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Environmental pollution in a growing economy with endogenous structural change

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Abstract

In this paper we study the impact of environmental pollution in an endogenous growth model that allows for structural change. The model is based on doubly-differentiated R&D where newer, less polluting technologies gradually replace older ones. The analysis shows that the presence of environmental externalities stimulates structural change but reduces the growth rate of the economy. Further, comparing the models with and without structural change demonstrates that the latter implies stronger environmental damages and, consequently, a lower growth rate than the first one. Finally, levying a tax on the polluting output speeds up structural change, thus, reducing environmental pollution and spurring economic growth. This can give new support for the double dividend hypothesis.

JEL classification: Q55, O31, O44

Keywords: Environmental Damages, Endogenous Growth, Creative Destruction, Endogenous Structural Change, Double Dividend Hypothesis

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1 Introduction

The concept of Green Growth as termed by OECD rests on the assumption that the combination of positive economic growth rates and the switch to green technologies is feasible. Thus, it is of interest to formally analyze how this Green Growth can be implemented. One way to achieve this is to use progressively cleaner technologies in production while dropping older dirtier ones. However, since newer technologies need time to be developed the speed of such transformation of the economy is subject to discussion. One example is the transition to renewable energy generation being implemented in some OECD countries. While all the parties agree with the associated environmental benefits, sometimes it is argued that too fast a transition may be harmful for growth.

In this paper we present a formal model of the transition to progressively cleaner technologies where positive economic growth goes along with a decrease in environmental damages, associated with industrial production. This happens despite the fact that newer technologies have zero productivity at the start. Positive growth is achieved through more abundant financial assets of the economy, which allow a faster development of newer technologies compared to older ones.

The question of how economic evolution and environmental degradation are interrelated has a long tradition in economics. Seminal work in this field has been undertaken by Forster (1973), Mäler (1974) or Gruver (1976), for example. Forster analyzed the Ramsey growth model where environmental pollution occurs as a by-product of capital accumulation and can be reduced by abatement spending. He shows that this model is characterized by a stationary state in the long-run with all variables being constant, unless the economy is hit by an exogenous shock. Mäler analyzes several aspects associated with environmental degradation in different frameworks, such as a general equilibrium model and a model of economic growth with environmental damages. However, in contrast to Forster, he is less interested in the long-run evolution of the economy but assumes a finite time horizon.

With the emergence of endogenous growth theory in the late 1980's and 1990's, the research focus has moved to the interrelation between environmental policies, such as taxes and quotas, on the one hand, and the long-run growth rate and welfare, on the other hand. Examples of such studies are Bovenberg and Smulders (1995), Smulders and Gradus (1996), Greiner (2005) or Grimaud and Rougey (2014).[1] In those papers, the economy is characterized by ongoing growth with the long-run growth rate being an endogenous variable. That property results from the fact that the marginal product of capital does not decline as capital grows which, for its part, may be a result of human capital accumulation, of the creation of new technologies or from productive public investment, for example. However, to our knowledge none of those contributions deals with the relationship between environmental pollution and endogenous strucural change in a growth context.

In this paper we analyze the effects of environmental pollution within an endogenous growth model allowing for structural change that results from the introduction of new technologies that make old ones obsolete, giving rise to creative destruction as already described by Schumpeter (1942). Starting point of our analysis is the model without environmental pollution presented in Bondarev and Greiner (2016). There, new technologies are permanently developed as a result of R&D investment replacing old technologies. Simultaneously, existing technologies are improved through vertical innovations as in the seminal paper by Aghion and Howitt (1992). Newer technologies have a higher productive potential and, therefore, can attain a higher productive efficiency although initially all new technologies are identical, as in the model by Peretto and Conolly (2007).

We take up the benchmark model by Bondarev and Greiner (2016) and extend this model by assuming that goods production implies negative environmental externalities that are a purely public good (or bad) that exerts a negative impact on the production of each sector in the economy. Further, the emissions intensity of each new technology is smaller than the one of the preceeding technolgy implying that newer technologies are less polluting than older ones. Our goal, then, is to compare the effects of environmental degradation in the growth model with structural change to those obtained in a model

[1]For a survey, see also the book by Greiner and Semmler (2008).

without structural change. Further, we analyze the effects of environmental pollution by contrasting the benchmark model, where environmental considerations are absent, with the model including environmental damages. Finally, we integrate an ad-valorem tax on revenues of the manufacturing firm, with the tax rate equal to the emissions intensity, and study its effects on the growth rate of the economy and its implications with respect to the environment.

The rest of the paper is organized as follows. The next section briefly presents the structure of the growth model and shows how the environment has been integrated into the benchmark model. Section 2.3 gives the solution of the model and section 3 derives the impacts of environmental pollution. Section 4, finally, concludes.

2 The growth model with environmental pollution

We briefly describe the structure of the growth model with environmental degradation. For more details concerning the model without the environment, which serves as the benchmark model, the reader is referred to Bondarev and Greiner (2016). A condensed exposition of this model can be found in the Appendix.

2.1 The economy

The economy is decentralised with a household sector, a productive sector and a R&D sector that invests in horizontal and vertical innovations. The representative household maximizes[2]

$$J^H = \int_0^\infty e^{-\rho t} \ln C \, dt, \tag{1}$$

with ρ the discount rate and C a continuum of differentiated products from existing sectors,

$$C = \left[\int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

with ε the elasticity of substitution between goods and N_{max} is the range of manufacturing sectors with positive operating profit and N_{min} is the range of sectors, which

[2]We delete the time argument t as long as no ambiguity arises.

disappeared from the economy up to time t. The range of developed sectors is growing over time reflecting the expansion in the variety of products. However, the range of existing sectors, given by $N_{max} - N_{min}$, may grow decrease or stay constant in time, depending on the characteristics of the process of variety expansion of technologies, $\dot{N}$, with N the total number of technologies that have been invented up to time t. The budget constraint of the household is given by,

$$\dot{a} = ra + \int_{N_{min}}^{N_{max}} L_i di - E, \tag{3}$$

with E denoting consumption expenditures, a assets, r return to assets, the wage rate serves as the numéraire , $w \equiv 1$, and $\int_{N_{min}}^{N_{max}} L_i di$ is total (employed) labor, bounded by the (exogenously fixed) labor force L, normalized to 1:

$$\int_{N_{min}}^{N_{max}} L_i di \leq L = 1. \tag{4}$$

Environmental spillovers, however may prevent the economy to operate at full employment level, hence inequality sign.

