Universität Basel
Wirtschaftswissenschaftliche Fakultät

WWZ forum

January 2018

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WWZ Working Paper 2018/01

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Robust policy schemes for R&D games with asymmetric information*

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January 16, 2018

Abstract

We consider an abstract setting of the differential r&d game, where participating firms are allowed for strategic behavior. We assume the information asymmetry across those firms and the government, which seeks to support newer technologies in a socially optimal manner. We develop a general theory of robust subsidies under such one-sided uncertainty and establish results on relative optimality, duration and size of different policy tools available to the government. It turns out that there might exist multiple sets of second-best robust policies, but there always exist a naturally induced ordering across such sets, implying the optimal choice of a policy exists for the government under different uncertainty levels.

Keywords: technology lock-in, technological change, strategic interaction, uncertainty, robust policy sets, uncertainty thresholds, robust welfare improving policy, axiom of choice

JEL classification: C02, C61, O31, O38

*This research is part of the activities of SCCER CREST (Swiss Competence Center for Energy Research), which is financially supported by the Swiss Commission for Technology and Innovation (CTI).

1 Introduction

For many environmental problems, a shift to green technologies is considered to be a promising long-term solution. A prominent example is climate change, where much hope rests on a transition from fossil fuel based technologies to renewable energy sources. Another example is traffic-related air pollution, where cleaner engines or e-mobility provide opportunities to reduce pollution levels substantially.

In this context, a crucial question is whether and to what extent a government should interfere with technological change. It is obvious that an internalization of environmental externalities is important to provide incentives for developing clean technologies. Arguably, competition among technologies will seek out the best technological solutions once environmental damages are correctly priced. But many countries use considerably more fine-grained approaches to steer details of technological change. A prominent example are feed-in tariffs for renewables. By using different tariffs for different technologies, many countries make sure that a broad set of technologies is developed and used. Often this approach eliminates competition among technologies (as less efficient technologies are subsidized to an extent that ensures their use) and thus replaces market-based technology selection with politically set targets for technology development and diffusion. Subsidies for different new transport technologies (fuel cells, e-mobility) work in a similar way.

Not surprisingly, many economists are skeptical regarding this approach and argue that governments might lack the necessary information to ensure efficient investments in different options for green technologies. However, there are also economic arguments in favor of detailed incentive schemes. Numerous studies have shown that almost unavoidable market failures can lead to a technology lock-in; typical examples are lock-ins caused by market power that is due to patents for new technologies (see, e.g., Krysiak (2011)) or externalities caused by network effects in technology adoption (see Arrow (1962), Arthur (1989), (Unruh, 2000), or (Unruh, 2002)). In such cases, it is not sufficient to only set a price for environmental damages to ensure that the best clean technologies are developed; more specific incentives are necessary (Krysiak, 2011).

The size and duration of such specific interventions will typically depend strongly on different cases of market failures. For example, the development of a new promising technology might only be delayed or it could be prevented completely, rendering different interventions necessary.

However, in many cases it is not easy to assess the type and scope of market failures that might require an intervention. This holds in particular, as the potential of yet to be developed technologies cannot be predicted with certainty. It is often hard to say whether a new technology is not developed, because market actors expect that it is an inferior solution (and thus do not invest) or because some actors with incumbent technologies use their power to forestall the development of a superior competition. Furthermore, it is hard to assess whether a development is forestalled or only delayed.

In this paper, we investigate how qualitatively different types of market failures arising in technological change require different policy actions. We use a fairly general model where incumbent firms might or might not have an incentive to keep new technologies out of the market or to delay its entrance. A government could, in addition to internalizing an externality, provide specific support for the new technology. We show that different cases of market failure can arise and require different levels and duration of an intervention. We provide a set of general results on the multiplicity and ordering of potential subsidizing schemes and elaborate on the criteria for selection of the most appropriate one. The novelty of our approach is that it does not require any specific assumptions and is based solely on the notion of the choice function over sets.

Robust control has been used in a number of applications in environmental and energy economics. Studies on climate negotiations use robust optimization, as in (Babonneau et al., 2013) or (Ben-Tal, El Ghaoui and Nemirovski, 2009). The robust control approach has been used to investigate government interventions in environmental problems, in particular related to the precautionary principle, as, for example, in (Athanassoglou and Xepapadeas , 2012) or (Vardas and Xepapadeas, 2010). Other applications are found, for example, in asset management, see (Vardas and Xepapadeas, 2015).

These studies are based on the minmax approach, where a planner tries to minimize a threat and the realization of uncertain variables is chosen by a malevolent nature to maximize damages. One particularly interesting formal analysis of the robustness approach is given by (Todorov, 2009), where a Kullback-Leibler entropy measure is used. Our analysis does not explicitly account for the minmax problem on behalf of the government, but the results on robustness are still following the same pattern. In particular we answer the question, what is the crucial level of noise (uncertainty) which makes a given policy rule ineffective.

