rho-1}}{1-\varphi C_{-ii}^{\rho-1}} = \frac{p_{ii}}{p_{-ii}} = 1 \tag{A.3}$$

Solving the equation for C_{-ii} in terms of C_{ii} we get:

$$C_{-ii} = C_{ii}\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi} \tag{A.4}$$

Remember that $\chi = \frac{1}{1-\rho}$, then substituting C_{-ii} into the expenditures we get

$$pC_{ii} + pC_{-ii} = pC_{ii} + pC_{ii}\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi} = E_i \tag{A.5}$$

$$C_{ii} = \frac{E_i}{p + p\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi}} = \frac{(p/\varphi)^{-\chi}}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}E_i \tag{A.6}$$

By symmetry the demand function for the foreign goods consumed at home is given by

$$C_{-ii} = \frac{(p/(1-\varphi))^{-\chi}}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}E_i \tag{A.7}$$

The corresponding indirect utility function is therefore given by

$$\overline{\mathcal{U}} = \frac{E_i}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}} \tag{A.8}$$

$$\max_{C} \int_0^{\infty} \mathrm{e}^{-\rho t} \ln\left[\frac{E_i}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}\right] dt; \tag{A.9}$$

subject to (1) for the home country and to (2) for the foreign country. Optimality conditions give the standard Euler equation in which at each moment in time the rate of growth of expenditures on consumption is equal to the difference between the instantaneous interest rate and the rate of time preference.

$$\frac{\dot{E}_i}{E_i} = r_i - \rho,\ i \in \{h, f\}. \tag{A.10}$$

Assuming equal interest rates across countries, $r_h = r_f$ yields equal growth rates of expenditures.

B Derivations for final producers

The (current value) Hamiltonian functions for final producer in country i is:

$$\mathcal{H}_i = pY_i - \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - p_R R_i - \lambda_i \left(s \left[a - p - Y_i(t) - Y_{-i}(t) \right] \right), \tag{B.1}$$

substituting in it for Y_i from (6) and normalizing $s = 1$ for simplicity

$$\mathcal{H}_i = p \left[R_i^\gamma \int_0^{N_i} Q_{ij}^\eta dj \right] - \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - p_R R_i + \lambda_i \left[a - p - \left(R_i^\gamma \int_0^{N_i} Q_{ij}^\eta dj \right) - \left(R_{-i}^\gamma \int_0^{N_{-i}} Q_{-ij}^\eta dj \right) \right] \tag{B.2}$$

and then for Q_{ij} from (21) one may write down explicitly Hamiltonian as a function of resource for both firms :

$$\mathcal{H}_i = p \left[R_i^\gamma \int_0^{N_i} \left(\left[\frac{\eta}{p_Q} R_i^\gamma \right]^{\frac{1}{1-\eta}} \right)^\eta dj \right] - \int_0^{N_i} p_{Q_{ij}} \left(\left[\frac{\eta}{p_Q} R_i^\gamma \right]^{\frac{1}{1-\eta}} \right) dj - p_R R_i + \lambda_i \left[a - p - \left[R_i^\gamma \int_0^{N_i} \left(\left[\frac{\eta}{p_Q} R_i^\gamma \right]^{\frac{1}{1-\eta}} \right)^\eta dj \right] - \left[R_{-i}^\gamma \int_0^{N_{-i}} \left(\left[\frac{\eta}{p_Q} R_{-i}^\gamma \right]^{\frac{1}{1-\eta}} \right)^\eta dj \right] \right]. \tag{B.3}$$

Assuming similar prices for all the intermediates, $p_{Q_{ij}} = p_Q$ we can integrate over and obtain

$$\mathcal{H}_i = pR_i^{\frac{\gamma}{1-\eta}} \left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} N_i - R_i^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^{\frac{1}{\eta}}}{p_Q} \right)^{\frac{\eta}{1-\eta}} N_i - p_R R_i + \lambda_i \left[a - p - R_i^{\frac{\gamma}{1-\eta}} \left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} N_i - \right. \tag{B.4}$$

$$\left. R_{-i}^{\frac{\gamma}{1-\eta}} \left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} N_{-i} \right]. \tag{B.5}$$

Taking F.O.C for resource demand:

$$\frac{\partial \mathcal{H}_i}{\partial R_i} = \frac{\gamma}{1-\eta} pR_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} - \frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} \left(\frac{\eta^{\frac{1}{\eta}}}{p_Q} \right)^{\frac{\eta}{1-\eta}} N_i - p_R + \lambda_i \left[-\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} \right] \tag{B.6}$$