Expenditures are given by,

$$E = \int_{N_{min}}^{N_{max}} P_i C_i di, \tag{5}$$

with P_i the price of good i.

The solution of this optimization problem leads to the standard Euler equation,

$$\frac{\dot{E}}{E} = r - \rho. \tag{6}$$

For details see Appendix A.

The market form of the manufacturing sector is characterized by monopolistic competition where firms produce different goods, Y_i, with the help of a patented technology i from the available spectrum. Firms use technology, A_i, and labor for production and there is a negative effect from environmental pollution that is a pure public good (or bad),

$$Y = \int\limits_{N_{min}}^{N_{max}} Y_i di,\ Y_i = \left(\frac{1}{1+T}\right) A_i^{\alpha} L_i, \tag{7}$$

with $\int Y_i di$ aggregate output and T gives environmental damages, with $T = 0$ standing for the unpolluted state of the nature.[3] Profits of firms in the manufacturing sector are,

$$\Pi_i = P_i Y_i - L_i - \Psi, \tag{8}$$

with Ψ a fixed operating cost. Profit maximization of firms, then, determines prices and labor demands in a standard way as in the benchmark model, see Appendix B.

The technology is described by vertical and horizontal innovations undertaken by the R&D sector with investments set optimally by R&D firms. There are two types of R&D. Productivity-improving innovations and variety-expanding innovations. Both types of R&D use only financial capital as the only input in a proportional fashion as in the basic model of Romer (1990) with proportionality factor set to one for simplicity. Thus the total sum of both kinds of R&D investments at any time forms the demand for assets:

$$u(t) + \int\limits_{N_{min}(t)}^{N(t)} g(i,t) di = a^D(t) \tag{9}$$

where

- $u(t)$ are horizontal innovations investments at time t;
- $g(i,t)$ are vertical innovations investments at time t for technology i within the range of invented and not outdated technologies, $[N_{min}(t), N(t)]$;
- $a^D(t)$ is the total demand for assets.

The dynamics of vertical, A_i and horizontal, N innovations is represented by linear

[3]For example, T could be interpreted as the deviation of the average surface temperature on earth from its pre-industrial level.

processes:

$$\dot{A}_i = \gamma g_i - \beta A_i; \tag{10}$$

$$\dot{N} = \delta u. \tag{11}$$

The R&D activities are unaffected by the state of the environment and are identical to those in the benchmark model, see Appendix C for details. Hence, the overall influence of the environment on the economy consists solely in the symmetric reduction of output of all existing sectors in this economy.

2.2 The environment

The natural environment is affected by aggregate output in a usual fashion, as in Bréchet et al. (2011) for example,

$$\dot{T} = -\mu T + eY, \tag{12}$$

where:

- T is some aggregate measure of the environment (deviation from the average global surface temperature);
- μ is the regeneration rate of the environment;
- e is the intensity of emissions, defined by the state of technology;
- Y is the aggregate output of the economy.

The intensity of emissions is a function of an effective mix of technologies being used for production at a given point in time.

We assume that each of the technologies has a different intensity of emissions or environmental impact. In fact we condense the usual two-equations form of environmental spillover (output produces carbon emissions due to the fact of usage of fossil fuels and the GHG concentration affects temperature increase, which in turn decreases overall production efficiency) into one for simplicity. However our main focus is to analyze the influence of emissions intensity e on environmental system and thus we assume that there is one-to-one correspondence between output and GHG concentration, neglecting possible regenerative capacity of the atmosphere. Indeed, this regenerative capacity is much smaller than the industrial influence as reported by IPCC, for example and in our qualitative analysis this may be omitted.

For simplicity we assume a hyperbolic decrease of the emissions intensity across the space of technologies (since a linear decrease is not applicable to the unrestricted space $\mathbf{N}$):

$$\forall i \in \mathbf{N} : \ \iota(i) = 1/i; \tag{13}$$

where $\iota(i)$ is the function of the environmental impact for technology i.

We thus assume newer technologies to be cleaner than older ones and treat technical progress as the one directed on cleaner technologies. Indeed, manufacturing technologies being developed recently are more energy efficient and energy generation capacity mix is moving towards cleaner (renewable and nuclear) sources. Thus we analyze the scenario of "green" transformation of the economy.

Observe that in (7) A_i is the state of technology in sector i, increasing this sector's output and environmental damages in turn. However, every sector has limited operational time and each new sector has lower emissions intensity e_i. Thus, there are two opposite channels how economy influences environment in (12): higher productivity leads to higher output and thus higher damages, but faster technical change replaces older technologies with cleaner ones, reducing damages.

The overall environmental impact of the actual technology mix includes the fraction of output being generated with the use of a certain technology. Such a function cannot be computed without knowing the evolution of output of every sector and is done next. Thus, the total emissions intensity is formulated as,

$$e(t) = e_0 \frac{\int_{N_{min}}^{N_{max}} (1/i) y_i di}{\mathcal{O}} \tag{14}$$

where y_i is the share of each technology output relative to total output:

$$y_i = \frac{Y_i}{Y}, \tag{15}$$

with output given by (7). The overall dynamics of the joint system, then, is given by:

- Assets accumulation, (3);
- Productivities evolution for each sector, (10);
- Expansion of a variety of technologies, (11);
- Evolution of the environment, (12);
- Evolution of emissions from all existing sectors, (14).

2.3 Solution of the model

The solution procedure follows the same steps as for the benchmark model: we derive R&D dynamics, then prices, labor demand evolution and household expenditures, resulting in assets dynamics and output for each sector. At last we derive environmental degradation associated with the economy. We denote with the superscript O the quantities associated with no-pollution case.

2.3.1 R&D sector

Horizontal innovations are linear and proportional to the expected profit of the next technology:

$$u^* = \delta \pi^R(i)|_{i=N}. \tag{16}$$

Since the technologies are homogeneous, the expected profit is the same for all technologies and the variety expansion is a linear function of time:

$$N(t) = \delta^2 \pi^R t + N_0. \tag{17}$$

The obsolescence of technologies and the entrance of new technologies into the profitable phase are also linear processes yielding a constant size of the core $\mathcal{O}$ similar to the benchmark model:

$$N_{min} = \delta^2 \pi^R (t - t_{min}) + N_0; \tag{18}$$

$$N_{max} = \delta^2 \pi^R (t - t_{max}) + N_0, \tag{19}$$

with

- $t_{min} = N_{min}^{-1}(i)$, the time when product (technology) i becomes outdated;
- $t_{max} = N_{max}^{-1}(i)$, the time when product (technology) i becomes profitable;
- $t_0 = N^{-1}(i)$, the time when product (technology) i is invented.

