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Temporary and permanent technology lock-ins in the quality-differentiated Bertrand competition*

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Abstract

We consider a setting where strategic behavior of r&d firms can lead to different types of a technology lock-in, permanent or temporary, in an eventually inferior technology. The simple setting with one incumbent and one potential entrant may lead to a wide variety of possible strategic regimes. We study conditions on relative market strength of the incumbent and the entrant which lead to different strategic actions and demonstrate, that such a strategic behavior is not always socially sub-optimal, since it may lead to faster development of the existing technology due to persistent threat of the potential entrant. We further elaborate on the selection of support tools which may induce the development of new technology in the second-best world and establish criteria for these tools to be social welfare improving ones.

Keywords: technology lock-in, technological change, strategic interaction, r&d policy, multiple regimes

JEL classification: C61, O31, O38

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1 Introduction

For many environmental problems, a shift to green technologies is considered to be a promising long-term solution. A prominent example is climate change, where much hope rests on a transition from fossil fuel based technologies to renewable energy sources. Another example is traffic-related air pollution, where cleaner engines or e-mobility provide opportunities to reduce pollution levels substantially.

In this context, a crucial question is whether and to what extent a government should interfere with technological change. It is obvious that an internalization of environmental externalities is important to provide incentives for developing clean technologies. Arguably, competition among technologies will seek out the best technological solutions once environmental damages are correctly priced. But many countries use considerably more fine-grained approaches to steer details of technological change. A prominent example are feed-in tariffs for renewables. By using different tariffs for different technologies, many countries make sure that a broad set of technologies is developed and used. Often this approach eliminates competition among technologies (as less efficient technologies are subsidized to an extent that ensures their use) and thus replaces market-based technology selection with politically set targets for technology development and diffusion. Subsidies for different new transport technologies (fuel cells, e-mobility) work in a similar way.

Not surprisingly, many economists are skeptical regarding this approach and argue that governments might lack the necessary information to ensure efficient investments in different options for green technologies. However, there are also economic arguments in favor of detailed incentive schemes. Numerous studies have shown that almost unavoidable market failures can lead to a technology lock-in; typical examples are lock-ins caused by market power that is due to patents for new technologies (see, e.g., Krysiak (2011)) or externalities caused by network effects in technology adoption (see Arrow (1962), Arthur (1989), (Unruh, 2000), or (Unruh, 2002)). In such cases, it is not sufficient to only set a price for environmental damages to ensure that the best clean technologies are developed; more specific incentives are necessary (Krysiak, 2011).

The size and duration of such specific interventions will typically depend strongly on different cases of market failures. For example, the development of a new promising technology might only be delayed or it could be prevented completely, rendering different interventions necessary.

However, in many cases it is not easy to assess the type and scope of market failures that might require an intervention. This holds in particular, as the potential of yet to be developed technologies cannot be predicted with certainty. It is often hard to say whether a new technology is not developed, because market actors expect that it is an inferior solution (and thus do not invest) or because some actors with incumbent technologies use their power to forestall the development of a superior competition. Furthermore, it is hard to assess whether a development is forestalled or only delayed.

There are numerous models analyzing r&d competition between firms in a differential game context. This literature dates back to patent races (D'Aspremont and Jacquemin, 1988). Some more recent contributions are (Dawid et al., 2010) and (Bondarev, 2014). These studies concentrate on the r&d dynamics in a rather similar setup but do not go into analysis of government policies. Other studies focus on the evolution of the market structure as in (Hinloopen et al., 2013), where one firm may leave the market due to having a weaker position. In this paper the dynamic market structure is allowed for but the government policy is not studied. The paper (Ben-Youssef and Zaccour, 2014) considers a dynamic r&d duopoly and government regulation, but does not allow for strategic pricing behavior. Our contribution is the development of a framework which accounts both for strategic behaviour of duopolists and government interventions. We specifically focus on the market failure that could lead to a monopolization of a market and ways to avert this problem, thus combining the effects of government policy with an evolving market structure.

In this paper, we investigate how qualitatively different types of market failures can arise in technological change and what kind of policy intervention is required to cope with them. We use a simple model where, depending on the efficiency of a new technology, an incumbent might or might not have an incentive to keep the new technology out of the market or to delay its entrance. A government could, in addition to internalizing an externality, provide specific support for the new technology. We show that different cases of market failure can arise and require different levels and duration of an intervention.

The structure of the rest of the paper is as following: Section 2 introduces the model, in Section 3 we describe the multiplicity of arising r&d regimes, Section 4 describes the social welfare and subsidizing schemes for different regimes of r&d, Section 5 concludes. Most lengthy computations may be found in the Appendices section.

2 The model

We consider a setting with a production sector, where firms decide which out of two technologies to use, and an r&d sector, where two firms develop these technologies.

In the production sector, there is perfect competition. But in the r&d sector, the firms get a patent for their developments and are thus monopolistic suppliers of their technology. One of the firms has an initial advantage (its technology being somewhat more developed initially) and thus might act strategically to forestall the use and development of the second technology.

Both r&d firms know with certainty technology characteristics of each other. In our analysis we abstract from further market imperfections such as environmental externalities, assuming it is already taken care about by proper remuneration schemes in case technologies at hand are dirty and clean ones or both are green. By doing so we apply our study to the case of general innovations setting with green technologies being a specific (but rather important) example of those.

2.1 The production sector

Firms in the production sector can choose among two technologies, labeled $j \in \{A, B\}$. Each firm can invest in one unit of technology. The technologies differ regarding their quality $q_j(t)$ and their price $p_j(t)$, which can change over time. Thereby the quality influences the amount of output generated with one unit of equipment. The remuneration per unit of output is z and is fixed over time[1].

[1] it is possible to carry out the analysis with time-varying final product price, but analytical derivations become much more challenging without altering the main results of the paper

The firms have different locations, which also influence the achievable output. For simplicity, we use a single location parameter x that influences the output achievable with both technologies, but in different directions (one technology being more suited at a location than the other). For example, in the case of renewable energy, some locations might be better suited for wind power whereas others are more suitable for PV. We assume that there is a continuum of locations x and that there is exactly one firm at each location.

The profit that this firm can obtain by using technology A or B is given by

$$\pi_A^{Prod} = z\,(q_A(t) - x) - p_A(t), \tag{1}$$

$$\pi_B^{Prod} = z\,(q_B(t) + x) - p_B(t). \tag{2}$$

We assume $x \in [-\bar{x}_A, \bar{x}_B] \subset \mathbb{R}$. Thus, depending on the choice of $\bar{x}_A, \bar{x}_B$, locations could be on average better suited for technology A or for technology B.

To calculate the demand for each technology, we take into account that each firm buys one unit of equipment and each locations hosts a single firm. As long as $z\,q_j > p_j$ holds for $j = A, B$, all locations are used and thus the demand for technology j is determined by the distance between $\bar{x}_j$ and the location where a firm is indifferent between both technologies. This implies the following demand functions

$$N_A^{Prod} = \bar{x}_A - \frac{1}{2}\left(\frac{p_A - p_B}{z} - q_A + q_B\right), \tag{3}$$

$$N_B^{Prod} = \bar{x}_B - \frac{1}{2}\left(\frac{p_B - p_A}{z} - q_B + q_A\right). \tag{4}$$

We assume that $\bar{x}_j \leq q_j/2$ for $j = A, B$, which implies that in the equilibrium derived later, the condition $z\,q_j > p_j$ for $j = A, B$ will always hold. Thus these demand functions characterize the case where both technologies are available.

If only technology A is available (which will be the case in some settings), demand for this technology is determined by the distance between $\bar{x}_A$ and the location where a firm receives a profit of zero when using technology A. In this case, demand for technology A is given by

$$N_{A,-B}^{Prod} = \bar{x}_A - \left(\frac{p_A}{z} - q_A\right). \tag{5}$$

where subscript $A, -B$ denotes the demand for A in the absence of B.

2.2 The r&d sector

In the r&d sector, firms can invest in r&d and set prices for their technologies. Owing to their patent, they are the sole suppliers of their respective technologies.

A firm's r&d efforts improve the quality of its technology

$$\dot{q}_j(t) = g_j(t)\, Q_j - q_j(t), \tag{6}$$

with Q_j being a measure of the efficiency of r&d for this technology and thus (implicitly) of the long-term quality (potential) that the technology might eventually achieve.

The objective of both firms is to maximize their discounted stream of profits (value) for a given discount rate r and with N_j being given by (3),(4)

$$J_j = \max_{p_j, g_j} \int_0^\infty \mathrm{e}^{-rt} \left\{ p_j(t)\, N_j(t) - \frac{1}{2}\, g_j^2(t) \right\} dt. \tag{7}$$

As we allow for dynamic price adjustments, the state of both technologies influence both firms and we thus have a differential game setup. To reduce dimensionality, we introduce the distance between technologies as a state variable (referred to as technological gap throughout the rest of the paper). States of both technologies and distance between them are linked via

$$\dot{\delta}(t) = \dot{q}_A(t) - \dot{q}_B(t) = g_A(t)\, Q_A - g_B(t)\, Q_B - \delta(t). \tag{8}$$

The differential game thus consists of two firms maximizing the functionals (7) subject to the common dynamic constraint (8) and the demand functions given by (3),(4), which can also be written as functions of $\delta(t)$. The initial condition for the game is the distance between technologies at time 0, which we denote by $\delta(0)$.

We want to analyze the most interesting case, where an older and thus somewhat more refined technology A could potentially prevent the development of a currently less refined technology B that, however, has the potential to become the better technology. We thus assume $\delta(0) > 0$, that is, technology A has the better initial quality[2]. Furthermore, we assume $Q_B > Q_A$, that is, technology B has the better long-run potential.