We contribute to this literature by using a generalized robustness approach, where the robustness criteria naturally follows from the choice function over the set of outcomes. The closest paper to our approach is (Brock et al., 2014), where the notion of hot spots is introduced to mark the cases where uncertainty may break down the regulation or lead to instability of the underlying system. Our approach is somewhat more general yet simpler than in the aforementioned paper. The decision rules we develop are related to hotspots type I and type III of that paper, but we do not elaborate on stochastic differential systems and their treatment here by assuming r&d firms to be fully informed on capabilities of the competitor. Our study is rather concentrated on obtaining the simplest possible policy rules, separating qualitatively different outcomes.

We consider the problem where the government has limited information, which makes it impossible to discern clearly what type of intervention is required. To gain insight into this problem, we use a robustness approach, that is, we assume that the government knows an upper and a lower boundary to the potential of the new technology, but it does not know a probability distribution over possible potentials. Given this information, the government aims to use a robust policy, that is, a policy that works well even in the most adverse possible cases. Using this approach, we show that more extensive uncertainty will lead to higher and longer subsidies for the new technology as long as an intervention still remains rationale.

This analysis shows that there is indeed some economic rationale for a time-limited intervention in the process of technological change, even if government cannot fully ascertain what level and duration of support is required. In the following we set up the abstract model, describe policy schemes necessary to prevent different types of lock-in and obtain robust policy sets, evaluating associated welfare costs of uncertainty.

2 The model

In this paper we pursue the development of a general yet simple theory of robust decision making on behalf of the social planner in the environment of one-sided uncertainty. We thus need an underlying dynamics of the (fully informed) market participants which provide the information to social planner concerning the state of the economy. To specialize, we study the underlying market as an r&d differential game in the spirit of (Ben-Youssef and Zaccour, 2014), (Bondarev, 2014). We thus neglect any production side dynamics, which can be easily incorporated as in (Krysiak, 2011).

In particular we assume that in the production sector, there is perfect competition and final producers are price takers. In the r&d sector, the firms get a patent for their developments and are thus monopolistic suppliers of their technology. Some of the firms have an initial advantage (their technologies being somewhat more developed initially) and thus might act strategically to forestall the use and development of the new technologies.

All r&d firms know with certainty technology characteristics of each other, but the government experiences some uncertainty over the potential of some of the technologies (the newer ones). The government may implement a subsidizing scheme to prevent the strategic behavior of the more developed technology owners, but does so only if this is welfare improving.

In our analysis we abstract from further market imperfections such as environmental externalities, assuming it is already taken care about by proper remuneration schemes in case technologies at hand are dirty and clean ones or both are green. By doing so we apply our study to the case of general innovations setting with green technologies being a specific (but rather important) example of those.

2.1 The general r&d game

In the r&d sector, there is a finite $N \subset \mathbb{N}$ number of firms. Each firm $j \in N$ can invest in r&d and set prices for its own technology. Owing to the patent, each firm is the sole supplier of its technology, thus the market is monopolistically competitive one. We assume the free entry condition with no sunk costs of entry.

We assume the evolution of technology j may depend on own firm's investments as well as on investments of other firms and on the current state of own technology and technologies of other firms:

$$\forall j \in N: \ \dot{q}_j(t) = f_j(q_j(t), q_{-j}(t), g_j(t), g_{-j}(t)), \tag{1}$$

where we assume all f_j are continuous in their arguments, $-j$ index denotes all other firms except j. The dynamics of all technologies (1) forms a controlled N-dimensional ODE system.

The objective of every firm is to maximize its discounted stream of profits (value) for a given discount rate r choosing optimally price schedule and investments:

$$J_j = \max_{p_j, g_j} \int_0^\infty \mathrm{e}^{-rt} \left\{ \Pi_j(p_j(t), q_j(t), p_{-j}(t), q_{-j}(t)) - c_j(g_j(t)) \right\} dt. \tag{2}$$

where we additionally assume that profit of firm j, Π_j may be a function of prices of all firms as well as of technologies, but costs associated with development of technology j depend only on firm's j own investments (but can be heterogeneous across firms).

Given some final producers' demand system

$$\forall j \in N: \ Q_j^D = F_j(p_{j,-j}, q_{j,-j}) \tag{3}$$

for N firms present at the market, we get N reaction functions for prices of technologies j and as a result an N-dimensional system for price schedules as functions of technology states of all the firms, $\forall j \in N: \ p_j = w_j(q_j, q_{-j})$ where we assume only w_j to be continuous functions of $q_{j,-j}$.

We thus reduce the problem of N firms given by (1), (2) to the differential game over technologies states q_j with controls g_j. To keep the constructive nature of the exposition we limit ourselves to open-loop solution concept, since closed loop one does not always exist. As long as the controlled system (1) permits for the solution q_j^* we denote

$$V_j^* = \max_{g_j} \int_0^\infty \mathrm{e}^{-rt} \left\{ \Pi_j(p_j(q_{j,-j}^*), q_{j,-j}^*(t), p_{-j}(q_{j,-j}^*)) - c_j(g_j(q_{j,-j}^*)) \right\} dt = V_j^*(q_{j,-j}(0)) \tag{4}$$

value function of the firm j as a function of initial states of all technologies $j \in N$ with superscript $*$ denoting the simultaneous development regime (solution of N-players differential game).