Optimal R&D investments for each sector are proportional to the assets stock minus horizontal investments [4]:

$$g_i^* = \frac{a - u}{N - N_{min}}. \tag{20}$$

Details are to be found in Appendix C for R&D derivations and Appendix D for constant core proof.

2.3.2 Markets clearing

Recall that the output of each sector is affected uniformly by environmental pollution, see Eq. (7). Prices and labor demand are proportional to no-pollution case (see Appendices A, B):

$$\begin{aligned} P_i &= \left(\frac{\epsilon}{\epsilon - 1}\right) A_i^{-\alpha}(1+T) = (1+T) P_i^O, \\ L_i &= E\left(\frac{\epsilon - 1}{\epsilon}\right)\left(\frac{1}{1+T}\right) \frac{A_i^{-\alpha(1-\epsilon)}}{\int_{N_{min}}^{N_{max}} A_j^{-\alpha(1-\epsilon)} dj} = \left(\frac{1}{1+T}\right) L_i^O, \end{aligned} \tag{21}$$

[4]we make use of the resource constraint (9) and capital market clearing condition $a^D = a^S$ here

where P_i^O is given by (B.9), L_i^O is given by (B.10) leaving expenditures unchanged relative to the benchmark model:

$$E = \int_{N_{min}}^{N_{max}} P_i Y_i di = \frac{\epsilon}{\epsilon - 1} \int_{N_{min}}^{N_{max}} \left(A_i^{-\alpha}(1+T) \frac{L_i A_i^{\alpha}}{1+T} \right) di = \left(\frac{\epsilon}{\epsilon - 1} \right) L = E^O. \tag{22}$$

with E^O given by (B.12).

However, the total labor income changes since the total output of the economy is lower because of the environmental degradation. Consider the labor market clearing condition:

$$L^E = \int_{N_{min}}^{N_{max}} L_i di = \frac{1}{1+T} \int_{N_{min}}^{N_{max}} L_i^O di = \left(\frac{1}{1+T} \right) L < L. \tag{23}$$

where L^E denotes the total employed labor under environmental spillover.

The assets dynamics is thus obtained by substituting (23) into (3):

$$\dot{a} = ra - E + \frac{1}{1+T} \tag{24}$$

2.3.3 Output and environment

The state of the environment depends on output and on the technology mix. We start with computing the share of each technology in total output. The output of each individual sector is given by,

$$Y(i,t) = \left(\frac{1}{1+T} \right) \frac{A(i,t)^{\alpha\epsilon}}{\int_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}} \tag{25}$$

and the fraction of output of each (operational) technology is:

$$y_i = \frac{Y_i}{Y} = \frac{A(i,t)^{\alpha\epsilon} \left(\int_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)} \right)^{-1}}{\int_{N_{min}}^{N_{max}} Y(i,t) di} = \frac{A(i,t)^{\alpha\epsilon}}{\int_{N_{min}}^{N_{max}} A(i,t)^{\alpha\epsilon} di}, \tag{26}$$

where $A(i,t)$ is the productivity level of technology i at time t.

Thus, the evolution of the environment can be expressed as a function of productivities and of the environment:

$$\dot{T} = -\mu T + \frac{\int_{N_{min}}^{N_{max}} (1/i) y_i di}{\mathcal{O}} Y(t) = \frac{1}{\mathcal{O}} \left(\frac{1}{1+T} \right) \frac{\int_{N_{min}}^{N_{max}} \left(\frac{1}{i}\right) A(i,t)^{\alpha\epsilon} di}{\int_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}} - \mu T =$$

$$= \frac{1}{\mathcal{O}} \left(\frac{1}{1+T} \right) \int_{N_{min}}^{N_{max}} \left(\frac{1}{i} \right) A(i,t)^{\alpha} di - \mu T. \tag{27}$$

Equation (27) shows that the larger the operational range of technologies $\mathcal{O}$ (core) is, the lower is the environmental impact in the economy. The economic intuition behind this fact is as follows: the higher the range of technologies, the lower is the fraction of output produced by each of them and, consequently, the lower is the share of dirty technologies. Since the assets are distributed evenly across all technologies (they are homogeneous in this respect), a rise in the operating range of technologies is always shifting assets usage towards cleaner technologies, thus raising the relative share of less polluting sectors.

Next, since output is affected identically by the environment in all sectors, the output growth with environmental pollution is

$$\left(\dot{Y}/Y\right)^{E} = \alpha \left(\frac{1}{1+T} \right) \frac{\dot{\bar{A}}}{\bar{A}} (N_{max} - N_{min}) - \frac{\dot{T}}{(1+T)^2}, \tag{28}$$

where the first component is the same as in the benchmark model (see Appendix E) multiplied by $1/(1+T)$ and the second is determined by environmental degradation given in (27).

A full analytic solution for our model economy cannot be obtained, but we can analyze the behaviour of its main variables compared to the model with a fixed range of sectors, i.e. without horizontal innovations, to illustrate the impact of structural change on the environment and to the model with structural change but without pollution, to illustrate the impact of environmental spillover. That is done in the next section.

3 Analysis of the model

3.1 Comparison with the model without structural change

First, we compare the evolution of the environment with and without structural change. The economy without structural change is identical to the one with structural change but operates with a fixed range of sectors. This implies that newer technologies just replace older ones as in the quality ladders model of Aghion and Howitt (1992) and $\dot{N} = 0$. Without horizontal innovations the emissions intensity in the economy, $e(t)$, is constant and determined by the existing composition of the technology.

It is straightforward to see that without structural change there is no slowdown of environmental degradation in the economy at all. Consider the differential equation describing environmental degradation for the case where all technologies grow at the same speed as the average technology $\bar{A}$, i.e. $\dot{A}_i = \dot{\bar{A}}$. The rate of environmental degradation is then determined by the state of the average technology $\bar{A}$:

$$\dot{T} = \frac{1}{\mathcal{O}}\left(\frac{1}{1+T}\right)\int\limits_{N_{min}}^{N_{max}}\left(\frac{1}{i}\right)A(i,t)^{\alpha}di - \mu T = \frac{\bar{A}^{\alpha}\ln(N_{max}/N_{min})}{\mathcal{O}^2(1+T)} - \mu T, \tag{29}$$

In the economy without structural change the term $\ln(N_{max}/N_{min})$ is constant, while under structural change it decreases over time such that the emissions intensity declines. To see this, consider that

$$\begin{aligned}\frac{d}{dt}\ln(N_{max}/N_{min}) &\overset{\dot{N}=0}{=} 0,\\ \frac{d}{dt}\ln(N_{max}/N_{min}) &\overset{\dot{N}>0}{<} 0.\end{aligned} \tag{30}$$

since $\dot{N} = \dot{N}_{max} = \dot{N}_{min}$ under linear variety expansion.