[2] if on the contrary, $\delta(0) < 0$ technology B has the head start and all analysis is repeated with interchanging A and B

2.3 Government

We introduce the social welfare into the model to make comparisons across different regimes of the r&d game. However we restrain from formulating any subsidies/taxes at this stage, to keep the analysis focused on strategic behavior of the firms. The net social benefit consists of a marginal benefit β attached to each unit of production with both technologies[3] minus locational costs, minus the costs of developing the technologies. For simplicity, we assume that the social planner uses the same discount rate r as the r&d firms. Social welfare is thus given by:

$$W := \int_0^\infty \mathrm{e}^{-rt} \left\{ \beta \left(N_A(t)\ \left(q_A(t) + \Xi_A(t)\right) + N_B(t)\ \left(q_B(t) + \Xi_B(t)\right)\right) - \frac{1}{2}\ \left(g_A^2(t) + g_B^2(t)\right) \right\} dt, \tag{9}$$

where $\Xi_j(t)$ denotes the average effect of used locations on output for technology j. In case that both technologies are available, these costs are given by[4]

$$\Xi_A(t) = \frac{\bar{x}_A^2}{2} - \frac{\left(p_A(t) - p_B(t) - z\,(q_A(t) - q_B(t))\right)^2}{8\,z^2}, \tag{10}$$

$$\Xi_B(t) = \frac{\bar{x}_B^2}{2} - \frac{\left(p_A(t) - p_B(t) - z\,(q_A(t) - q_B(t))\right)^2}{8\,z^2}. \tag{11}$$

In case only technology A is used, we get

$$\Xi(t) = \frac{\bar{x}_A^2}{2} - \frac{1}{2\,z^2}\ \left(p_A(t) - z\,q_A(t)\right)^2. \tag{12}$$

In Section 4 we compute W associated with different outcomes of the game and study what ranking is induced across social welfare by different strategic actions of the incumbent firm.

3 Regimes of the r&d sector

To assess whether the government should subsidize the initially weaker technology, it is important to analyze the dynamics of technological development. The model admits several qualitatively different cases: Both firms might develop their technologies simultaneously, firm B might decide not to enter the market, firm A could use strategic pricing

[3]For simplicity, we assume that both technologies induce the same marginal benefit.

[4]This follows directly from (3)–(4).

to delay development of technology B (temporary technology lock-in), and firm A might keep firm B from ever developing its technology (permanent technology lock-in).

To prepare the analysis of policy interventions, we first investigate these cases sequentially and then show under which conditions which case will emerge as the solution of the game. Throughout the exposition we move all intermediate calculations and results to appendices referred to throughout the main text.

3.1 Simultaneous play: both technologies are present

In case both technologies are available, (3)–(4) describe the relevant demand system. The optimal price schedules for both firms as functions of the technological gap are then

$$p_A(t) = z\,\bar{x}_A + \frac{z\,\delta(t)}{2} + \frac{p_B(t)}{2},\; p_B(t) = z\,\bar{x}_B - \frac{z\,\delta(t)}{2} + \frac{p_A(t)}{2}; \tag{13}$$

$$p^*_A(t) = \frac{z}{3}\,\left(4\,\bar{x}_A + 2\,\bar{x}_B + \delta(t)\right), \tag{14}$$

$$p^*_B(t) = \frac{z}{3}\,\left(4\,\bar{x}_B + 2\,\bar{x}_A - \delta(t)\right). \tag{15}$$

where the superscript $*$ denotes optimally chosen prices. Thus revenue for each firm is a function of the quality difference between technologies only:

$$\begin{aligned} p^*_A(t)\,N^*_A(t) &= S^A_1\,\delta^2(t) + S^A_2\,\delta(t) + S^A_3, \\ p^*_B(t)\,N^*_B(t) &= S^B_1\,\delta^2(t) + S^B_2\,\delta(t) + S^B_3, \end{aligned} \tag{16}$$

with $S^A_1 = S^B_1 = \frac{z^2}{18}$, $S^A_2 := \frac{z\,(4\,\bar{x}_A+2\,\bar{x}_B)}{9}$, $S^B_2 := \frac{z\,(2\,\bar{x}_A+4\,\bar{x}_B)}{9}$, $S^A_3 := \frac{(4\,\bar{x}_A+2\,\bar{x}_B)^2}{18}$, and $S^B_3 := \frac{(2\,\bar{x}_A+4\,\bar{x}_B)^2}{18}$. With the above price choices, the firms still have to choose their r&d efforts. Given the above revenues, firm A has an incentive to increase $\delta(t)$, whereas firm B wants to reduce the quality difference. This constitutes a standard non-cooperative differential game[5]:

$$
\begin{aligned}
&J_A = \max_{g_A(\bullet)} \int_0^\infty \mathrm{e}^{-rt} \left\{ p_A^* \, N_A^* - \frac{1}{2}\, g_A^2 \right\} dt, \\
&J_B = \max_{g_B(\bullet)} \int_0^\infty \mathrm{e}^{-rt} \left\{ p_B^* \, N_B^* - \frac{1}{2}\, g_B^2 \right\} dt, \\
&s.t. \\
&\dot{\delta} = g_A \, Q_A - g_B \, Q_B - \delta, \\
&\delta(0) = q_A(0) - q_B(0) = \delta_0, \\
&g_A, g_B \in [0; \infty) \subset \mathbb{R}_+
\end{aligned}
\tag{17}
$$

where the last condition ensures nonnegative controls.

An application of Maximum Principle[6] yields optimal investments of both firms and state dynamics[7]:

$$
\delta^* = \delta_0 \mathrm{e}^{\frac{1}{2}\left(r - \sqrt{(r+2)^2 - \frac{4}{9} z^2 (Q_A^2 + Q_B^2)}\right) t} + \frac{(S_2^A Q_A^2 + S_2^B Q_B^2)(\mathrm{e}^{\frac{1}{2}\left(r - \sqrt{(r+2)^2 - \frac{4}{9} z^2 (Q_A^2 + Q_B^2)}\right) t} - 1)}{\frac{1}{9} z^2 (Q_A^2 + Q_B^2) - (1 + r)}
\tag{18}
$$

$$
g_A^* = 4 \mathrm{e}^{\frac{1}{2}(r - \sqrt{X})\, t} \left(F_1^A \delta_0 + F_2^A \right) - F_3^A, \tag{19}
$$

$$
g_B^* = -4 \mathrm{e}^{\frac{1}{2}(r - \sqrt{X})\, t} \left(F_1^B \delta_0 + F_2^B \right) - F_3^B, \tag{20}
$$

Details of derivation may be found in Appendix A and definitions of coefficients in Appendix F.

The time t^*, when technology B catches up with technology A, is therefore (from $\delta^*(t^*) = 0$):

$$
t^* = 2 \frac{\ln \left(\frac{(S_2^A Q_A^2 + S_2^B Q_B^2)}{\delta_0 (\frac{1}{9} z^2 (Q_A^2 + Q_B^2) - (1+r)) + (S_2^A Q_A^2 + S_2^B Q_B^2)} \right)}{r - \sqrt{(r+2)^2 - \frac{4}{9} z^2 (Q_A^2 + Q_B^2)}}. \tag{21}
$$

The illustration of the typical simultaneous development of both technologies is given by Figure 1. We denote by δ_*^j such initial technology gaps, that simultaneous play yields

[5] From now on we omit time argument to condense notation everywhere where it is possible.

[6] we derive only the open-loop solution for this game and mean everywhere by solution the open-loop one. Main results hold for the closed loop also, but with substantial analytical complications.

[7] assuming smooth interior solution $\forall t$, $\forall j : g_j^* > 0$ with both firms remaining operative infinitely long, condition for that see Appendix A

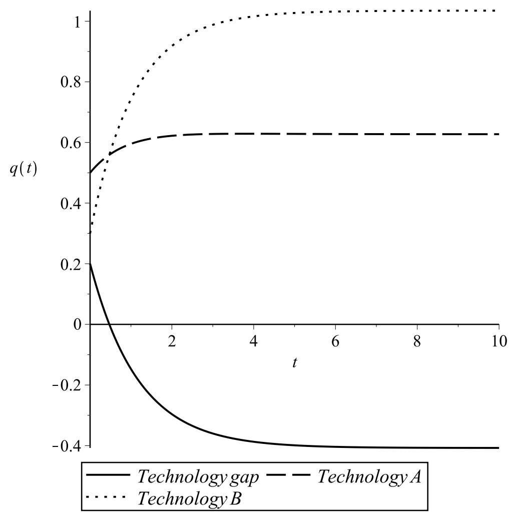


Figure 1: Evolution of rival technologies with simultaneous play

zero value for firm j once $\delta_0 = \delta_*^j$.

3.2 Sole innovator: Uncontested monopoly and strategic pricing

We next describe two regimes, where only firm A is present on the market, but with differing price schedules.

3.2.1 Unrestricted monopoly

If firm B does not enter the market, because the initial gap between technologies is so large that firm B's value becomes negative (i. e. $\max\{\delta_*^B\} > \delta_0 > \min\{\delta_*^B\}$ see Appendix C for details), we obtain the case of unrestricted monopoly of firm A.

In this case, we get the usual monopolistic price and revenue functions:

$$p_A^M = \frac{z\,\left(\bar{x}_A + \delta^M\right)}{2},\; p_A^M(t)\; N_A^M(t) = S_1^M \delta^2(t) + S_2^M \delta(t) + S_3^M \tag{22}$$

with $\delta^M(t) = q_A^M(t) - q_B(0)$, depending on the state of technology A only, and $S_1^M = \frac{z}{4}$,$S_2^M = \frac{z\,\bar{x}_A}{2}$, and $S_3^M = \frac{z\,\bar{x}_A^2}{2}$.

The monopolist'a problem is a standard optimal control problem:

$$
\begin{aligned}
& J_A^M = \max_{g_A(\bullet)} \int_0^\infty \mathrm{e}^{-rt} \left\{ p_A^M(t)\, N_A^M(t) - \frac{1}{2}\, g_A^2 \right\} dt, \\
& s.t. \\
& \dot{\delta} = g_A\, Q_A - \delta, \\
& g_A \in [0; \infty) \subset \mathbb{R}_+
\end{aligned} \tag{23}
$$

The Maximum Principle yields the following technology development and investments:

$$
\begin{aligned}
\delta^M(t) &= \delta_0 \mathrm{e}^{\frac{1}{2}\left(r - \sqrt{(r+2)^2 - 8 S_1^M Q_A^2}\right)t} + \frac{S_2^M Q_A^2 (\mathrm{e}^{\frac{1}{2}\left(r - \sqrt{(r+2)^2 - 8 S_1^M Q_A^2}\right)t} - 1)}{2 S_1^M Q_A^2 - (1+r)} \\
g_A^M &= F_1^M \delta_0 \mathrm{e}^{\frac{1}{2}(r - \sqrt{X_m})t} + F_2^M \mathrm{e}^{\frac{1}{2}(r - \sqrt{X_m})t} - F_3^M
\end{aligned} \tag{24}
$$

with $X_m := (r+2)^2 - 8 S_1^M Q_A^2$ (and real-valued solution exists only if $X_m > 0$) and with $F_{1,2,3}^M$ being functions of monopolist's demand parameters and efficiency of investments Q_A specified in the Appendix F.