Without loss of generality we further denote $\mathcal{O}$ the set of all possible outcomes of this game and by $\mathcal{F}$ the set of feasible outcomes (under given initial conditions and parameters vectors). We note that these sets have finitely many elements as long as the number of players N is finite. Examples of those elements are issues of strategic behaviour of one or more firms on the market, leading to temporary/permanent prevention of entry of new firms. We denote $m \in \mathcal{O}$ an arbitrary outcome of the r&d game and by $q_j^m(t)$ the state of technology j in regime m at time t. We further assume that there exist (unique) equilibrium state vector $\vec{q}^m$ of the r&d game, to which technologies converge in the long run in regime m[1].

2.2 Government

The government has the objective of maximizing the net social benefit from all the technologies. This net social benefit consists of a marginal benefit β_j attached to each unit of production with technology j minus locational costs, minus the costs of developing the technologies. For simplicity, we assume that the social planner uses the same discount rate r as the r&d firms. Social welfare is thus given by:

$$W_m := \int_0^\infty \mathrm{e}^{-rt} \sum_{j=1}^{N} \left\{ \beta_j Q_j^m(t) \left(q_j^m(t) + \Xi_j^m(t) \right) - c_j \left(g_j^m(t) \right) \right\} dt, \tag{5}$$

where $\Xi_j(t)$ denotes the average effect of used locations on output for technology j. In general these are functions of prices and technologies' states, $p_{j,-j}, q_{j,-j}$.

The government may use two policy instruments. First, there is the remuneration for green production (measured by β_j). Second, it might subsidize the initially disadvantaged technologies for some time.

These instruments have to correct three market failures, two of which are well known. First, without the remuneration, there would be no green production. Second, as r&d

[1]of course this vector may include infinite elements if some technologies do not have a steady state, but we restrict the analysis to those cases one parameters' space allow only for finite values of this vector.

firms have market power, they will set socially suboptimal prices and thus green technologies will be used less than in the social optimum. Finally, the firms developing the initially more advanced technologies might use their market power to set prices that keep the other firms from investing into developing the newer technologies.

In this paper, we will focus on this third problem, because this is new and could be particularly detrimental, as the development of a technology with high potential might be prevented indefinitely. We thus directly consider the point raised in the introduction: Should a government only provide a general incentive for using green technologies (such as a price for GHG emissions) or should it also steer technological change by using technology-specific subsidies?

2.3 One-sided uncertainty

To investigate a setting that is both scientifically interesting and practically relevant, we focus on the case, where the government knows initial technology vector $\vec{q}(0)$ which is invariant across regimes of the game, but does not know the long-run potential of some of the new technologies $i \in I \subset N$. Firms themselves are fully informed over characteristics of both old and new technologies. This is often the case in real industries, since industry players put more efforts into learning their competitors capabilities than the regulating authority. Without loss of generality we further assume that at any moment there is exactly one new technology $i \in N$ which is entering the market and the planner does not know its potential, but has some limited information about boundaries of this potential[2]. Since we do not specify the dynamics of technologies, (1) we also restrain from specifying where this uncertainty comes from. It is sufficient for our purposes to assume that at any moment in time government knows the state of the new technology i with some certainty:

$$q_i(t) \in [q_i(t) - \epsilon, q_i(t) + \epsilon],\ \varepsilon \overset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon),\ \sigma_\varepsilon = \epsilon^2 \tag{6}$$

[2]indeed this is not a binding assumption, since in the case of simultaneous entry of several firms the problem may be decomposed into the sequence of problems with single entry, see discussion on the sequence of pairwise games of the leader and the follower in Bondarev and Greiner (2017) and in the current paper later in Sec. 3

and the government is not able to learn the true state over time (otherwise the problem becomes trivial).

As we will show, an increasing value of ε implies that the government is less able to differentiate between different cases of strategic and non-strategic behavior of the incumbent firms and thus to ascertain that and how long a subsidy is required. We thus look for a robust policy that can cope with several cases at once.