Next, consider assets accumulation and productivity growth. Since environmental degradation is less drastic in the economy with structural change, assets accumulation is faster. Given a higher total stock of financial capital, the available assets are larger.

Therefore, productivity growth will also be higher.[5] Thus, we can establish

Proposition 1 (Effects of the structural change)

In the economy with endogenous structural change the following holds true:

1. *The environmental degradation is slower than in the economy with a constant range of technologies;*

2. *The economy exhibits a higher assets accumulation and a higher productivity growth because of lower environmental damages.*

To illustrate that proposition, we consider a numerical example with some plausible parameter values given in table 1.

Table 1: Parameters used in illustrations

Parameter	Value
α	0.4
δ	0.5
β	0.1
μ	0.2
r	0.05
N_0	1

Given these parameter values, the evolution of the environment is illustrated in figure 1 for the economy with structural change and without, i.e. for a fixed range of technologies. The state of the environment under structural change is stabilized in the medium-run because of the introduction of cleaner technologies and because of the out-dating of older ones. In the long-run, however, the environmental pollution rises again because the effect of cleaner technologies is dominated by the strong increase in the (average) productivity and the ensuing output growth that exerts a negative impact on the environment.

[5] In the model without structural change, horizontal innovations are absent, tending to raise investment in productivity growth. However, the negative effect of a rising environmental degradation will always dominate sooner or later since it increases over time.

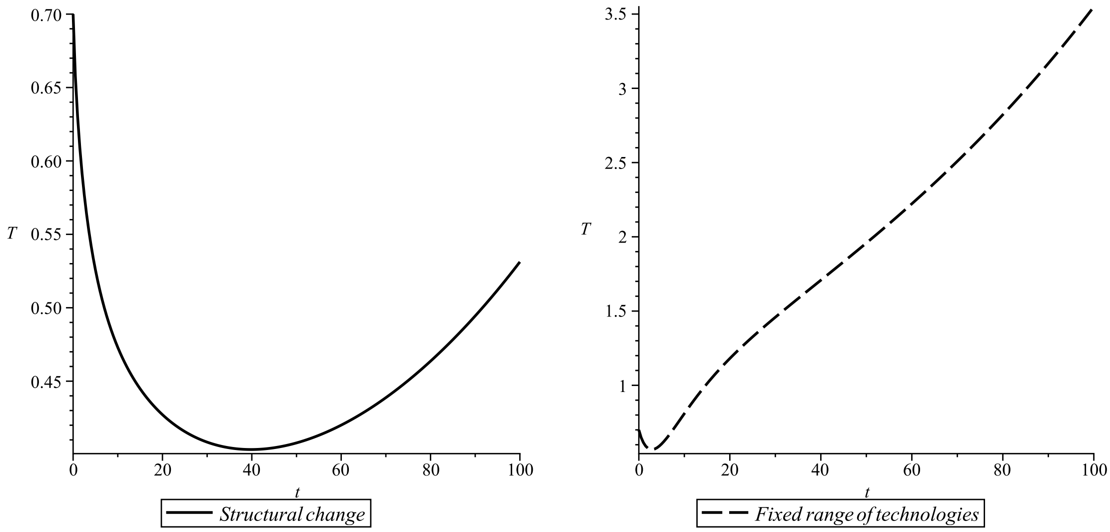
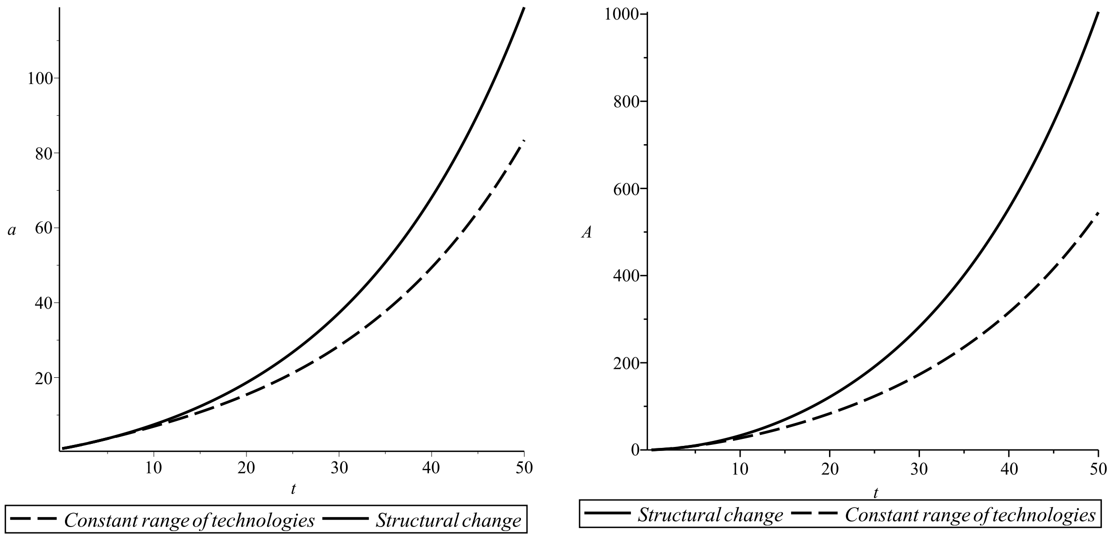


Figure 1: Influence of structural change on the environment

The economic evolution is shown in figure 2 where the evolution of the capital stocks and of the average productivities are depicted. It can be seen that both capital and productivity are higher in the case of structural change. It is then straightforward to conclude that the output growth is also higher in the economy with structural change compared to the economy with a fixed range of sectors. It should be noted that this is not the consequence of a different size of the economies in terms of the spectrum of technologies used (as this is constant in both cases), but rather a result of the composition of this range determined through the speed of structural change.