3.2.2 Strategic pricing

If firm B position is strong enough to enter the market, firm A may have an incentive to prevent its entry. If firm A has a sufficiently strong advantage, it can keep the other firm off the market. To do so, it has to set the price of its technology in such a way, that firm B does not gain by entering the market. More precisely, firm A has to set its price so that, even with its best response, firm B cannot achieve a total discounted profit stream that is strictly greater than zero.

This strategic price, ensuring firm B does not enter the market is

$$
p_A^S = z\ (\delta - 2\, \bar{x}_B)\,. \tag{25}
$$

Details of derivation are in Appendix B.

If strategic pricing is implemented (conditions of Lemma 8 from Appendix B hold) permanently, firm B never enters the market. In this case, the resulting technology state and investments are derived by the same Maximum principle application as for uncontested monopoly case albeit with price given by (25):

$$\delta^S = \delta_0 \mathrm{e}^{-t} + \frac{S_1^S Q_A^2 (1 - \mathrm{e}^{-t})}{(1+r)} \tag{26}$$

$$\bar{\delta}^S = \frac{S_1^S Q_A^2}{1+r} \tag{27}$$

$$g_A^S = Q_A S_1^S \tag{28}$$

where $\delta^S = q_A^S(t) - q_B(0)$ and $S_1^S = z(\bar{x}_A + 2\bar{x}_B)$.

3.3 Piecewise solutions: temporary strategic pricing

Observe that solution given by (26) is valid only if strategic pricing is implemented indefinitely long. Still, it might be that firm A will be able only to delay the entry of firm B via strategic pricing, that is, firm A might switch from strategic pricing to the pricing analyzed in subsection 3.1. Moreover it might be the case that temporary strategic pricing is sufficient to permanently prevent B's entry and firm A switches to monopolistic behavior studied in subsection 3.2.

3.3.1 Temporary strategic pricing with permanent technology lock-in: (un)contested monopoly

First consider the case when firm A is able to develop its technology to such a level, that it is no longer profitable for firm B to enter the market even in the absence of strategic pricing. This is possible, if there exists a value of technology gap δ_*^B, after reaching which at some time t^P the value for the firm B upon entrance is non-positive in the simultaneous development regime. Condition for such permanent lock-in are stated by Lemma 9 in Appendix C.

If this is the case, the objective for firm A is:

$$J_A^P = \max_{g_A(\bullet)} \left\{ \int_0^{t^P} \mathrm{e}^{-rt} \left\{ p_A^S N_A^S - \frac{1}{2}\left(g_A^S\right)^2 \right\} dt + \int_{t^P}^{\infty} \mathrm{e}^{-rt} \left\{ p_A^M N_A^M - \frac{1}{2}\left(g_A^M\right)^2 \right\} dt \right\}. \tag{29}$$

with P superscript denoting piecewise strategic-monopolistic regime. We label it by *uncontested monopoly* to contrast it with the *contested monopoly* described further.

The optimal control problem solution for the case (29) is also obtained via Maximum Principle and the resulting state dynamics is a piecewise system[8]:

$$\dot{\delta}^P(t) = \begin{cases} \dot{\delta}^S(t), t < t^P, \\ \dot{\delta}^M(t), t \geq t^P, \delta^M(t^P) = \delta_*^B. \end{cases} \tag{31}$$

where $\dot{\delta}^S, \dot{\delta}^M$ are dynamical systems associated with permanent strategic pricing and unrestricted monopoly respectively and δ_*^B is the threshold technology gap value.

Figure 2a illustrates possible (un)contested monopoly cases. As firm A anticipates the change in the strategy from g^S to g^M, the solution differs from the monopolistic and from strategic ones. It is always the case that

$$\delta^M(t) \leq \delta^P(t) \leq \delta^S(t) \tag{32}$$

The longer it takes to prevent the entry of firm B, the closer the resulting technology evolution is to δ^S and vice versa: the sooner the strategic pricing stops, the closer the evolution is to the monopolistic one.

3.3.2 Temporary strategic pricing with temporary technology lock-in: delay

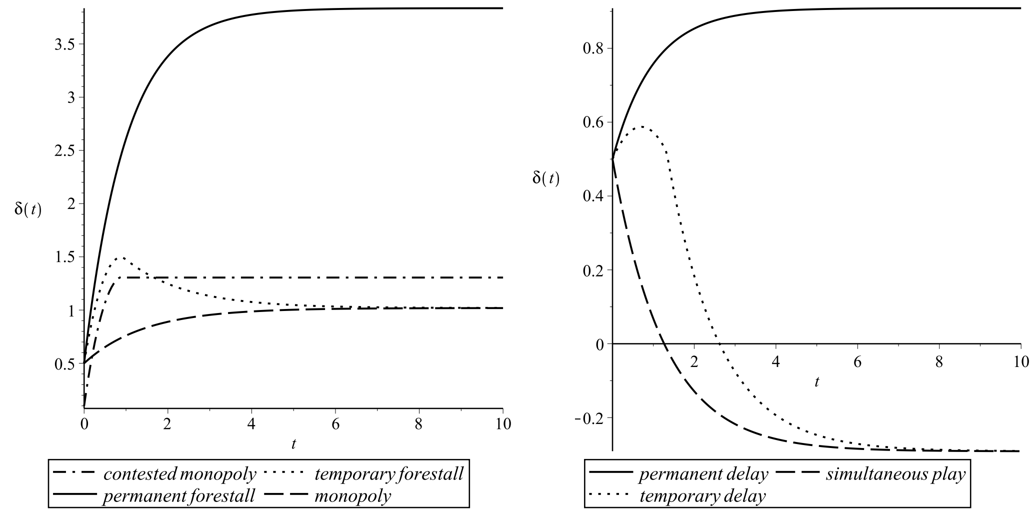
Next we study the case when position of firm A is not strong enough to prevent entrance of firm B forever, but it still finds it profitable to price strategically for some time, delaying firm's B entry. We label it as a *delay*.

In this case there exists a threshold value of technology gap δ_d upon reaching which at a time t^d it is no longer profitable for firm A to continue with strategic pricing and

[8] in fact, as long as $\bar{\delta}^S > \bar{\delta}^M$ two cases are possible: either $\bar{\delta}^M > \delta_*^B$ or vice versa. In the first case the dynamics is described by (31), but in the other case it is not possible for the firm A to switch to the uncontested monopoly from subsection 3.2. It can still stop strategic pricing and price monopolistically, but has to sustain the threshold level of technology gap δ_{*B}. We refer to this case as *contested* monopoly and it is described by

$$\dot{\delta}^C(t) = \begin{cases} \dot{\delta}^S(t), t < t^P, \\ 0, t \geq t^P, \delta(t) = \delta_*^B. \end{cases} \tag{30}$$

with superscript C denoting contested monopoly regime.



(a) Contested monopoly for $t^P = 1$ (b) Delay for $\delta_d = 0.8$

Figure 2: Piecewise regimes of the model

it allows the entry of firm B, while firm B still finds it profitable to enter the market (conditions for that stated by Lemma 12 in Appendix E). We thus have a mixed optimal control-differential game for firm A with the objective

$$J_A^d = \max_{g_A(\bullet)} \left\{ \int_0^{t^d} \mathrm{e}^{-rt} \left\{ p_A^S N_A^S - \frac{1}{2} \left(g_A^S\right)^2 \right\} dt + \int_{t^d}^{\infty} \mathrm{e}^{-rt} \left\{ p_A^* N_A^* - \frac{1}{2} \left(g_A^*\right)^2 \right\} dt \right\}. \quad (33)$$

and differential game for firm B starting at time t^d at initial gap δ_d with the objective

$$J_B^d = \max_{g_B(\bullet)} \left\{ \int_{t^d}^{\infty} \mathrm{e}^{-rt} \left\{ p_B^* N_B^* - \frac{1}{2} \left(g_B^*\right)^2 \right\} dt \right\}. \quad (34)$$

The application of standard technique results in the piecewise state dynamics:

$$\dot{\delta}^d = \begin{cases} \dot{\delta}^S, \ t < t^d, \\ \dot{\delta}^*, \ t \geq t^d, \delta^*(t^d) = \delta_d. \end{cases}$$

Observe that in the case of temporary delay the solution again is piecewise-defined and hence fully informed players have dynamics different from both the strategic and simultaneous play regimes, as illustrated by the Figure 2b.

It always holds that $\delta^*(t) < \delta^d(t) < \delta^S(t)$ in analogue with strategic piecewise solution described above.

We thus observe that the simple framework with two firms allows for rich set of possible outcomes if no entry costs are assumed. We next move to classifying and characterizing conditions, under which these cases may arise.

3.4 Choosing a game to play

So far, we have analyzed a number of possible outcomes. To answer the question which case realizes we compare values of firms under different outcomes. Define by Π_j^m the value of firm $j \in \{A, B\}$ in regime $m \in \{*, M, S, P, C, d\} = \mathcal{O}$ computed at time 0, where labels in set $\mathcal{O}$ denote associated regimes described above. Denote further $\mathcal{F} \subseteq \mathcal{O}$ the set of feasible outcomes of the game.

It turns out that value of both firms in all regimes can be represented as polynomials of at most 2nd degree (See Appendix C for details). Denote by δ_m^j *roots* of polynomials type Π_j^m in δ_0.

We compare the strategic pricing and the monopoly cases from the perspective of firm A. The value function of firm A under permanent strategic pricing and under monopoly (given in Appendix C) have roots δ_S^A, δ_M^A. Value under monopoly is greater than under strategic regime[9], thus firm A will try to switch from strategic regime to monopoly as soon as possible.

Next, we inquire whether strategic pricing is feasible and profitable. Feasibility is worked out by Lemma 10, and profitability condition is given by Lemma 11, both from Appendix D. If conditions of both lemmas hold, firm A behaves strategically for the time t^P defined in Lemma 9 (see Appendix C) and switches to monopolistic behavior afterwards (contested or uncontested).

At last, if there is no option to permanently deter the entrance by temporary strategic pricing ($\delta_*^B > \bar{\delta}^S$), it might be still possible (and profitable) to delay the entrance of firm B. This case is worked out in Lemma 12 in Appendix E.

[9]since it is always the case that $p_A^M > p_A^S$ and monopoly is profit maximizing while strategic forestaller is not

We thus observe three different types of strategic behavior, with two of them leading to permanent lock-in and another one to temporary lock-in on the industry in an inferior technology A. We need thus a unified characterization of all those cases of strategic behavior. We state this result next.