3 Analysis of the model

3.1 Characterization of the r&d game outcomes

We first characterize the multiplicity of outcomes of the underlying r&d game under full information on behalf of competing firms. Once the solution for the game exists, it defines the vector of value functions of participating firms as a function of initial states of all the technologies, (4). Those firms, which have initial advantage, $l \in N : q_l(0) > q_j(0)$ may choose strategic behavior to keep competitors out of the market, creating the multiplicity of outcomes. Whether or not such strategic behavior is optimal depends on comparison of values generated by competitive and strategic behavior for every such firm l. Assume for certainty there is a ranking of initial technologies states, such that:

$$q_1(0) \geq q_2(0) \geq .. \geq q_N(0) \tag{7}$$

so that the firm 1 has initial advantage over all other firms and the next firm has advantage over the rest of $N-2$ firms, etc. Then firm 1 decides whether or not to implement strategic pricing and at which level as following: it may set the price at the level $p_1^{S(1)}$ such that the profit for all other firms is zero if they enter the market and keep this schedule for some time. If this turns to be not feasible or not profitable, it can set the price at the level $p_1^{S(2)}$ as to keep all competitors except the closest one out of the market. Continuation of this argument yields a descending sequence of strategic prices for firm 1, $p_1^{S(1)} < p_1^{S(2)} < .. < p_1^{S(N)}$ such that the latest strategic price keeps off the market only the firm N.

If the first firm sets the strategic price $p_1^{S(i)}$ allowing for entrance of $i < N$ firms, those firms upon entrance may play the best response price or again act strategically. However the leader under full information may predict actions of all the followers and sets the strategic price effectively determining the number of competitors. We thus may reduce the problem to the case of only two firms, since every next competitor just repeats the decision process of preceding firms upon the strategic price setting.

We thus denote the incumbent firm A and the new entrant B. We do not specify the number of potential strategic regimes other than assuming them to be finite. Denote by Π_j^m the value generated by the outcome m of the underlying r&d game for the firm j. Observe next that from the above discussion it follows that it is the leader (firm with maximal initial state of technology) which defines the regime of the game. At last note that value function of any firm j is a function of initial states of technologies and parameters only. Denote by $\delta(0) = q_1(0) - q_2(0)$ the initial technological gap between the leader and the closest competitor (potential entrant). Denote further by $\delta_m^j(0)\{z\}$ the z-th root yielding to zero the difference in values generated by the game for the firm j across outcomes i, j[3].

We then have:

Proposition 1 (Algebraization of r&d game outcomes)**.**
The outcome $i \in \mathcal{F}$ of the r&d game is individually optimal if for the leader j there exists the choice function:

$$\begin{aligned}
&\Theta(\mathcal{F}) : \delta(0) \in: \bigcup_{m \in \mathcal{F}} \bigcap_{m \in \mathcal{F}} [\delta_m^j(0)\{z\}; \delta_m^j(0)\{z+1\}] : \\
&\Pi_j^i - \Pi_j^m \geq 0 \implies \\
&\Theta(\mathcal{F}) = \arg\max_{m \in M} \Pi_j^m = i. \qquad (8)
\end{aligned}$$

Proof. The proof simply follows from the defintion of value functions Π_j^m and the fact that the choice function and the selector exist for any finite (and countable) collection of arbitrary sets satisfying the axiom of choice. $\square$

[3]we assume the value functions of all potential regimes has finitely (or countably many) complex roots, which is the case for any value function represented via some analytic function over $\delta(0)$

We now move to the uncertain part of the problem. Since government experiences uncertainty over the true potential of technology B, it cannot assign the first-best subsidy as usual[4]. We thus start with definition of social policy in our setup, then define social optimality under uncertainty and work out what we call robust policy schemes preventing strategic behavior.

3.2 Social welfare under uncertainty

We assume from now on initial level of technology A being fixed as well as other parameters of the model except for $q_B(0), Q_B$.

First we define the social optimality measure under robustness ϵ for given q_B[5]:

Definition 1 (Social optimality under uncertainty)**.**
The outcome m of the r&d game $s \in \mathcal{O}$ is (weakly) social welfare improving over the outcome m with robustness level ϵ if

$$\forall \varepsilon \in [-\epsilon, \epsilon] : \min_{\varepsilon \in [-\epsilon, \epsilon]} \{W_s^\varepsilon(q_B)\} \geq \min_{\varepsilon \in [-\epsilon, \epsilon]} \{W_m^\varepsilon(q_B)\} \tag{9}$$

it is strongly welfare improving if

$$\forall \varepsilon \in [-\epsilon, \epsilon] : \min_{\varepsilon \in [-\epsilon, \epsilon]} \{W_s^\varepsilon(q_B) - W_m^\varepsilon(q_B)\} \geq 0 \tag{10}$$

where $W_{s,m}^\varepsilon(q_B)$ are given by integral Eq. (5) with $Q_B = Q_B + \varepsilon$.

The outcome s is (weakly) socially optimal with robustness level ϵ if

$$\forall \varepsilon \in [-\epsilon, \epsilon], \forall m \in \mathcal{O} : \arg\max_{m \in \mathcal{O}} \min_{\varepsilon \in [-\epsilon, \epsilon]} \{W_m^\varepsilon(q_B)\} = s \tag{11}$$

it is strongly welfare optimal if

$$\forall \varepsilon \in [-\epsilon, \epsilon], \forall m \in \mathcal{O} : \arg\max_{m \in \mathcal{O}} \min_{\varepsilon \in [-\epsilon, \epsilon]} \{W_s^\varepsilon(q_B) - W_m^\varepsilon(q_B)\} = s \tag{12}$$

[4] in the first-best case the government plays a Stackelberg differential game being the first mover and setting the subsidy size and duration such as to maximize social welfare

[5] We omit time argument in integral quantities W, Π and understand $q_B = q_B(0), \delta = \delta(0)$ in what follows if this does not lead to confusion.