(a) Assets (b) Average productivity

Figure 2: Influence of structural change on the economy

Thus, it can be stated that the economy with structural change is characterized by slower environmental degradation, by a faster assets accumulation and by a higher productivity growth leading to higher output growth.

It should also be pointed out that environmental degradation continues in the long-run as output grows unless additional resources are used for abatement. The simplest way to achieve a constant level of environmental pollution would be to levy a lump-sum tax and to use the tax revenue for abatement, for example. The question of how environmental pollution can be stabilized in growing economies has been the subject of a great many studies (see e.g. the models in Greiner and Semmler, 2008). Therefore, we do not treat this problem but, rather, focus on the relation between structural change, economic growth and environmental pollution with the latter determined by the economic system alone, neglecting abatement activities.

3.2 Comparison with the model without environment

To consider the impact of environmental pollution on the economy with structural change we compare the benchmark economy of Bondarev and Greiner (2016) with the one described in this paper.

First, it follows from (23) that employment in the economy under environmental pollution is lower compared to the benchmark model. This gap seems to follow quite naturally the notion of *environmental unemployment*:

$$U^E = L - L^E = \left(1 - \frac{1}{1+T}\right) L \overset{L=1}{=} \left(\frac{T}{1+T}\right). \tag{31}$$

Second, it should be noted that the discrepancy between labor demand in the benchmark economy and in the economy with environmental spillovers will rise in time if environmental degradation continues. This will decrease the labor income of the households and, thus, slow down assets accumulation:

$$\dot{a} = ra - E + \frac{1}{1+T} < \dot{a}^O = ra - E + 1 \tag{32}$$

The lower stock of financial capital decreases productivity growth but does not affect variety expansion. The latter is linear and depends on the potential profit of the next technology. Let us consider the creation of new technologies in the environmental spillovers model compared to the benchmark economy. This is governed by the profits resulting from the development of a new technology,

$$\pi^R(i,t) = p_A(i) - \frac{1}{2}\int_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} g^2(i,t)dt, \tag{33}$$

which depends on the price of the patent, $p_A(i)$, and on accumulated investments.

The prices of patents will be higher, since the lower output is counterbalanced by the higher prices, and the manufacturing sector profits are larger than in the benchmark model due to lower labor costs,

$$\Pi_i^E = P_i^E Y_i^E - L_i^E - \Psi = \left(\frac{\epsilon}{\epsilon-1} - \frac{1}{1+T}\right) L_i^O - \Psi; \tag{34}$$

$$\Pi_i^O = P_i^O Y_i^O - L_i^O - \Psi = \left(\frac{\epsilon}{\epsilon-1} - 1\right) L_i^O - \Psi; \tag{35}$$

$$T > 0: \ \Pi_i^E > \Pi_i^O \rightarrow p_i^E(A) > p_i^O(A). \tag{36}$$

Thus, the patent price under environmental degradation will be higher and depends on the state of the environment at the time when technology i becomes operational and on the time when it becomes outdated. But, this factor affects all technologies in the same way and also influences investments (through assets accumulation).

Accumulated investments at the same time are lower for every technology due to slower financial capital accumulation compared to the benchmark model:

$$\int_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} \left((g^E(i,t))^2\right) dt = \int_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} \left(\frac{a^E - u^E}{N^E - N^E_{min}}\right)^2 dt \tag{37}$$

Assuming the same linear variety expansion process for the economy with environmental spillovers, it follows that the dynamics is governed by the a^E term which is always lower than the capital in the benchmark model, see (32). Then, the accumulated investments into productivity development (vertical innovations) are indeed lower for every technology by the factor of environmental damages, $1/(1+T)$. This gives

$$\pi^{R,E} > \pi^{R,O}, \tag{38}$$

and variety expansion (and thus structural change) is boosted in the economy with environmental spillovers, Thus, we obtain

$$\dot{N}^E > \dot{N}^O \rightarrow \mathcal{O}^E > \mathcal{O}^O. \tag{39}$$

Thus it can be pointed out that the productivity growth is slower in the economy with environmental pollution, but the structural change is faster.

As long as the environmental degradation continues, that is as long as $\dot{T} > 0$, the output growth is slower than without environmental spillover. However, what distinguishes our model from other endogenous growth models is that the environmental degradation slows down because of structural change since the latter implies that the emissions intensity declines. Thus, after some point in time the environment starts to regenerate and it is possible to have $\dot{T} < 0$. This happens when the core of the economy includes only technologies with very small environmental impact, $i \in \mathcal{O} : \iota(i) \rightarrow 0$, and the regeneration rate μ of the environment is higher than the impact of emissions. Hence, in the medium-run the economic growth of the economy under environmental spillovers may be even higher, than that of the benchmark model.

However, in the long-run the output will slow down, since the temperature starts to increase again due to the higher productivity growth rate that exceeds the decrease of emissions intensity. The length of the period during which the recovery of the environment is observed depends on the relationship between μ, the regeneration rate of the environment, and α, the elasticity of output with respect to technology that is the same for all sectors. Figure 1 illustrates the stabilization and the regeneration of the environment during 50 years (periods) for the model with structural change.

Substituting equation (29) into (28) provides the foundation for the comparison of output growth rates:

$$\left(\dot{Y}/Y\right)^E - \left(\dot{Y}/Y\right)^O = \frac{\alpha}{1+T}\frac{\dot{\bar{A}}^E}{\bar{A}^E}\mathcal{O}^E - \frac{(\bar{A}^E)^\alpha \ln(N^E_{max}/N^E_{min})}{(\mathcal{O}^E)^2(1+T)^2} + \frac{\mu T}{1+T} - \alpha\frac{\dot{\bar{A}}^O}{\bar{A}^O}\mathcal{O}^O =$$
$$\frac{\mu T}{1+T} - \frac{(\bar{A}^E)^\alpha \ln(N^E_{max}/N^E_{min})}{(\mathcal{O}^E)^2(1+T)^2} + \frac{\mathcal{O}^E - \mathcal{O}^O}{1+T}\left(\frac{\dot{\bar{A}}^E}{\bar{A}^E}\right) - \alpha\mathcal{O}^O\left(\frac{\dot{\bar{A}}^O}{\bar{A}^O} - \frac{1}{1+T}\frac{\dot{\bar{A}}^E}{\bar{A}^E}\right) \quad (40)$$

As long as the environmental state is stabilized, $\dot{T} \leq 0$, the regeneration rate of the temperature is equal to or higher than emissions from output and the growth rate of the economy is actually boosted. In the long-run, when the growth of productivity and, thus, of total output dominates the effects of cleaner technologies, the degradation of the environment starts again and the output growth diminishes to zero. The following proposition summarizes our results.