Proposition 1 (Strategic regimes algebraization)**.**

- *If* $\max\{\delta_*^B\} > \delta_0 > \min\{\delta_*^B\}$ *and* $\max\{\delta_M^A\} > \delta_0 > \min\{\delta_M^A\}$*, the unrestricted monopoly of firm A earning* Π_A^M *realizes;*

- *If Lemma 8 holds, strategic pricing is feasible for firm A and, if in addition* $\min\{\delta_d\} \leq \delta_0$*, strategic pricing is also profitable.*

- *If Lemmas 9, 10 and 11 hold, strategic pricing is implemented for* t^P*, given by* (C.6)*, and firm B never enters the market. Firm A switches to the uncontested monopoly regime after* t^P *and earns* Π_A^P *if* $\min\{\delta_*^B\} < \bar{\delta}^M$ *or to contested monopoly preserving the threshold level* $\min\{\delta_*^B\}$ *otherwise earning* Π_A^C*.*

- *If Lemma 9 does not hold, but Lemma 12 holds and Condition* (E.4) *is met, strategic pricing is implemented for* $t \in [0, t^d[$ *(given by* (E.2)*) and firm B enters at time* t^d*. Firm A earns* Π_A^d *and firm B earns* $\Pi_B^d|_{\delta_0=\delta_d}$*.*

- *If Condition* (E.4) *does not hold, but Condition* (E.5) *holds, the delay option results in permanent strategic pricing with firm A earning* Π_A^S*.*

Proof. The first point follows from assumption on leading coefficients of value functions to be of opposite signs and definitions of δ_m^j values. Other points follow from lemmas contained in Appendix. □

This proposition shows that, even in our rather simple model, qualitatively different cases of strategic behavior can arise. It is possible that the development of technology B is only delayed, it can be prevented for ever by permanent strategic pricing, or firm A could attain an advantage after some time, where firm B would never enter the market, even if firm A stops strategic pricing. More importantly it relates the cases of strategic behavior with a single observable parameter δ and its threshold values.

It is straightforward to translate the result of Proposition 1 into the comparison of potentials of competing technologies (as these are the main parameters of interest).

Corollary 1 (Impact of technologies' potentials on values of r&d)**.**
For all polynomials type Π_j^m *it holds that*

$$\frac{\partial|\delta_m^j|}{\partial Q_{-j}} \leq 0, \; \frac{\partial|\delta_m^j|}{\partial Q_j} \leq 0, \tag{35}$$

thus the higher is the potential of both technologies, the lower is the range of initial gap, for which simultaneous play may happen.

Proof. Showing (35) amounts to differentiating the roots of polynomials given in Appendix w.r.t. Q_j.

Next, observe that (35) characterizes the decrease in the length of the interval in δ axis, for which the simultaneous game takes place. If initial gap is inside of the interval $[\min\{\delta_*^B\}, \max\{\delta_*^B\}]$, firm B will not engage into the competitive development game. Firm A will not engage into the game once the initial gap is outside the interval $[\min\{\delta_*^A\}, \max\{\delta_*^A\}]$. Thus the r&d game will take place only if δ_0 lies in the intersection of $[\min\{\delta_*^A\}, \max\{\delta_*^A\}]$ with one of $(-\infty, \min\{\delta_*^B\}]$, $[\max\{\delta_*^B\}, +\infty)$. Denote this by $I(*)$, interval of r&d game realization. Since we limited exposition to $\delta_0 > 0$ it follows that with increase in potentials the intersection of the $I(*)$ with positive range of δ becomes smaller. □

4 Social welfare

4.1 Social welfare comparisons

Proposition 1 shows that there are several distinct cases in which a technology lock-in occurs; the incumbent (firm A) uses its advantage to prevent or delay the entry of the competing firm B.

The first question is under which conditions this is socially suboptimal, that is, when should technology B be developed. To this end, we have to evaluate and compare social welfare, as defined by (9), for the different cases of simultaneous development, delayed development, forestalled development and unrestricted monopoly.

Denote by W^m the social welfare resulting from outcome $m \in \mathcal{F}$ of the r&d game above and further denote

$$D_m(W) = W^* - W^m \tag{36}$$

the social welfare difference between the given regime m and the simultaneous development regime[10] e. g. for $m = M \in \mathcal{F}$ we have:

$$D_M(W) \overset{def}{=} W^* - W^M =$$
$$= \int_0^\infty \mathrm{e}^{-rt} \left\{ \beta \left(N_A^*(t)\ \left(q_A^*(t) + \Xi_A^*(t)\right) + N_B^*(t)\ \left(q_B^*(t) + \Xi_B^*(t)\right)\right) - \frac{1}{2}\ \left((g_A^*(t))^2 + (g_B^*(t))^2 \right) \right\} dt -$$
$$\int_0^\infty \mathrm{e}^{-rt} \left\{ \beta N_A^M(t)\ \left(q_A^M(t) + \Xi_A^M(t) \right) - \frac{1}{2} (g_A^M(t))^2 \right\} dt \tag{37}$$

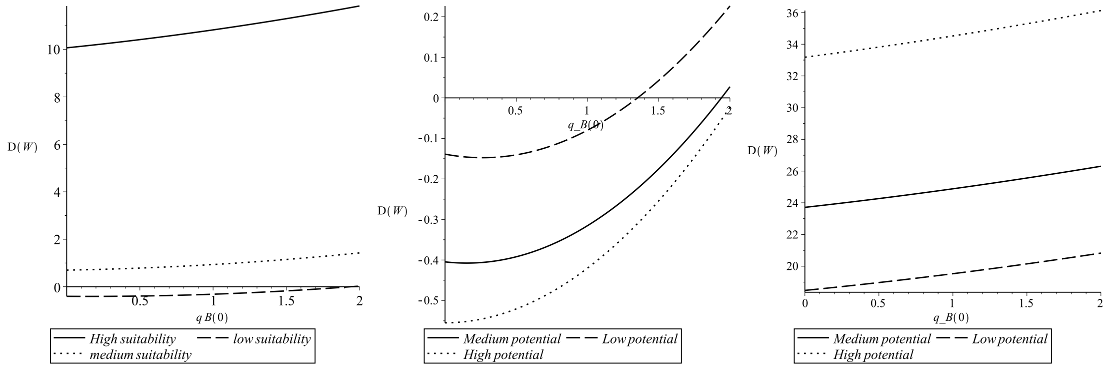
This expression depends on potentials of both technologies, initial conditions and the characteristic of locations better suited to one or the other technology. If we take characteristics of technology A $(Q_A, q_A(0), \bar{x}_A)$ fixed, the social welfare difference is positive and increases in $Q_B, q_B(0)$ as long as $\bar{x}_B \gg \bar{x}_A$ reflecting the high market potential of technology B (it is better suited for more sites). However, as soon as $\bar{x}_B < \bar{x}_A$, the monopoly of technology A might be social welfare improving. This is illustrated by Figure 3.

It is interesting to note, that the higher is the potential of technology B, the lower is the social welfare under simultaneous development, provided the condition $\bar{x}_B \ll \bar{x}_A$ holds (see Figure 3b). At the same time as soon as $\bar{x}_B \gg \bar{x}_A$, the higher is the potential of B, the higher is welfare under simultaneous development (see Figure 3c).

The difference in social welfare (37) is the 3d degree polynomial in $q_B(0)$ of the form

$$W_1^M q_B(0)^3 + W_2^M q_B(0)^2 + W_3^M q_B(0) + W_4^M \tag{38}$$

[10] of course we could define more generally $D_{m,k}(W) = W^k - W^m$, $\forall \{m, k\} \in \mathcal{O}$, but we limit exposition to comparisons with simultaneous behavior here for brevity reasons



(a) Different $\bar{x}_B$, $Q_B > Q_A$ (b) Different Q_B,$\bar{x}_B \ll \bar{x}_A$ (c) Different Q_B ,$\bar{x}_B \gg \bar{x}_A$

Figure 3: Social welfare under simultaneous development and monopoly

where superscript M denotes that difference is taken with respect to monopoly. Its leading coefficient W_1^M is always negative[11]:

$$W_1^M = \frac{z\beta\left(-4\,z{Q_A}^2 - 4\,Q2_B{}^2 z + \sqrt{-4\,z{Q_A}^2 - 4\,{Q_B}^2 z + 9\,r^2 + 36\,r + 36}r + 9\,r^2 + 36\,r + 36\right)}{432\,\sqrt{-4\,{Q_B}^2 z - 4\,z{Q_A}^2 + 9\,(r+2)^2}\left(\left({Q_A}^2 + {Q_B}^2\right)z - 2\,r^2 - 9\,r - 9\right)} \tag{39}$$

and does not depend on initial technologies' states, thus the resulting welfare is higher under monopoly starting from some $q_B(0)$ which is given by the maximal root of the equation (38) and always higher for competitive case for $q_B(0)$ values below the minimal root of that polynomial.

There are potentially up to three values of technology B initial state (being functions of demand potentials and potentials for development of both technologies), separating regimes where monopoly is preferred to simultaneous development and vice versa. Denote roots of the polynomial (37) in $q_B(0)$ by $\hat{q}_B^M\{1,2,3\}$ with superscript denoting the difference with monopoly and indexed such that $\hat{q}_B^M\{1\} < \hat{q}_B^M\{2\} < \hat{q}_B^M\{3\}$. We have then the following:

[11] indeed, since square root has to be positive, and expression has the form $\frac{(x-y)+\sqrt{x-y}}{\sqrt{x-y}\cdot\alpha(y-x)}$, $\alpha < 1$ and $y < x$ to yield real values

Lemma 1. *Simultaneous development is social welfare improving over the monopoly of technology A as long as:*

1. *Either* $0 \leq q_B(0) < \hat{q}_B^M\{1\}$
2. *Either* $0 \leq \hat{q}_B^M\{2\} < q_B(0) < \hat{q}_B^M\{3\}$

Proof. Follows from the negative leading coefficient in (38) and shape of 3d degree polynomials algebraic curves. □

Next consider the social welfare under infinitely forestalled development of technology B and simultaneous development.

The difference in social welfare is defined equivalently to (37) as

$$D_S(W) \stackrel{def}{=} W^* - W^S = W_1^S q_B(0)^3 + W_2^S q_B(0)^2 + W_3^S q_B(0) + W_4^S \tag{40}$$

with the help of objective functional (9) and associated solutions to the original two-states problem in q_A, q_B. This is again the 3d degree polynomial in $q_B(0)$ with coefficients denoted $W_{1,2,3,4}^S$. Denote its roots by $\hat{q}_B^S\{1, 2, 3\}$. Then we have the same result as for monopoly case:

Lemma 2. *As long as* $W_1^S > 0$*, simultaneous development is socially welfare improving over strategic forestall by firm A if* $q_B(0) > \hat{q}_B^S\{3\} > 0$ *or* $0 < \hat{q}_B^S\{1\} < q_B(0) < \hat{q}_B^S\{2\}$.