The definition of social optimality (11) requires to obtain minimum possible welfare for every regime m over realization of the noise ε and then to take the maximum across regimes. The regime which provides maximal welfare under the most unfavorable circumstances ($\min_\varepsilon$) is socially optimal with certainty (robustness) level ϵ, if its minimum strictly dominates other minima.

Strong optimality requires that regime s has higher welfare under the most unfavourable circumstances then other regimes have under the most favourable ones, since $\min\{x-y\} = \min\{x\} - \max\{y\}$. Apparently the strong optimality holds if there are no intersections of welfare functionals as functions of $\varepsilon \in [\pm\epsilon]$ and corresponds to the case when uncertainty is *inessential*.

It is straightforward that under $\epsilon \to 0$ the full certainty social welfare difference $\min_\varepsilon\{W_s^\varepsilon(q_B)\} - \min_\varepsilon\{W_m^\varepsilon(q_B)\} = W_s(q_B) - W_m(q_B)$ is recovered.

To establish social optimality it thus suffices to consider the differences in social welfare across different regimes of the r&d game.

$$\forall \in \mathcal{O} : \; D_{s,m}(W) \stackrel{def}{=} W_s(q_B) - W_m(q_B) \tag{13}$$

and their robust counterparts as:

$$D_{s,m}^\epsilon(W) \stackrel{def}{=} \min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_s^\varepsilon(q_B)\} - \min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_m^\varepsilon(q_B)\} \tag{14}$$

The regime which is robust welfare optimal would yield positive differences with all other regimes m, but we cannot apply max operator over these differences to select the best outcome as in Def. 1.

To obtain such a procedure to select the socially optimal robust outcome we establish the result concerning social welfare. To this end we make use of the algebraization tools applied to the difference in minima, (14).

First observe that robust social welfare values, $W_m^\epsilon(q_B) = \min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_m^\varepsilon(q_B)\}$ may be treated as an analytic function in $q_B(0)$ since min operator gives a unique ε value, which is simply

$$\varepsilon_m^\epsilon = \arg\min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_m^\varepsilon(q_B)\} \tag{15}$$

by the theorem on the average value of a function. However, these values are in general different for different m, thus $D^{\epsilon}_{s,m}(W)$ depends on both $\varepsilon^{\epsilon}_{m}, \varepsilon^{\epsilon}_{s}$ values. Still it is analytic in $q_B(0)$ with coefficients depending on given robustness level ϵ, since $\varepsilon^{\epsilon}_{m,-m} \in [\pm\epsilon]$.

We denote associated roots of such functions for every pair $m, -m$ in $q_B(0)$ by $q_B^{m,-m}(\epsilon)\{z\}$ where z is the index of the root such that $q_B^{m,-m}(\epsilon)\{z+1\} > q_B^{m,-m}(\epsilon)\{z\}$. The full certainty case is recovered with $\epsilon \to 0$.

We next formulate the following Proposition:

Proposition 2 (Social welfare algebraization under uncertainty)**.**
The outcome $s \in \mathcal{F}$ of the r&d game is socially optimal among outcomes $\mathcal{F} \subseteq \mathcal{O}$ under robustness level ϵ if $q_B(0)$ lies in the union of intervals where social welfare is higher under outcome s than under any other $m \in \mathcal{F}$, i. e. there exists the choice function:

$$\begin{aligned} &\Psi_\epsilon(\mathcal{F}) : q_B(0) \in \bigcup_{m\in\mathcal{F}} \bigcap_{m\in\mathcal{F}} [q_B^{s,m}(\epsilon)\{z\}; q_B^{s,m}(\epsilon)\{z+1\}] : \\ &D^{\epsilon}_{s,m}(W) \geq 0 \implies \\ &\Psi_\epsilon(\mathcal{F}) = \arg\max_{m\in\mathcal{F}} \min_{\varepsilon\in[-\epsilon;\epsilon]} W^{\varepsilon}_{m}(q_B) = s. \end{aligned} \tag{16}$$

The outcome of the game s is robust welfare maximizing up to the level ϵ^W_s if $\forall\epsilon < \epsilon^W_s$: $\Psi_\epsilon(\mathcal{F}) = s$.