Proposition 2 (Environmental impact on the economy)
In the economy with structural change environmental spillovers lead to the following:

1. *The environment recovers in the medium-run boosting output growth;*
2. *Capital and productivity of each sector grow at a lower rate than in the benchmark model without environmental degradation;*
3. *The economic growth rate is almost always lower than in benchmark economy and*

becomes negative in the long-run;

4. *Structural change is faster than in the benchmark economy, and the diversity of operational sectors is larger.*

Proposition 2 demonstrates the consequences of the market failure in internalizing environmental spillovers under structural change. The decentralised economy responds to the environmental pollution by speeding up structural change, compared to the benchmark model, but the higher variety of technologies cannot offset the negative impact of environmental damages without any government intervention. Therefore, environmental policy, such as a tax on the polluting output, is still necessary to correct for the market failure. We discuss one of the ways to do so further on.

3.3 Environmental policy and impact on growth

Given the results from the previous sections, it is straightforward to note that the government should stimulate the rate of structural change to slow down environmental degradation. This can be done by internalizing the environmental impact caused by each technology. The latter can be achieved by levying a tax on the revenue of the firms in the manufacturing sector with the tax rate, τ^T, determined by the degree of environmental damages caused by the respective firm. Thus, the tax rate can be written as,

$$\tau^T(i) = \iota(i) = 1/i, \tag{41}$$

Here and further on the superscript T denotes the economy with environmental taxation. At this stage we do not study where the taxes are going to since competitive uses of environmental taxation (R&D subsidies, consumption subsidies, etc.) may constitute an interesting follow up study. Our main concern is to demonstrate that under such a tax system the resulting outcome is better both for the economy and for the environment.

Given the tax specified in (41) the profit function for the manufacturing firm is written as,

$$\Pi_i^T = (1 - \frac{1}{i})P_i Y_i - L_i - \Psi. \tag{42}$$

Then, the price demanded for the product i is obtained as,

$$P_i^T = \frac{\epsilon}{\epsilon - 1}(1+T)\frac{i}{i-1}A_i^{-\alpha} \tag{43}$$

and labor demand is proportionally reduced:

$$L_i^T = \frac{1}{1+T}\frac{i-1}{i}L_i^O. \tag{44}$$

This will change capital accumulation and, thus, total productivity growth by the factor $\mathcal{O}$ because labor and, therefore labor income, takes a different form:

$$L^T = \frac{1}{1+T}\int_{N_{min}}^{N_{max}} \frac{i-1}{i}L_i^O di = \frac{1}{1+T}\left(\mathcal{O} - \ln(N_{max}/N_{min})\right)L. \tag{45}$$

Depending on the dynamics of the operational range in this regulated economy, assets accumulation may be faster or slower than in the economy without taxation. Now, turn to the changes in profits of R&D. Since a higher index of the sector implies a lower tax burden, the profits for R&D are now increasing in i,

$$\frac{\partial \pi^R(i)}{\partial i} > 0. \tag{46}$$

Because of that, horizontal innovations are no longer constant but increase in time making variety expansion a non-linear convex function. Since the processes N_{min} and N_{max} are proportional to the variety expansion, they are also non-linear. The core $\mathcal{O}$ is then an increasing function of time and not constant any longer.

It is difficult to obtain the analytic form of optimal investments for each technology since the shadow costs of these investments are no longer identical. The reason for that is that shadow costs, which determine the investments into vertical innovations, depend on the derivative of the patent price with respect to productivity and are no longer constant across the technologies. Indeed, they now depend on i because profits are different across sectors:

$$\Pi^T(i) = P_i^T Y_i^T - L_i^T - \Psi = \left(\frac{\epsilon}{\epsilon - 1} - \frac{1}{1+T}\frac{i-1}{i}\right) L_i^O - \Psi; \tag{47}$$

$$p_A^T(i) = \int\limits_{t_{max}(i)}^{t_{min}(i)} \mathrm{e}^{-r(t-t_0(i))} \Pi^T(i) dt. \tag{48}$$

The shadow costs are then decreasing in i making investments into newer technologies more attractive. The resulting economy is characterized by a higher variety expansion speed and a higher productivity growth, while environmental pollution is significantly decreased because of an increasing core $\mathcal{O}$ over time.

We summarize the results in proposition 3.

Proposition 3 (Effects of environmental policy)
In the decentralised economy with structural change and environmental pollution, the introduction of environmental taxes $\tau^T(i) = \iota(i)$ will lead to the following:

1. *The operational range of sectors $\mathcal{O}$ is an increasing function of time, $\dot{\mathcal{O}} > 0$;*
2. *Environmental degradation is slowed down compared to the economy without taxation, $\dot{T}^T < \dot{T}^E$;*
3. *Economic growth is faster than in the deregulated economy:*

$$\frac{\dot{Y}^T}{Y^T} > \frac{\dot{Y}^E}{Y^E}. \tag{49}$$

The last statement follows from the fact that the core $\mathcal{O}$ is constant and environmental degradation is higher in the deregulated case, i.e. in the economy without environmental taxation.

Thus, proposition 3 demonstrates that the introduction of an environmental tax leads both to higher growth and to smaller environmental degradation. This shows that the internalization of the negative externalities does not go along with a reduction in production but rather leads to higher output. Hence, taxing the polluting output is clearly Pareto improving even if market economy reacts on the environmental spillover by fostering structural change by itself. It turns out that this speed up in the turnover of technologies is still insufficient from optimal viewpoint and government intervention is still unavoidable.

4 Conclusion

In this paper we have studied the consequences of environmental pollution in a growing economy taking into account endogenous structural change. The model with unified horizontal and vertical innovations allows us to consider different environmental damages caused by different technologies rather than positing an ad hoc emissions intensity in the economy. It turns out that the mix in the emissions intensity is crucial with respect to the environment and for the economy. The decentralised economy without regulation cannot cope with environmental degradation, even if newer and less polluting technologies are continously introduced, since it lacks sufficient incentives for boosting fundamental research that generates less polluting technologies. To achieve both positive long-run growth of the economy and to avoid a permanently deteriorating environment, it is necessary to speed up structural change, i.e. the expansion of the range of operational technologies. Such an expansion can counterbalance the negative influence of productivity growth on the environment and can be achieved through environmental taxation.