As long as $W_1^S < 0$*,simultaneous development is socially welfare improving over strategic forestall by firm A if* $0 < q_B(0) < \hat{q}_B^S\{1\}$ *or* $0 < \hat{q}_B^S\{2\} < q_B(0) < \hat{q}_B^S\{3\}$.

Proof. The same as for Lemma (1) but for positive and negative leading coefficients cases. □

If we compare the social welfare under monopoly and strategic permanent pricing, it turns out that the social welfare under monopoly can be higher or lower than under strategic forestall depending on the market potential of technology B, $\bar{x}_B$. This is illustrated by Figure 4, which is the result of different signs of leading coefficients in polynomials (38),(40).

The same logic applies for the delayed development: it may not necessarily lie in between the simultaneous development and strategic permanent forestall. Thus both

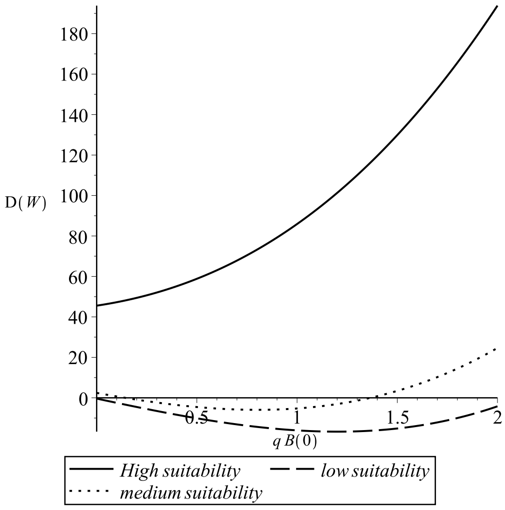


Figure 4: Difference in social welfare under monopoly and strategic forestall

$\delta^P(t), \delta^d(t)$ regimes are to be taken into account. We need some invariant measure of social optimality of simultaneous development case. For that we use the choice function defined as:

Definition 1. *A choice function (selector, selection) is a mathematical function f that is defined on some collection X of non empty sets and assigns to each set S in that collection some element $f(S)$ of S.*

It is provided by the following Lemma:

Lemma 3 (Selector for socially optimal regime)**.**
The simultaneous development of both technologies is socially optimal across all possible regimes $m \in \mathcal{O}$ iff $q_B(0)$ lies in the intersection of positive intervals of all respective polynomials, i. e. there exists a selector

$$\Psi(q_B(0)) : \prod_{m \in \mathcal{O}} D_m(W)|_{q_B(0)} > 0 \tag{41}$$

Proof. The product $\prod_{m \in \mathcal{O}} D_m(W)$ is positive only if all of components are positive, meaning simultaneous development is better than any other regime in social welfare terms. This product is a rational function (product of finitely many polynomials), and has finite number of intervals with changing sign. Thus as soon as $q_B(0)$ lies in one of such intervals, it is better for social planner to have simultaneous development than any other regime of the game. □

It is immediate to note that the same type of selectors may be obtained for social optimality of any other regime. We omit this since we are primarily interested in simultaneous development regime.We now can summarize our social welfare results in terms of choice functions over regimes as following:

Proposition 2 (Social welfare algebraization)**.**
The outcome $s \in \mathcal{F}$ of the r&d game is socially optimal among outcomes $\mathcal{F} \subseteq \mathcal{O}$ if $q_B(0)$ lies in the union of intervals where social welfare is higher under outcome s than under any other $m \in \mathcal{F}$, i. e. there exists the choice function:

$$\Psi(\mathcal{F}) : q_B(0) \in \bigcup_{m\in\mathcal{F}} \bigcap_{m\in\mathcal{F}} [\hat{q}_B^{s,m}\{z\}; \hat{q}_B^{s,m}\{z+1\}] :$$
$$D_{s,m}(W) \geq 0 \implies$$
$$\Psi(\mathcal{F}) = \arg\max_{m\in\mathcal{F}} W^m(q_B) = s. \tag{42}$$

In particular, the outcome $*$ is welfare-optimal if $* \in \mathcal{F}$ and $\Psi(\mathcal{F}) = *$.

Proof. follows from the direct computation of social welfare defined above and comparison of the roots of resulting polynomials in $q_B(0)$. □

The particular application of Proposition 2 to the case of simultaneous development yields combined results of Lemmas 1, 2

Corollary 2 (Social optimality of simultaneous development)**.**
One of the following cases hold:

1. *As long as both* $\max\{\hat{q}_B^M\} < 0, \max\{\hat{q}_B^S\} < 0$ *or* $\forall j \in 1,2,3 :\ q_B(0) > \max\{\hat{q}_B^S\{j\}, \hat{q}_B^M\{j\}\} > 0$ *it is never socially optimal to allow monopoly or strategic behavior of firm A;*

2. *If* $\max_j\{\hat{q}_B^M\{j\}\} > q_B(0) > \max_j\{\hat{q}_B^S\{j\}\} > 0$ *it is socially optimal to allow monopoly regime but not the strategic regime;*

3. *If* $\max_j\{\hat{q}_B^M\{j\}\} > \max_j\{\hat{q}_B^S\{j\}\} > q_B(0) > 0$ *it is socially optimal to allow strategic pricing regime and turn it into monopoly by proper subsidizing technology A;*

4. If $\max_j\{\hat{q}_B^S\{j\}\} > \max_j\{\hat{q}_B^S\{j\}\} > q_B(0) > 0$ *it is socially optimal to allow permanent strategic pricing*

Proof. Application of Proposition 2 to the case $s = *$. □

It should be noticed that cases 2, 3, 4 above can be realized only for low $\bar{x}_B \ll \bar{x}_A$ new technology market potential. We thus assume in the rest of the paper that case 1 holds, i. e. parameters are set up in such a way, that it is socially optimal to have both technologies being developed.

Corollary 3 (Impact of technologies' potential on welfare)**.**
There exists $\bar{x}_j^* > \bar{x}_{-j} : \forall \bar{x}_j > \bar{x}_j^*$ *it holds:*

$$\frac{\partial \hat{q}_j^m}{\partial Q_j} \leq 0, \ \frac{\partial \hat{q}_j^m}{\partial Q_{-j}} \geq 0 \tag{43}$$

In particular if case 1 of Corollary 2 takes place, the higher is the potential of B*, the less initial* $q_B(0)$ *suffices for socially optimal simultaneous development.*

Proof. Amounts to computing derivatives of roots of $D_m(W)$ polynomials with respect to potentials. Since as long as $q_B(0)$ is higher than any of the roots $\hat{q}_B^S\{j\}, \hat{q}_B^M\{j\}$ it is optimal to develop both technologies, increase of Q_B decreases these roots and thus increases the range of $q_B(0)$ for which development of B is socially desirable. □

Using Proposition 2 it is straightforward to compute social welfare under different policy schemes and compare it with sole development of A regimes. This is done by replacing q_B^*, q_A^* terms in $D_m(W)$ polynomials by $q_j^{\sigma_k}$ terms, which are optimal open loop solutions of the associated differential game of r&d firms under given subsidy level σ_k. For that we first derive some policy schemes and then conclude with statements over social optimality of subsidized r&d regimes.

4.2 Policy schemes preventing strategic behavior

To prevent strategic pricing, the government can use a subsidy that is paid for each unit of technology B. We do not ask the optimality, but only feasibility of such subsidies at this stage[12]. Such a subsidy σ alters the reaction functions of both firms:

$$p_A(t) = z\,\bar{x}_A + \frac{z\,\delta(t)}{2} + \frac{p_B(t)}{2},\; p_B(t) = z\,\bar{x}_A - \frac{z\,\delta(t)}{2} + \frac{p_A(t)}{2} - \frac{1}{2}\sigma; \tag{44}$$

The resulting equilibrium prices for both firms are

$$\begin{aligned} p_A^\sigma &= \frac{1}{3}\,\left(z\,\left(4\,\bar{x}_A + 2\,\bar{x}_B + \delta(t)\right) - \sigma\right) = p_A^* - \frac{1}{3}\sigma, \\ p_B^\sigma &= \frac{1}{3}\,\left(z\,\left(2\,\bar{x}_A + 4\,\bar{x}_B - \delta(t)\right) - 2\,\sigma\right) = p_B^* - \frac{2}{3}\sigma. \end{aligned} \tag{45}$$

In the following, we discuss subsidies for the different cases of strategic behavior described by the Proposition 1. We thereby derive different bounds for the subsidy and its duration.

Given the above reaction functions, the strategic price of firm A that will reduce the profit of firm B to zero is now subsidy-dependent:

$$p_A^{-\sigma,S} = z\,\delta - 2\,z\,\bar{x}_B - \sigma. \tag{46}$$

Thus the subsidy level that makes strategic pricing infeasible (reducing firm A's revenues to zero) follows from $p_A^{-\sigma,S} = 0$ and is

$$\sigma_{max} = z\,\delta(t) - 2\,z\,\bar{x}_B = p_A^S. \tag{47}$$

Given that $\delta(t)$ declines over time when both firms develop their technology, the maximal required subsidy size is thus

$$\sigma_+ \stackrel{def}{=} z\,\delta_0 - 2\,z\,\bar{x}_B. \tag{48}$$

This maximal subsidy is constant and thus easy to implement. However, the state-dependent (and thus declining) subsidy σ_{max} suffices to prevent strategic pricing in all regimes.

The corresponding duration of the subsidy can be derived from the condition in Lemma 8, as this is a necessary and the least demanding condition for strategic pricing. Denoting the solution of the simultaneous development game with the subsidy by δ^σ, we get the following result.

[12]the first-best subsidy is always defined within a Stackelberg game with government playing a leader. However this first-best subsidy is not computationally feasible even in our simple setup and we characterize several simpler subsidizing schemes being aware of there second-best optimality.

Lemma 4 (Maximal size and duration of subsidy)**.**
To prevent strategic pricing of firm A, it is sufficient to set the subsidy level at σ_{max} and pay it no longer than

$$t^{max} : \delta^{\sigma}(t^{max}) = 2\,\bar{x}_B. \tag{49}$$

This duration is always shorter than the time required for technology B to catch up to technology A.