Proof. As long as the worst-case outcome s is better than the worst-case outcome m for given ϵ, it follows that $D^{\epsilon}_{s,m}(W) \geq 0$. These objects are analytic functions in $q_B(0)$, depending on robustness ϵ thus roots $q_B^{s,m}(\varepsilon^{\epsilon}_{s,m})\{z\}$ form a sequence of intervals where outcome s is social welfare improving or not over m. Select those ranges of $q_B(0)$ which yield positive value for this polynomial. Repeat this process for all $m \in \mathcal{F}$. Outcome s is better than any collection of other outcomes from $\mathcal{F}$ as long as all differences $D^{\epsilon}_{s,m}(W)$ are positive (since Def. 1). Ranges of $q_B(0)$ where this condition remains valid are given by the union of all intervals associated with positive difference for all $D^{\epsilon}_{m,s}(W)$. Union of those intervals gives the total range, where outcome s is better than any $m \in \mathcal{F}$ hence (16). The last claim is just an observation that choice function depends on the uncertainty level: once we change ϵ, it could be the case that $q_B(0)$ no longer lies in intervals of positive sign and outcome s is no longer maximizing worst-case welfare (although it still can be optimal in full certainty case). □

This proposition gives the criteria for comparing any regimes in terms of social welfare of the R&D game for fixed ϵ: we need to compute values W_m^ϵ , and then compare them for the given $q_B(0)$. As long as the difference between $m, -m$ functions W is positive, regime m is robust welfare improving over regime $-m$. Thus computing roots of this difference provides the range of $q_B(0)$, for which this ordering holds. Since ϵ is fixed, these roots are functions of parameters and uncertainty level. Thus for any given error size we can establish the ordering or regimes of the underlying r&d game in terms of social welfare. Formally speaking the relationship (16) provides a selector function in the space of functions type $D^\epsilon_{m,-m}(W)$: once the condition of positive difference is fulfilled, it checks whether given $q_B(0)$ falls into one of the provided intervals.

Observe also, that the choice of robust optimal regime depends both on the size of the set $\mathcal{F}$ and the noise level ϵ: it could be the case that competitive outcome is better than strategic forestall for any ϵ, but not so if comparing with the monopolistic development.

We also need the robust criteria of individual optimality. It is done in the same way as for social welfare, albeit for profit functionals of the players. Denote by $\Pi_j^m(\varepsilon)$ total profit of player j in regime m under uncertainty level ε. Observe that this is valid only for social planner, since players do not experience uncertainty at all.

Definition 2 (Robust outcome of the r&d game)**.**
The outcome i of the r&d game is believed to be (weakly) individually optimal across (feasible) outcomes $\mathcal{F} \subseteq \mathcal{O}$ for the leader j with robustness ϵ, if

$$\arg\max_{m\in\mathcal{F}} \min_{\varepsilon\in[\epsilon;\epsilon]} \Pi_j^m(\varepsilon) = i \tag{17}$$

It coincides with actual realization (the belief is robust)

$$\arg\max_{m\in\mathcal{F}} \Pi_j^m = i = \arg\max_{m\in M} \min_{\varepsilon\in[\epsilon;\epsilon]} \Pi_j^m(\varepsilon) \tag{18}$$

if it is strongly individually optimal:

$$\arg\max_{m\in\mathcal{F}} \min_{\varepsilon\in[\epsilon;\epsilon]} \{\Pi_j^i(\varepsilon) - \Pi_j^m(\varepsilon)\} = i \tag{19}$$

This definition provides the criteria for a planner, how to define which regime of the game to expect in the absence of regulation. Still, as the second part points out, the believed regime is not always the actual one, so there is a room for mistake which is that higher, the higher is ϵ.

Corollary 1 (Algebraization of robust outcomes of the r&d game)**.**
The outcome $i \in \mathcal{F}$ of the r&d game is expected to realize among outcomes $M \subseteq \mathcal{O}$ with robustness level ϵ if $\delta(0)$ lies in the union of intervals where worst-case profit is higher under outcome s than under any other $m \in M$ for player j (denoted as the leader), i. e. there exists the choice function:

$$\begin{aligned}
&\Theta_\epsilon(\mathcal{F}) : \delta(0) \in \bigcup_{m\in\mathcal{F}} \bigcap_{m\in\mathcal{F}} [\delta^j_m(\epsilon)\{z\}; \delta^j_m(\epsilon)\{z+1\}] : \\
&\min_{\varepsilon\in[-\epsilon;\epsilon]} \{\Pi^i_j(\varepsilon)\} - \min_{\varepsilon\in[-\epsilon;\epsilon]} \{\Pi^m_j(\varepsilon)\} \geq 0 \implies \\
&\Theta_\epsilon(\mathcal{F}) = \arg\max_{m\in M} \min_{\varepsilon\in[-\epsilon;\epsilon]} \Pi^m_j(\varepsilon) = i. \qquad (20)
\end{aligned}$$

The outcome i is the robust realization of the r&d game with certainty level ϵ^O_i if $\forall \epsilon < \epsilon^O_i :\ \Theta_\epsilon(\mathcal{F}) = i$.