In particular, we have seen that environmental pollution enhances structural change but reduces output growth. On the other hand, allowing for structural change weakens the negative impact of pollution, thus, fostering economic growth. In the medium-run, environmental pollution can even decline because structural change leads to the replacement of older more polluting technologies by newer and cleaner ones. However, in the long-run that effect is dominated by the productivity increase that leads to a rising output that pollutes the environment. Finally, taxing the polluting output is both beneficial for the environment as well as for economic growth and, therefore, yields a Pareto superior outcome compared to the economy without taxation.

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Appendices

A Households derivations

The objective function of the household is

$$J^H = \int_0^\infty \mathbf{e}^{-\rho t} U(C) dt \; . \tag{A.1}$$

with $U(C) = \ln C$ being the utility function from composite consumption C consisting of the continuum of products,

$$C = \left[\int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \; . \tag{A.2}$$

with $1 < \varepsilon < \infty$ being the elasticity of substitution between goods.

The Lagrangian of the household is

$$L = \left[\int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \left(\int_{N_{min}}^{N_{max}} P_i C_i di - rK + \dot{K} + W \right) \; . \tag{A.3}$$

The first order condition for consumption good i is

$$C_i^{-\frac{1}{\varepsilon}} C^{\frac{1}{\varepsilon}} = \lambda P_i \; . \tag{A.4}$$

Taking the F.O.C. for i and for j and substituting in yields

$$C_i = C_j \left(\frac{P_i}{P_j} \right)^{-\varepsilon} \; . \tag{A.5}$$

Substituting this back into the equation for expenditure, equation (5) yields

$$C_j \left(\frac{1}{P_j}\right)^{-\varepsilon} \int_{N_{min}}^{N_{max}} P_i^{1-\varepsilon} di = E \; , \tag{A.6}$$

which can be rearranged to yield

$$C_i = E \frac{P_i^{-\varepsilon}}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj} \; . \tag{A.7}$$

and the standard Euler equation implies that the optimal growth rate for expenditure is given by

$$\frac{\dot{E}}{E} = r - \rho \; . \tag{A.8}$$

B Production sector derivations

Output of good i is given by:

$$Y_i = A_i^{\alpha} L_i \; . \tag{B.1}$$

where $0 < \alpha < 1$ is the productivity of technology in production. The maximization problem of firm i is

$$\max_{P_i} \Pi_i = P_i Y_i - L_i - \Psi \; , \tag{B.2}$$

where Ψ is a fixed operating cost.

The only use for output of all goods i is consumption, so that $C_i = Y_i$. The only product used for investments is financial capital a and is excluded from this spectrum.

The output by an individual firm Y_i equals to the consumption of that good C_i, so that we can insert equation (A.7) into the profit function:

$$\begin{aligned} \Pi_i &= P_i Y_i - L_i - \Psi && \text{(B.3)} \\ &= P_i Y_i - Y_i A_i^{-\alpha} - \Psi && \text{(B.4)} \\ &= P_i E \frac{P_i^{-\varepsilon}}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj} - E \frac{P_i^{-\varepsilon}}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj} A_i^{-\alpha} - \Psi; . && \text{(B.5)} \end{aligned}$$

We use further the assumption of zero mass of each individual product in the price index

$$\frac{\partial \int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj}{\partial P_i} = 0 \tag{B.6}$$

which is usual when the continuum of goods is employed.

Maximizing profit with respect to the price under this non-atomic assumption yields

$$\frac{\partial \Pi_i}{\partial P_i} = \frac{E}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj}(1-\varepsilon)P_i^{-\varepsilon} - \frac{E}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj} P_i^{-\varepsilon-1}(-\varepsilon)A_i^{-\alpha} = 0\ . \tag{B.7}$$

The price is thus

$$P_i = \frac{\varepsilon}{\varepsilon - 1} A_i^{-\alpha}\ . \tag{B.8}$$

All products out of the range $N_{max} - N_{min}$ have zero prices:

$$P_i = \begin{cases} 0,\ t < \tau_{max}(i), t_{max}(i) : \Pi_i = 0, \dot{\Pi}_i > 0; \\ \frac{\varepsilon}{\varepsilon-1} A_i^{-\alpha},\ \tau_{max}(i) < t \leq t_{min}(i), t_{min}(i) : \Pi_i = 0, \dot{\Pi}_i < 0; \\ 0, t > t_{min}(i). \end{cases} \tag{B.9}$$

Labour employed in sector i is thus a function of the relative productivity of labour in sector i. Repeating arguments being made for the price formation we have piecewise-defined labour demand:

$$L^D(i) = \begin{cases} 0,\ t < t_{max}(i), t_{max}(i) : \Pi_i = 0, \dot{\Pi}_i > 0; \\ \frac{\epsilon-1}{\epsilon} E \frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{-\alpha(1-\epsilon)} dj},\ t_{max}(i) < t \leq \tau_{min}(i), t_{min}(i) : \Pi_i = 0, \dot{\Pi}_i < 0; \\ 0, t > t_{min}(i). \end{cases} \tag{B.10}$$

The patent for each new technology has a price

$$p_A(i) \stackrel{def}{=} \int\limits_{t_{max}}^{t_{min}} \mathrm{e}^{-r(t-\tau_0)} \Pi_i dt. \tag{B.11}$$

which does not depend on time (only on integration limits).

The markets clearing implies constant expenditures:

$$E(t) = \int\limits_{N_{min}}^{N} P(i,t)C(i,t)di = \int\limits_{N_{min}}^{N} P(i,t)Y(i,t)di = \frac{\epsilon}{\epsilon - 1} \int\limits_{N_{min}}^{N} L(i,t)di = \frac{\epsilon}{\epsilon - 1} L. \tag{B.12}$$

C R&D derivations

The incentive for horizontal innovation is the potential profit from selling the (improved) technology to manufacturing firms. Thus, the value of horizontal R&D consists solely in expected future profits from vertical innovations:

$$V_N = \max_{u(\bullet)} \int\limits_{0}^{\infty} \mathrm{e}^{-rt} \left(\delta \pi^R(i)|_{i=N} u(t) - \frac{1}{2} u^2(t) \right) dt \tag{C.1}$$

Here profit of developing next technology $i = N$ is the value of vertical innovation into technology i, which is given by:

$$\pi^R(i)|_{i=N} = p_A(N) - \frac{1}{2} \int\limits_{t_0(N)}^{t_{min}(N)} \mathrm{e}^{-r(t-t_0)} g^2(N,t) dt. \tag{C.2}$$

with $g(N,t)$ being investments costs of developing technology N during the patent duration.