As a second step, we consider the subsidy level and duration that render strategic pricing *non-profitable*. Let us first consider the case of permanent strategic pricing. The condition that strategic pricing is not profitable in this case, implies that the profit streams under permanent strategic pricing and simultaneous development starting at level δ_σ are equal:

$$\delta_\sigma : \Pi^S_{A,\infty}|_{\delta_0=\delta_\sigma} = \Pi^*_{A,\infty}|_{\delta_0=\delta_\sigma}. \tag{50}$$

With (C.1) and (C.4), this results in a quadratic equation for δ_σ, which is similar to (E.1). Comparing this condition with those of Lemma 11, we observe that δ_σ is given by the same polynomial as δ_d (see Appendix D). The minimal duration of a subsidy is thus given by

$$t^{min} : \delta^{\sigma}(t^{min}) = \delta_\sigma = \min\{\delta_d\}. \tag{51}$$

The subsidy level that is necessary to prevent permanent strategic pricing is defined by the condition

$$\sigma_{min} : \pi^S_{A,\infty}|_{p_A=p_A^{-\sigma,S}} = \pi^*_{A,\infty}|_{p_{A,B}=p^{\sigma}_{A,B}}, \tag{52}$$

which compares the revenue streams at each time $t < t^{min}$ for firm A under the strategic and the simultaneous development regimes. The value of σ that reduces this difference to zero, is given by the larger root of the following second degree polynomial:

$$p_A^{-\sigma,S}(\sigma)N_A^S - p_A^{\sigma}N_A^{\sigma} = 0 \rightarrow \sigma_{min} = z\,\left(\delta(t) - 4\,\bar{x}_B - 2\,\bar{x}_A\right). \tag{53}$$

This value depends on the state $\delta(t)$. As the state is decreasing function of time under the subsidy regime, the corresponding constant subsidy is always higher:

$$\sigma_{-} \overset{def}{=} z\ (\delta_0 - 4\,\bar{x}_B - 2\,\bar{x}_A) \geq \sigma_{min}. \tag{54}$$

The minimal state-dependent subsidy is always lower than the maximal one for any δ, as it can be inferred from comparison of (47) and (53).

Now, we turn to non-permanent strategic pricing, as described by Lemma 9. If this lemma holds, the strategic pricing regime lasts only for t^P. Thus the profitability of strategic pricing for firm A could be higher and is limited by the monopolistic profit. Thus the subsidy should continue for

$$t^{suff} : \delta^{\sigma}(t^{suff}) = \delta_{\sigma} = \min\{\delta_{-}\}, \tag{55}$$

whenever $t^P < \infty$. Obviously, $t^{suff} > t^{min}$, as $\min\{\delta_M\} < \min\{\delta_d\}$.

In this case, the level of the subsidy that is sufficient to prevent strategic pricing is given by the condition

$$\sigma_{suff} : \pi^M_{A,\infty}|_{p_A = p_A^{-\sigma,M}} = \pi^*_{A,\infty}|_{p_{A,B} = p^{\sigma}_{A,B}}. \tag{56}$$

where

$$p_A^{-\sigma,M} = \frac{z\ \left(\bar{x}_A + \delta^M\right)}{2} - \sigma \tag{57}$$

Repeating our arguments regarding σ_{min}, we see that σ_{suff} has to be the maximal root of polynomial over σ given by the difference in revenue streams between the monopoly and the simultaneous development regime:

$$\begin{aligned} & p_A^{-\sigma,M}(\sigma) N_A^M - p_A^{\sigma} N_A^{\sigma} = 0 \rightarrow \\ & \sigma_{suff} = \frac{z}{36+z} \left(3X_{suff} + 3\delta + 6\bar{x}_A + \left(\bar{x}_A + 2\bar{x}_B - \frac{1}{2}\delta \right) z \right) \overset{\delta \leq \delta_2}{\geq} \sigma_{min}, \end{aligned} \tag{58}$$

where X_{suff} is specified in the Appendix, and where δ_2 is the value of δ above which the sufficient subsidy becomes lower than the minimal one.

As above, we can complement this state-dependent subsidy with a constant, sufficient subsidy that might be easier to implement by replacing the time-varying difference between the states of the technologies with the initial gap:

$$\sigma_o \stackrel{def}{=} \frac{z}{36+z}\left(3X_{suff}|_{\delta=\delta_0} + 3\delta_0 + 6\bar{x}_A + \left(\bar{x}_A + 2\bar{x}_B - \frac{1}{2}\delta\right) z\right) \geq \sigma_{suff},$$
$$\sigma_o \stackrel{\delta_0 \leq \delta_2}{\geq} \sigma_-. \tag{59}$$

As long as both firms are present on the market, δ is always decreasing till the steady state level. Thus, for the minimal duration t^{min} to be lower than the maximal duration t^{max} it is necessary that

$$t^{min} \leq t^{suff} \leq t^{max} : \delta_d \geq \delta_- \geq 2\,\bar{x}_B. \tag{60}$$

Note that the size of subsidies is not always ordered in the same way:

$$\delta_2 > \delta > \delta_1 \rightleftarrows \sigma_{max} > \sigma_{suff} > \sigma_{min} \tag{61}$$

where δ_1 is defined by the intersection of σ_{max} and σ_{suff} and δ_2 is defined by the intersection of σ_{suff} and σ_{min}. Figure 5 illustrates the relationship between different levels of subsidizing.

Altogether, we have proven the following result.

Lemma 5 (Minimal size and duration of the subsidy)**.**
To prevent strategic pricing of firm A, it is necessary to set the subsidy level at σ_{min} *and pay it at least* t^{min}. *It is sufficient to pay the subsidy at the level* σ_{suff} *during* t^{suff}

It holds that $t^{min} < t^{suff} < t^{max}$ *and* $\sigma_{min} < \sigma_{suff} < \sigma_{max}$ *as long as* (60), (61) *hold.*

The exact size and duration lie within the following boundaries

$$\sigma_{min} \leq \sigma(t^P) \leq \sigma_{suff}, \; \frac{\partial\sigma(t^P)}{\partial t^P} < 0;$$
$$t^{min} \leq t^{\sigma}(t^P) \leq t^{suff}, \; \frac{\partial t^{\sigma}(t^P)}{\partial t^P} < 0,$$
$$\lim_{t^P\to\infty} \sigma(t^P) \to \sigma_{min}, \lim_{t^P\to 0} \sigma(t^P) \to \sigma_{suff},$$
$$\lim_{t^P\to\infty} t^{\sigma}(t^P) \to t^{min}, \lim_{t^P\to 0} t^{\sigma}(t^P) \to t^{suff}. \tag{62}$$

Finally, let us consider the last remaining case, that is, delayed development. Here, Lemma 9 does not hold, that is, it is not possible to keep firm B permanently out of the

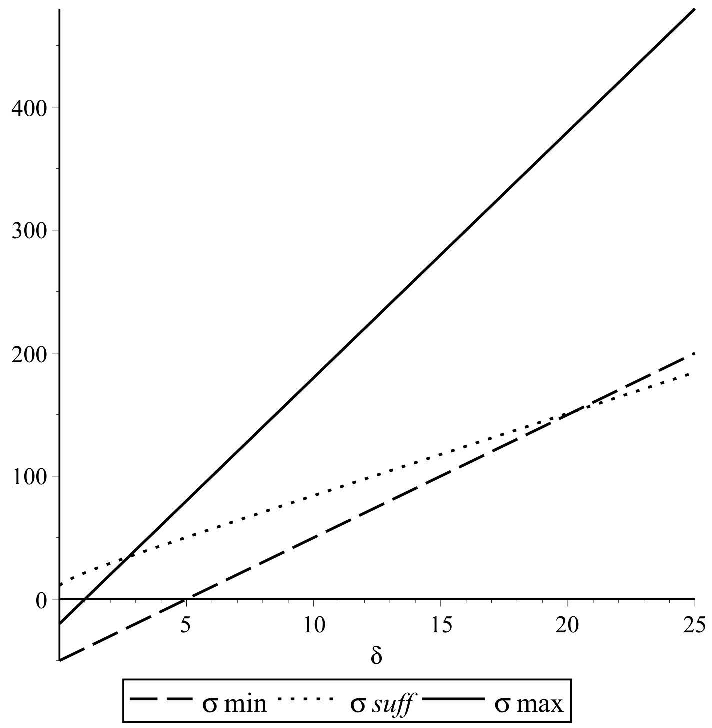


Figure 5: Relative subsidies sizes as functions of the state

market, but Lemma 12 holds, so that there is an option for delaying the entrance of firm B.

In this case, the subsidy duration t^{min} is sufficient to prevent the delay. Indeed, we have $\delta_\sigma = \delta_d$ and thus if no $\delta_{*B} \leq \bar{\delta}^S$ exists, the duration t^{min} is both necessary and sufficient, irrespective of whether the strategic delay is temporary or permanent.[13] This yields the following result

Lemma 6 (Subsidy size and duration for the case of strategic delay)**.**
In conditions of Lemma 12 it is both necessary and sufficient to set the subsidy at σ_{min}, t^{min} *levels to prevent the strategic delay option.*

Summarizing our results on preventing strategic pricing under full information, we get the following proposition.

[13]This holds, as the delay duration is defined by $\max\{\delta_d\}$, whereas the subsidy duration is defined by $\min\{\delta_d\}$.

Proposition 3 (Subsidy to prevent strategic pricing)**.**

1. *If Lemmas 8, 9, and 10 hold, the subsidy is set at the level* σ_{suff} *and paid till* $t = t^{suff}$*, if* (60)–(61) *holds, or at* σ_{max} *till* $t = t^{max}$ *otherwise;*

2. *If Lemma 9 does not hold, but Lemma 12 holds, the subsidy is set at* σ_{min} *and paid till* $t = t^{min}$*;*

3. *If one of the Lemmas 8 or 11 does not hold, there is no need for a subsidy.*

This proposition shows that the different cases of strategic behavior considered in the preceding section ask for different responses by the regulator.

4.3 Social welfare under subsidizing schemes

Previous subsection answered the question what kind of subsidy should be implemented to prevent each possible kind of strategic behaviour. The answer depends on the initial gap between both technologies. However the question whether it is socially welfare improving to implement a subsidy is still open.

We use the same approach as in subsection 4.1, deriving and comparing social welfare under different regimes. We have identified six different subsidizing schemes, three of them constant and three state-dependent. It turns out that the quantities $D(W)$ computed in the same manner as in (37), are 2nd degree polynomials in $q_B(0)$ for constant subsidies and 3d degree polynomials for state-dependent subsidies. As such, they allow to derive analytically the threshold levels, which indicate whether the given subsidy is socially optimal.