Equating the derivative to zero we get resource demand as a function of prices:

$$p_R = \frac{\gamma}{1-\eta} R^{\frac{\gamma}{1-\eta}-1} N_i \left[p \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - \lambda_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - \left(\frac{\eta^{\frac{1}{\eta}}}{p_Q}\right)^{\frac{\eta}{1-\eta}} \right],$$

$$p_R = \frac{\gamma}{1-\eta} R^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} [(p-\lambda_i)-\eta] \tag{B.7}$$

$$R_i^D = \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}} \left(\frac{(p-\lambda_i)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_i^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \tag{B.8}$$

which is (24). Expressing λ_i from (24) through resource demands yields

$$\lambda_i = p - \frac{p_R}{\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}} - \eta \tag{B.9}$$

giving

$$\dot{\lambda}_i = \dot{p} - \frac{\dot{p_R} \frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - p_R \frac{\gamma}{1-\eta}\left(\frac{\gamma}{1-\eta}-1\right) R_i^{\frac{\gamma}{1-\eta}-2} \dot{R}_i N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}}{\left(\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^2} \tag{B.10}$$

Costate equations are given by:

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial \mathcal{H}_i}{\partial p} = (r+1)\lambda_i - R_i^{\gamma \frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i \tag{B.11}$$

Given resource demand, (24), the evolution of shadow costs is defined by prices and itself:

$$\dot{\lambda}_i = (r+1)\lambda_i - \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}} \left(\frac{(p-\lambda_i)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_i^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} N_i, \tag{B.12}$$

and the same for λ_{-i}. At the same time we may make use of (B.9) and (B.10) to express dynamics of resource demand for each country from (B.11):

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial \mathcal{H}_i}{\partial p} = (r+1)\lambda_i - R_i^{\gamma \frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i =$$

$$(r+1)\left(p - \frac{p_R}{\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}} - \eta\right) - R_i^{\gamma \frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i,$$

$$\left(R_i^{\gamma \frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i = Y_i,\ \frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} = \frac{\partial Y_i}{\partial R_i}\right), \tag{B.13}$$

$$\dot{p} - \frac{\dot{p_R}\frac{\partial Y_i}{\partial R_i} - p_R \frac{\partial^2 Y_i}{\partial^2 R_i}\dot{R}_i}{\left(\frac{\partial Y_i}{\partial R_i}\right)^2} = (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) - Y_i,$$

$$\left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\left(\frac{\partial Y_i}{\partial R_i}\right)^2 = \dot{p_R}\frac{\partial Y_i}{\partial R_i} - p_R \frac{\partial^2 Y_i}{\partial^2 R_i}\dot{R}_i,$$

$$\dot{R}_i = \frac{\dot{p_R}\frac{\partial Y_i}{\partial R_i} - \left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{p_R \frac{\partial^2 Y_i}{\partial^2 R_i}},$$

$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}},$$

$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left((a - p - Y_i - Y_{-i}) - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}$$

$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left((a - p - Y_i - Y_{-i}) - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}} =$$

$$\frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(a - Y_{-i} - (2+r)p + (1+r)\frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}} =$$

$$\left(\frac{\dot{p_R}}{p_R} - (1+r)\right)\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(a - Y_{-i} - (2+r)p\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}. \tag{B.14}$$

Yielding the resource demand dynamics as a function of itself, final price and outputs of both producers (which are in turn also functions of resource demand).

Evolution of the price itself is:

$$\dot{p} = \left(a - p - R_i^{\gamma}\int_0^{N_i} Q_{ij}^{\eta} dj - R_{-i}^{\gamma}\int_0^{N_{-i}} Q_{-ij}^{\eta} dj\right) \tag{B.15}$$

C Resource price determination

The total demand for resource is

$$R = \left(\frac{\gamma}{1-\eta}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{(p-\lambda_h)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_h^{1-\frac{\gamma}{1-\eta}} + \left(\frac{(p-\lambda_f)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_f^{1-\frac{\gamma}{1-\eta}}\right) \tag{C.1}$$

The optimization for resource monopolist is carried out with respect to price of the resource, since demand is given and no market segmentation is assumed. Current value Hamiltonian is:

$$\mathcal{H}^R = p_R R - \lambda_R R \tag{C.2}$$

with R given by (C.1). Differentiation w. r. t. p_R yields price proportional to shadow costs of extraction:

$$\frac{\partial \mathcal{H}^R}{\partial p_R} = R + p_R \frac{\partial R}{\partial p_R} - \lambda_R \frac{\partial R}{\partial p_R} = 0, \tag{C.3}$$

with derivative of resource demand proportional to the demand itself

$$\frac{\partial R}{\partial p_R} =$$
$$-(1-\frac{\gamma}{1-\eta})\frac{1}{p_R}\left(\frac{\gamma}{1-\eta}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{(p-\lambda_h)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_h^{1-\frac{\gamma}{1-\eta}} +\right.$$
$$\left.\left(\frac{(p-\lambda_f)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_f^{1-\frac{\gamma}{1-\eta}}\right) =$$
$$-(1-\frac{\gamma}{1-\eta})\frac{1}{p_R}R,$$

we define the price of the resource:

$$-(1-\frac{\gamma}{1-\eta})R(1-\lambda_R/p_R) + R = 0, \tag{C.4}$$

$$\frac{\lambda_R}{p_R} \overset{R\neq 0}{=} \frac{\gamma}{\eta+\gamma-1}. \tag{C.5}$$