The profit of the single R&D firm developing technology i is:

$$\pi^R(i) = p_A(i) - \frac{1}{2} \int\limits_{t_0}^{t_{min}} \mathrm{e}^{-r(t-t_0)} g^2(i,t) dt. \tag{C.3}$$

with investments going into increase of productivity:

$$\dot{A}(i,t) = \gamma g(i,t) - \beta A(i,t). \tag{C.4}$$

where γ is the efficiency of investments into productivity increase (equal for all sectors) and β is the cost of supporting the productivity on the current level (this abstracts infrastructure and human capital, required for reproduction of the current level of technology).

The aggregate problem for vertical R&D reads:

$$V = \max_{g} \int_{0}^{\infty} e^{-rt} dt \int_{N_{min}(t)}^{N(t)} p_A(i) di - \int_{0}^{\infty} e^{-rt} \int_{N_{min}(t)}^{N(t)} p_A(i) \frac{1}{2} g^2(i,t) di dt; \tag{C.5}$$

$$s.t. \tag{C.6}$$

$$\forall i \in [N_{min}, N] \subset \mathbb{R}_+ : \dot{A}(i,t) = \gamma g(i,t) - \beta A(t) \tag{C.7}$$

$$\int_{N_{min}(t)}^{N(t)} g(i,t) di = a(t) - u. \tag{C.8}$$

Implementing Maximum Principle approach, we derive optimal investments for each R&D firm as a function of shadow costs of investments, $\psi(i,t)$, this last being the function of price of the patent:

$$\dot{\psi}(i,t) = r\psi(i,t) - \frac{\partial p_A(i)}{\partial A(i)},$$

$$g^*(i,t) = \gamma\psi(i,t) - \frac{\int_{N_{min}(t)}^{N(t)} \gamma\psi(i,t) di - G(t)}{N(t) - N_{min}(t)}. \tag{C.9}$$

it may be demonstrated, that shadow costs of investments are the same across all existing technologies:

$$\psi^*(i,t) = \psi^* = \frac{C}{r+\beta} \tag{C.10}$$

Then investments into productivities of all the existing technologies are symmetric:

$$g^*(t) = \frac{a-u}{N(t) - N_{min}(t)}, \tag{C.11}$$

but dynamics of productivities themselves differ by the depreciation rate:

$$\dot{A}(i,t) = \frac{a-u}{N(t) - N_{min}(t)} - \beta A(i,t). \tag{C.12}$$

D Constant core derivation

The fact that the size of the economy is constant follows from definitions of N_{min}, N_{max}

$$N_{min}(t): \frac{1}{\epsilon - 1} L \frac{A_{N_{min}}^{\alpha(\epsilon-1)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{\alpha(\epsilon-1)} dj} - \Psi = 0; \tag{D.1}$$

and

$$N_{max}(t): \frac{1}{\epsilon - 1} L \frac{A_{N_{max}}^{\alpha(\epsilon-1)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{\alpha(\epsilon-1)} dj} - \Psi = 0; \tag{D.2}$$

Now observe that maximum profit for any sector i is reached at the point of $\dot{\Pi}(i) = 0$. Then it follows, that growth of N_{min} and N_{max} is equal:

$$\dot{\Pi}(i) = 0 \Leftrightarrow \left(\frac{\dot{A}(i,t)}{A(i,t)} - \int\limits_{N_{min}}^{N_{max}} \frac{\dot{A}(j,t)}{A(j,t)} dj \right) = \frac{\Psi}{\alpha L} (\dot{N}_{max} - \dot{N}_{min}) \tag{D.3}$$

However, bracket in the lefthandside has to be equal to zero, since it equalizes growth rate of productivity of sector i and average growth rate of productivity in the economy. Since all the technologies are symmetric except for the time of their invention, it is straightforward to say that maximum profit for the given industry is reached at the point where its productivity grows at the average rate of the economy. Otherwise there will be still room for improvements of technology or the technology is already overdeveloped. From this it follows that $\dot{N}_{max} - \dot{N}_{min} = 0$ yielding constant range of operational sectors in the economy, $\mathcal{O} = const$.

E Growth rate of the benchmark economy

We start with the observation that (D.1) and (D.2) lead to

$$\forall i \in [N_{min}, N_{max}] : \frac{A(i,\tau_{max}(i))^{\alpha(\epsilon-1)}}{A(i,\tau_{min}(i))^{\alpha(\epsilon-1)}} = \frac{\int\limits_{N_{min}(\tau_{max}(i))}^{N_{max}(\tau_{max}(i))} A(j,\tau_{max}(i))^{\alpha(\epsilon-1)} dj}{\int\limits_{N_{min}(\tau_{min}(i))}^{N_{max}(\tau_{min}(i))} A(j,\tau_{min}(i))^{\alpha(\epsilon-1)} dj}. \tag{E.1}$$

Thus all technologies' productivities grow at the same average speed during the time period of operational activity of the technology,

$$\forall i \in [N_{min}, N_{max}], \forall t \in [\tau_{max}(i), \tau_{min}(i)] : \dot{A} = \frac{a-u}{N-N_{min}}. \tag{E.2}$$

To obtain output growth rate consider aggregate output:

$$Y = \int\limits_{N_{min}}^{N_{max}} \left(\frac{A(i,t)^{\alpha\epsilon}}{\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)}} \right) di = \frac{\int\limits_{N_{min}}^{N_{max}} A(i,t)^{\alpha\epsilon} di}{\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)} dj} \tag{E.3}$$

The direct calculation of output growth rates from (E.3) yields with the help of (E.2):

$$\frac{\dot{Y}}{Y} = \frac{\dot{\left(\int\limits_{N_{min}}^{N_{max}} A(i,t)^{\alpha\epsilon} di \right)}}{\int\limits_{N_{min}}^{N_{max}} A(i,t)^{\alpha\epsilon} di} - \frac{\dot{\left(\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)} dj \right)}}{\int\limits_{N_{min}}^{N_{max}} A(j,t)^{\alpha(\epsilon-1)} dj} \overset{\dot{N}_{min}=\dot{N}_{max}}{=} \alpha \frac{a-u}{N-N_{min}} \frac{N_{max}-N_{min}}{\bar{A}}$$

$$= \alpha \frac{\dot{\bar{A}}}{\bar{A}} (N_{max} - N_{min}) > 0. \tag{E.4}$$