For constant subsidizing schemes the social welfare decreases in the size of subsidy. Thus as long as the ordering of subsidies size $\sigma_+ > \sigma_o > \sigma_-$ holds, it also holds that $D_m(W)^{\sigma_-} > D_m(W)^{\sigma_o} > D_m(W)^{\sigma_+}$ where superscripts denote the implemented subsidizing scheme. Since all these polynomials are of the same degree it then follows, that the maximal roots of them in $q_B(0)$ also have the same ordering.

The same is true for state-dependent subsidies, but with 3d degree polynomials. We thus have:

Lemma 7. *The subsidies defined by the Proposition 3 are implemented only if the maximal root of the associated polynomial $D_m(W)^{\sigma}$ is lower than $q_B(0)$.*

Denote associated maximal roots by $\hat{\hat{q}}_m^{\sigma_{max}}, \hat{\hat{q}}_m^{\sigma_{suff}}, \hat{\hat{q}}_m^{\sigma_{min}}, \hat{\hat{q}}_m^{\sigma_+}, \hat{\hat{q}}_m^{\sigma_o}, \hat{\hat{q}}_m^{\sigma_-}$ where superscript denotes the polynomial $D_m(W)^{\sigma}$ from which the roots are taken. Denote further by $\Theta(\mathcal{F})$ the choice function (as in Lemma 3) for the individually-optimal outcome of the r&d game, defined via Proposition 1.

We then conclude our social welfare analysis with the following result:

Corollary 4 (Social welfare and policy schemes)**.**
There is a need to implement a subsidy only if $s = \Psi(\mathcal{F}) \neq \Theta(\mathcal{F}) = h$. In this case the policy scheme is assigned via Proposition 3 with the welfare ordering following Lemma 7. The subsidy scheme yielding $\max_{\sigma^k} D_{s,h}(W)^{\sigma_k}$ is implemented.

5 Conclusions

In this paper, we have investigated the question whether a government should interfere with green technological change by granting a technology-specific subsidy, which is a frequently observed practice (technology-specific feed-in tariffs for renewables being the most prominent example). We have studied a setting, where an incumbent firm might have an incentive to keep a new technology from the market or delay its entrance. We have shown that different cases of strategic behavior can arise and require different types of intervention.

Our results show that in many cases a time-limited interference with technological change is indeed socially optimal. As technological development requires patents to pay off and patents induce market power, there is a considerable danger of market failures. A firm that owns a patent for a technology with limited potential but an initial quality advantage can have an incentive to use this market power to prevent or delay the development of an ultimately superior competitor. It is socially optimal to correct this problem. However, it will often not be possible to use the best possible intervention, as it depends on the potential of the new technology, which is likely to be unknown to the government. Thus we have shown that even in cases of severe constraints of information, a government can and should interfere. Furthermore, if it should interfere, the level and duration of the subsidy increase with the uncertainty.

The paper extends prior studies in several points. First, using a simple but coherent model, we show how different cases of technology lock-in or development delay can arise from the existence of market power (patents) without any further imperfections. Second, we couple this market description with an analysis of interventions that could prevent such a lock-in.

From a more application-oriented perspective, our paper casts some new light on policies that aim to support green technological change. These policies are widely used and are often criticized by economists, as they eliminate competition among technological options. Our results show that there are cases where it is indeed reasonable to temporarily reduce the effects of competition via technology-specific subsidies. Most interestingly, a less informed government should subsidize new technologies more and longer, as long as it can still ascertain that developing the technology is socially desirable.

Appendices

A Solution for simultaneous game

Maximum principle yields (current value) Hamiltonians for both players:

$$\mathcal{H}_j = p_j^*(t)\, N_j^*(t) - \frac{1}{2}\, g_j^2 + \lambda_j\,(g_A\, Q_A - g_B\, Q_B - \delta) \tag{A.1}$$

with co-state dependent optimal investments of both players resulting from F.O.C.s:

$$g_A^* = \lambda_A\, Q_A,\; g_B^* = -\lambda_B\, Q_B \tag{A.2}$$

and co-state equations:

$$\dot{\lambda}_A = (1+r)\,\lambda_A - 2\,S_1^A\,\delta - S_2^A,\; \dot{\lambda}_B = (1+r)\,\lambda_B - 2\,S_1^B\,\delta - S_2^B. \tag{A.3}$$

The resulting canonical system of the game includes co-state equations (A.3) and the state equation:

$$\dot{\delta} = \lambda_A \, Q_A^2 + \lambda_B \, Q_B^2 - \delta. \tag{A.4}$$

Together, they form a three dimensional linear system of ODEs, which admits a closed-form solution. The resulting state evolution is given by Eq. (18).

Given the solution path for co-state equations, optimal investments of both firms can be written as functions of time and the demand parameters, given by Eqs. (19), (20), where we set $X := (r+2)^2 - \frac{4}{9}z^2(Q_A^2 + Q_B^2)$ (thus real-valued solution exists only if $X > 0$) and where $F_{1,2,3}^{A,B}$ are functions of demand parameters and investment efficiencies (see Appendix F for their definitions).

Observe also, that as long as $\delta(t)$ is monotonically decreasing in time, at some point t^E it could the case that the initial leader will exit the market as soon as $q_A(t^E) \leq 0$, resulting in the uncontested monopoly of the new technology after that time. The condition for that is

$$\bar{\delta}^* \leq -\bar{q}_B \tag{A.5}$$

where bars denote steady state values, and the steady state of q_B is given by the associated two-states problem. To simplify the analysis we further assume this is not the case, and the difference in technologies potentials is not too high, allowing the initial leader to stay on the market.[14]

B Strategic pricing case derivations

Formally, firm A sets the price so that it makes maximal profit of firm B non-positive:

$$\Pi_B|_{p_A(t)=p_A^S(t)} = \max_{p_B, g_B} \int_0^\infty \mathrm{e}^{-rt} \left\{ p_B(p_A^S, \delta) N_B - \frac{1}{2} g_B^2 \right\} dt \leq 0. \tag{B.1}$$

[14]otherwise the game becomes piecewise-defined and incentives for strategic behavior of firm A are further increased.

Using the revenue function of firm B and its best response to the price set by firm A given by (13), we get

$$p_B\,(p_A)\,N_B = \frac{p_A^2}{8\,z} + \frac{1}{2}(\bar{x}_B - \frac{\delta}{2})\,p_A + \frac{z\,\delta^2}{8} - \frac{z\,\bar{x}_B\,\delta}{2} + \frac{z\,\bar{x}_B^2}{2}. \tag{B.2}$$

Hence, the strategic price is

$$p_A^S = z\,\left(\delta - 2\,\bar{x}_B\right). \tag{B.3}$$

As long as the price A is set at most on this level, the investments of the firm B are zero, as the co-state equation (A.3) transforms into

$$\dot{\lambda}_B = (1+r)\lambda_B,\ \lim_{t\to\infty} \mathrm{e}^{-rt}\lambda_B(t) = 0. \tag{B.4}$$

This implies a co-state value of zero for all t and thus, by Eq. (A.2), an investment of zero. With zero investment of firm B, the distance between technologies increases, which in turn increases the level of the strategic price. Thus, if the strategic price can be implemented at time zero, it will stay at that value for all the time until the entry of the firm B is possible. This implies the following result.

Lemma 8 (Possibility of strategic pricing).
Whenever

$$\delta_0 > 2\,\bar{x}_B, \tag{B.5}$$

it is possible for the leading firm A to set the strategic price (25) *and prevent firm B from developing its technology.*

C Value functions

For strategic and monopoly regimes:

$$\Pi_A^S = \int_0^\infty \mathrm{e}^{-rt} \left\{ p_A^S N_A^S - \frac{1}{2} \left(g_A^S\right)^2 \right\} =$$
$$= \frac{S_1^S}{1+r}\delta_0 + H_A, \tag{C.1}$$
$$\Pi_A^M = \int_0^\infty \mathrm{e}^{-rt} \left\{ p_A^M N_A^M - \frac{1}{2} \left(g_A^M\right)^2 \right\} =$$
$$= \frac{S_1^M - \frac{1}{2}\left(F_1^M\right)^2}{\sqrt{X_m}}\delta_0^2 - 2\frac{\sqrt{X_m}\left(M^*\left(F_1^M - S_1^M Q_A\right) + \frac{1}{2}F_1^M F_2^M - S_2^M\right) + r\left(S_1^M M^* + \frac{1}{2}F_1^M F_2^M\right)}{\sqrt{X_m}(r+\sqrt{X_m})}\delta_0 + M_A \tag{C.2}$$

where the coefficients M_A, H_A, M^* are given in the Appendix F and depend only on the demand parameters and efficiency of investments of firm A. Denote the roots of the polynomials (C.1), (C.2) by δ_S^A, δ_M^A.

Under the simultaneous development regime values of both firms at initial time are:

$$\Pi_B^* = \int_0^\infty \mathrm{e}^{-rt} \left\{ p_B^* N_B^* - \frac{1}{2} \left(g_B^*\right)^2 + \lambda_B (g_A^* Q_A - g_B^* Q_B - \delta) \right\} dt \tag{C.3}$$
$$\Pi_A^* = \int_0^\infty \mathrm{e}^{-rt} \left\{ p_A^* N_A^* - \frac{1}{2} \left(g_A^*\right)^2 + \lambda_A (g_A^* Q_A - g_B^* Q_B - \delta) \right\} dt. \tag{C.4}$$

Again, all new parameters are given in the Appendix F. Denote the roots of the polynomials (C.3), (C.4) by δ_*^B, δ_*^A, respectively. We assume that parameters of the game are such that leading coefficients in all profit functions are positive for firm B and negative for firm A implying their profit functions depend on δ_0 in opposite directions.

The duration of a strategic pricing that is necessary to prevent the entry of firm B, is thus given by the time, when the δ resulting from strategic pricing ($\delta^S(t)$) equals δ_*^B. We denote this by t^P.

Firm A will be able to permanently prevent the follower from entering the market by temporary strategic pricing, whenever $t^P < \infty$. If the polynomial (C.3) has no positive real roots, only the permanent strategic pricing may prevent the enter of firm B to the market.

So far, our analysis has produced the following result.

Lemma 9 (Prevention of entry).

For temporary strategic pricing to result in a permanent prevention of development of technology B, it is necessary that the roots of polynomial (C.3) *are real and that*

$$\min\{\delta_*^B\} \leq \bar{\delta}^S. \tag{C.5}$$

Then firm A may prevent entrance of B by strategically pricing during

$$t^P = -\ln\left(\frac{S_1^S Q_A^2 + (1+r)q_B(0) - (1+r)\delta_{*B}}{S_1^S Q_A^2 - (1+r)\delta_0}\right). \tag{C.6}$$

Otherwise, a permanent prevention of entry is only possible via permanent strategic pricing.