The dynamics of the co-state is:

$$\dot{\lambda_R} = r\lambda_R - \frac{\partial \mathcal{H}^R}{\partial S} = r\lambda_R. \tag{C.6}$$

transversality condition is given by

$$\lim_{t\to\infty} p_R(t)R(t)e^{-\int_0^t r(\tau)d\tau} = 0 \tag{C.7}$$

Thus the growth of resource price is proportional to the interest rate:

$$\dot{p_R} = \frac{\eta+\gamma-1}{\gamma}\dot{\lambda_R} = r\frac{\eta+\gamma-1}{\gamma}\lambda_r \rightarrow \frac{\dot{p}_R}{p_R} = r\frac{\eta+\gamma-1}{\gamma}. \tag{C.8}$$

giving (29).

D Intermediate good price determination

The producer maximizes his profit (we assume $\xi = 1$):

$$\max_{p_{Q_{ij}}} \pi_{Q_{ij}}(Q_{ij}) = p_Q Q_{ij} - rK_{ij} \tag{D.1}$$

subject to

$$Q_{ij} = K_{ij}; j \in [0 : N_i]. \tag{D.2}$$

$$\frac{d\pi}{dQ} = \eta^2 R^\gamma Q_D^{\eta-1} - r = 0$$

$$r = \eta^2 R^\gamma Q_D^{\eta-1}$$

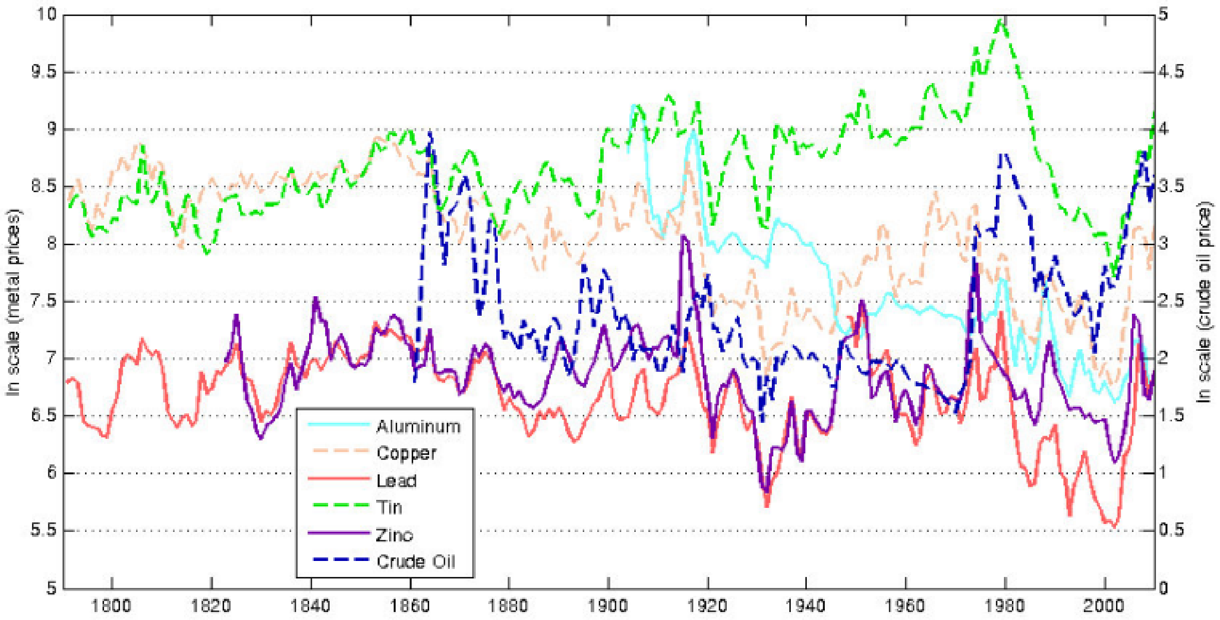
$$Q_D = \left(\frac{\eta^2}{r} R^\gamma\right)^{\frac{1}{1-\eta}}$$

$$p_{Q_i} = \eta R^\gamma \left(\left(\frac{\eta^2}{r} R^\gamma\right)^{\frac{1}{1-\eta}}\right)^{\eta-1}$$

$$p_{Q_i} = \frac{r}{\eta} \tag{D.3}$$

E Prices of Major Mineral Commodities

Figure 3: Real prices of major mineral commodities from 1790 to 2009 in natural logs.



Source:Stuermer (2013)