Proof. Amounts to application of results of Prop. 2 to the value functions of the underlying r&d game. □

3.3 Robust subsidies under uncertainty

The social planner may implement the policy scheme consisting of the subsidy and its duration to one of the players to prevent strategic behavior. Under full certainty the first-best subsidy might be implemented, but under the uncertain potentials of technologies this is not the case. The implementation of a subsidy follows multiple steps:

1. Social welfare is computed for all possible regimes of the game, and the best one in the sense of Prop. 2 is selected;

2. The expected regime of the game is defined via Cor. 1;

3. Subsidy is assigned to one of the players in such a way, as to incentivize players to switch to the desired regime;

4. Social welfare for the resulting regime (subsidized) is computed and checked against the otherwise realised non-perturbed regime and profit incentives of players in the resulting regime.

We thus require the subsidy to be robust and social-welfare improving, but not necessarily optimal. Under uncertainty the planner does not know profit incentives of players, but only expected ones subject to the error ε. Hence the definition of socially desirable robust subsidy:

Definition 3 (Robust welfare-optimal policy scheme).
We call a policy scheme the pair $\Sigma_k : \{\sigma_k, t^k\}$ which defines size and duration of the subsidy assigned to player B for certainty.

For each ϵ the robust welfare-improving policy scheme $\Sigma_k^\epsilon(i, s)$ switching the game from i to s is characterized by following:

1. *Regime i is expected to realize without the subsidy in the sense of Def. 2 but regime s is socially optimal in the sense of Def. 1*

2. *The policy scheme Σ_k^ϵ is (weakly) social welfare-improving under uncertainty level ϵ:*

$$\min_{\varepsilon \in [-\epsilon; \epsilon]} \{W_s^\varepsilon(q_B, \Sigma_k^\epsilon)\} - \min_{\varepsilon \in [-\epsilon; \epsilon]} \{W_i^\varepsilon(q_B)\} \geq 0 \tag{21}$$

   It is strongly welfare improving if

$$\min_{\varepsilon \in [-\epsilon; \epsilon]} \{W_s^\varepsilon(q_B, \Sigma_k^\epsilon) - W_i^\varepsilon(q_B)\} \geq 0 \tag{22}$$

3. *The policy scheme is (weakly) robust under uncertainty level ϵ, if this policy allows for the prevention of switching back from the subsidized regime in all cases considered in Proposition 1:*

$$\forall m \in \mathcal{F} : \min_{\varepsilon \in [-\epsilon; \epsilon]} \{\Pi_j^s(\Sigma_k^\epsilon)\} - \min_{\varepsilon \in [-\epsilon; \epsilon]} \{\Pi_j^m(\Sigma_k^\epsilon)\} \geq 0 \tag{23}$$

It is strongly robust if

$$\forall m \in \mathcal{F}: \min_{\varepsilon \in [-\epsilon;\epsilon]} \{\Pi_j^s(\Sigma_k^\epsilon) - \Pi_j^m(\Sigma_k^\epsilon)\} \geq 0 \tag{24}$$

The policy scheme Σ_*^ϵ is (weakly) optimal among all (weakly) robust welfare-improving policy schemes switching from i to s, $\Sigma_k^\epsilon \in \Sigma(i,s)$ if

$$\arg \max_{\Sigma_k^\epsilon \in \Sigma(i,s)} \min_{\varepsilon \in [-\epsilon;\epsilon]} \{W_s^\varepsilon(q_B, \Sigma_k^\epsilon)\} = \Sigma_*^\epsilon \tag{25}$$

It is strongly optimal if

$$\arg \max_{\Sigma_k^\epsilon \in \Sigma(i,s)} \min_{\varepsilon \in [-\epsilon;\epsilon]} \{W_s^\varepsilon(q_B, \Sigma_*^\epsilon) - W_s^\varepsilon(q_B, \Sigma_k^\epsilon)\} = \Sigma_*^\epsilon \tag{26}$$

where by the argument Σ_k^ϵ in social welfare we understand the social welfare obtained under policy scheme Σ_k^ϵ and by $\Pi_j^s(\Sigma_k^\epsilon)$ the value for r&d firms obtained under the given policy scheme in regime s[6].

Robustness is thus understood in this paper as the ability of a policy to perform a given task (prevention of strategic behavior) albeit crucial information is missing while preserving the social welfare at least not lower than in the worst-case scenario under alternative regimes. This concept of robustness is close to the usual min-max approach, since we use maximal confidence intervals for uncertainty and compare worst-case scenarios in terms of welfare. At the same time our concept allows for immediate application due to the the algebraization approach and can be used in the setting with many alternative regimes of the model.

Since there are multiple regimes of the game, the set of robust and welfare improving policy schemes will be different depending both on which regime is the target of subsidy (where the planner wants the game to switch to) and on the actual realization (which regime realizes in the absence of the planner), but policy schemes themselves are defined independently of regimes of the game.