This result shows what firm A has to do, in order to keep firm B permanently out of the market. The next question is whether it is optimal for firm A to act in this way.

D Piecewise monopoly dynamics

The profit stream under temporary strategic pricing Π_A^P, defined by Eq. (29), is bounded by (C.1) from below and by (C.2) from above. It is a decreasing function of t^P. Thus if $\Pi_A^S \geq 0$, the strategic pricing is feasible for any t^P. Moreover, even for negative Π_A^S, there might exist a $t^P < \infty$ such that strategic pricing is still feasible, if we have $\Pi_A^M > 0$. Therefore those two profit streams give a sufficient and necessary condition for the feasibility of strategic pricing.

Lemma 10 (Feasibility of strategic pricing).

For strategic pricing to be feasible, it is necessary that $\delta_0 \geq \min\{\delta_M\}$. It is sufficient that $\delta_0 \geq \delta_S$, where δ_S, δ_M are roots of equations (C.1), (C.2).

The next question is whether strategic pricing is also *profitable*. This question can be reduced to comparing (29) and (C.4). As long as both (29) and (C.4) are positive, firm A has an incentive for strategic pricing for the duration t^P, whenever

$$\exists 0 < t^P : \Pi_A^P - \Pi_A^* \geq 0. \tag{D.1}$$

Using Lemma 10, we can express this condition again in terms of roots of polynomials. Note that the differences in profit streams is the second degree polynomial in δ_0. The roots of this polynomial characterize the threshold levels of profit streams, for which the monopolistic and permanent strategic regimes are more profitable than the simultaneous development regime. As the temporary strategic pricing regime leads to a profit stream in between monopolistic and permanent strategic pricing, we get the following result.

Lemma 11 (Profitability of strategic pricing)**.**
For strategic pricing during $t^P > 0$ to be profitable, it is necessary that $\delta_0 \geq \min\{\delta_-\}$, and it is sufficient that $\delta_0 \geq \min\{\delta_d\}$, where δ_-, δ_d are the roots of polynomials $\Pi_A^M - \Pi_A^*$ and $\Pi_A^S - \Pi_A^*$, respectively.

E Temporary delay case

This depends on the roots δ_d of the polynomial $\Pi_{A,\infty}^S - \Pi_{A,\infty}^*$, which, by (C.4) and (C.1), can be written as

$$\begin{aligned} 0 = &-\frac{S_1^A - \frac{1}{2}\left(F_1^A\right)^2}{\sqrt{X}}\delta_d^2 + \frac{S_1^S}{1+r}\delta_d + \\ &+ 2\frac{\sqrt{X}\left(F_1^A\left(F_3^A + \frac{1}{2}F_2^A\right) + S_1^A G^* - S_2^A\right) - r\left(S_1^A G^* - \frac{1}{2}F_1^A F_2^A\right)}{\sqrt{X}(r+\sqrt{X})}\delta_d + H_A - G_A \end{aligned} \tag{E.1}$$

As long as at least one positive real root δ_d exists, it gives a criteria to stop strategic pricing, if it is implemented. The time till which market entrance of firm B is delayed is then given by

$$t^d = -\ln\left(\frac{S_1^S Q_A^2 + (1+r)q_B(0) - (1+r)\delta_d}{S_1^S Q_A^2 - (1+r)\delta_0}\right) \tag{E.2}$$

The incentive for a strategic delay of entrance is given by the discounted profit stream of firm A under switching from strategic price to the simultaneous development regime at time t^d and at the level of technology δ_d:

$$\Pi_{A,t^d}^S = \int_0^{t^d} \mathrm{e}^{-rt}\left\{p_A^S N_A^S - \frac{1}{2}\left(g_A^S\right)^2\right\}dt + \int_{t^d}^{\infty} \mathrm{e}^{-rt}\left\{p_A^* N_A^* - \frac{1}{2}\left(g_A^*\right)^2\right\}dt \geq \Pi_{A,\infty}^* \tag{E.3}$$

We conclude this case with the following result.

Lemma 12 (Strategic delay).
For strategic pricing to result in a temporary delay of firm B entry, it must hold that

$$\min\{\delta_d\} \leq \delta_0 < \max\{\delta_d\} \leq \bar{\delta}^S < \min\{\delta_{*B}\} \tag{E.4}$$

with delay duration t^d.

If

$$\min\{\delta_d\} \leq \delta_0 \leq \bar{\delta}^S < \max\{\delta_d\},\ \bar{\delta}^S < \min\{\delta_{*B}\} \tag{E.5}$$

only permanent strategic pricing is profitable and effective.

Proof. As long as (E.4) is met, it is initially profitable for firm A to use strategic pricing. But it cannot shift to monopolistic pricing, as δ never becomes large enough to prevent firm B's entry altogether. Firm A will thus stop strategic pricing, if it is no longer profitable compared to the simultaneous development case. In contrast, if the value of δ at which this occurs is higher than the value required by (E.5), the strategic pricing regime will hold forever. □

F List of coefficients

Optimal investments parameters competitive case:

$$\begin{aligned}
F_1^A &= \frac{Q_A S_1^A}{r+2+\sqrt{X}},\ F_2^A = \frac{Q_A S_1^A (S_2^A Q_A^2 + S_2^B Q_B^2)}{(r+2+\sqrt{X})(2S_2^A Q_A^2 + 2S_2^B Q_B^2 - (1+r))},\\
F_3^A &= \frac{Q_A(2S_1^A S_2^B Q_B^2 + S_2^A(1+r-2S_1^B Q_B^2))}{(2S_2^A Q_A^2 + 2S_2^B Q_B^2 - (1+r))(1+r)},\\
F_1^B &= \frac{Q_B S_1^B}{r+2+\sqrt{X}},\ F_2^B = \frac{Q_B S_1^B (S_2^A Q_A^2 + S_2^B Q_B^2)}{(r+2+\sqrt{X})(2S_2^A Q_A^2 + 2S_2^B Q_B^2 - (1+r))},\\
F_3^B &= \frac{Q_B(-2S_1^B S_2^A Q_A^2 + S_2^B(2S_1^A Q_A^2 - 1 - r))}{(2S_2^A Q_A^2 + 2S_2^B Q_B^2 - (1+r))(1+r)}
\end{aligned} \tag{F.1}$$

Optimal investments parameters monopolistic case:

$$\begin{aligned}
F_1^M &= \frac{\left(S_1^M Q_A^2 - \frac{1}{2}(1+r)\right)(2+r-\sqrt{X_m})}{Q_A(2S_1^M Q_A^2 - (1+r))},\ F_2^M = \frac{1}{2}\frac{S_2^M Q_A^2 (2+r-\sqrt{X_m})}{Q_A(2S_1^M Q_A^2 - (1+r))},\\
F_3^M &= \frac{S_2^M Q_A}{2S_1^M Q_A^2 - (1+r)}
\end{aligned} \tag{F.2}$$

Competitive profits coefficients:

$$
\begin{aligned}
G^* &= \frac{(S_2^A Q_A^2 + S_2^B Q_B^2)}{2(S_1^A Q_A^2 + S_1^B Q_B^2) - (1+r)}, \\
G_B &= \frac{1}{2}\frac{2S_1^B (G^*)^2 X - 4S_1^B (G^*)^2 \sqrt{X} r + 2S_1^B (G^*)^2 r^2 - 2S_2^B G^* X + 2S_2^B G^* \sqrt{X} r}{r\sqrt{X}(r+\sqrt{X})} + \\
&\frac{1}{2}\frac{2S_3^B \sqrt{X} r + 2S_3^B X - 4F_2^B F_3^B \sqrt{X} r - (F_2^B)^2 r(\sqrt{X}+r)}{r\sqrt{X}(r+\sqrt{X})}, \\
G_A &= \frac{1}{2}\frac{2S_1^A (G^*)^2 X - 4S_1^A (G^*)^2 \sqrt{X} r + 2S_1^A (G^*)^2 r^2 - 2S_2^A G^* X + 2S_2^A G^* \sqrt{X} r}{r\sqrt{X}(r+\sqrt{X})} + \\
&\frac{1}{2}\frac{2S_3^A \sqrt{X} r + 2S_3^A X - 4F_2^A F_3^A \sqrt{X} r - (F_2^A)^2 r(\sqrt{X}+r)}{r\sqrt{X}(r+\sqrt{X})}
\end{aligned} \tag{F.3}
$$

Strategic profit coefficient:

$$
\begin{aligned}
&H_A = \\
&\frac{1}{2}\,\frac{(x_A + 2x_B)\left((-4x_B + 6q_B(0))\, r^2 + (-8x_B + 12q_B(0))\, r + {Q_A}^2 z x_A + 2\,{Q_A}^2 z x_B - 4x_B + 6\,q_B(0)\right) z}{r\,(r+1)^2}
\end{aligned} \tag{F.4}
$$

Monopolistic profit coefficients:

$$
\begin{aligned}
M^* &= \frac{S_2^M Q_A}{2S_1^M Q_A^2 - (1+r)}, \\
M_A &= \frac{1}{2}\frac{\left(2\,S_1^M\,(M^*)^2 {Q_A}^2 + 2\,S_2^M M^*\,Q_A + 2S_3^M\right) X_m}{r\sqrt{X_m}(r+\sqrt{X_m})} - \\
&\frac{4\,\left(S_1^M (M^*)^2 {Q_A}^2 + \left(1/2 S_2^M Q_A + F_2^M\right) M^* + \frac{1}{4}(F_2^M)^2 - S_3^M/2\right) r\sqrt{X_m}}{r\sqrt{X_m}(r+\sqrt{X_m})} + \\
&\frac{r^2\left(2\,S_1^M (M^*)^2 (Q_A)^2 - (F_2^M)^2\right)}{r\sqrt{X_m}(r+\sqrt{X_m})}
\end{aligned} \tag{F.5}
$$

Coefficient for sufficient subsidy:

$$
\begin{aligned}
X_{suff} &= \sqrt{X_1 \delta^2 + X_2 \delta + X_3} \\
X_1 &= z^2 + 32z + 54 \\
X_2 &= (4z^2 + 152z - 272)x_A + 32x_B(z-6) \\
X_3 &= (4z^2 + 116z - 608)x_A^2 - 64x_A x_B(z+6) - 64x_B^2 z
\end{aligned} \tag{F.6}
$$

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