Thus to find an optimally robust subsidy in terms of Def. 3 social planner has to define both the socially desirable outcome with the help of Prop. 2 and the regime of the

[6]this is necessary since introduction of the subsidy changes both $\delta(t), q_{A,B}(t)$ dynamics and thus values for the planner and firms differ from those in regime m without subsidies

game which would actually realize in a non-distorted case. Prop. 2 and Cor. 1 provide tools necessary to define the starting position of a subsidy: where the system would go in unperturbed case and where the planner wants it to go as well as the criteria for optimality and robustness of it:

Corollary 2.
There is a need to subsidize regime s only if $s = \Psi_\epsilon(\mathcal{F}) \neq \Theta_\epsilon(\mathcal{F}) = i$ at $\epsilon < \epsilon_s^W$. Policy scheme Σ_k^ϵ is optimal and robust in switching from i to s at the level ϵ_s^W in the sense of Def. 3 only if

$$\Psi_\epsilon(\mathcal{F}) = \Psi_\epsilon(\mathcal{F}_{\Sigma_k^\epsilon}) = \Theta_\epsilon(\mathcal{F}_{\Sigma_k^\epsilon}) = s \tag{27}$$

where $\mathcal{F}_{\Sigma_k^\epsilon}$ denotes the set of feasible outcomes under the policy scheme Σ_k^ϵ.

In our model it is not the case that simultaneous development of both technologies is always socially desirable. We thus need a general criteria to select the appropriate policy scheme among suitable (welfare-improving and robust) ones.

Moreover, so far we defined robust social welfare, profit incentives and criteria of choice for policy schemes for a fixed level on uncertainty ϵ. It might happen that at some level ϵ^* one of the choice functions changes, i. e. predicts different outcome of the game as socially optimal and/or profit maximizing. We thus arrive to robust policy thresholds:

Proposition 3 (Selection and ordering of robust policy schemes)**.**

1. *If there exists $\epsilon_k^R(i,s) < \epsilon_s^W$ such that $\Theta_{\epsilon > \epsilon_k^R(i,s)}(\mathcal{F}_{\Sigma_k^\epsilon}) \neq \Theta_{\epsilon < \epsilon_k^R(i,s)}(\mathcal{F}_{\Sigma_k^\epsilon}) = s$, the policy scheme Σ_k^ϵ is robust in switching i to s up to level of uncertainty $\epsilon_k^R(i,s)$, otherwise it is globally robust for the pair i, s.*

2. *If there exists $\epsilon_k^S(i,s) < \epsilon_s^W$ such that $\Psi_{\epsilon > \epsilon_k^R(i,s)}(\mathcal{F}_{\Sigma_k^\epsilon}) \neq \Psi_{\epsilon < \epsilon_k^R(i,s)}(\mathcal{F}_{\Sigma_k^\epsilon}) = s$, the policy scheme Σ_k^ϵ is welfare improving for s up to level of uncertainty $\epsilon_k^S(i,s)$, otherwise it is globally improving for the pair i, s.*

3. *Policy scheme Σ_k^ϵ is admissible for the pair $\{i,s\}$ only for $\epsilon \leq \epsilon_k^*(i,s) = \min\{\epsilon_k^R(i,s), \epsilon_k^S(i,s)\}$.*

4. *At any given* $\epsilon \leq \epsilon_s^W$*, if the set* $\Sigma_\epsilon(i,s) \stackrel{def}{=} \{\Sigma_k^\epsilon\}, k \in K$ *of admissible policy schemes switching the game from* i *to* s *is non-empty, then there exists the choice function*

$$\exists\Lambda(\Sigma_\epsilon(i,s)):\ q_B(0) \in \bigcup_{\Sigma_k^\epsilon \in \Sigma_\epsilon(i,s)} \bigcap_{\Sigma_k^\epsilon \in \Sigma_\epsilon(i,s)} [q_B^{x,k}(\epsilon)\{z\}; q_B^{x,k}(\epsilon)\{z+1\}]:$$
$$\min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_s^{\Sigma_x}\} - \min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_s^{\Sigma_k}\} > 0 \implies$$
$$\Lambda(\Sigma_\epsilon(i,s)) = \arg\max_{\Sigma_k^\epsilon \in \Sigma_\epsilon(i,s)} \min_{\varepsilon\in[-\epsilon;\epsilon]}\{W_s^{\Sigma_k^\epsilon}\} = \Sigma_x^\epsilon \tag{28}$$

selecting the best policy scheme Σ_x^ϵ *among those welfare-improving and robust ones at the level* ϵ*.*

5. *Denote* $\epsilon_1^*(i,s) = \min_{k\in K}\{\epsilon_k^*(i,s)\}$*, the uncertainty threshold of policy scheme* $\Sigma_x^\epsilon = \Lambda(\Sigma_{\epsilon<\epsilon_1^*(i,s)}(i,s))$*. There exists an increasing sequence* $\Omega^*(i,s) = \{\epsilon_1^*(i,s), .., \epsilon_k^*(i,s), .., \epsilon_K^*(i,s)\}$ *of uncertainty thresholds for all policy schemes in* $\Sigma_\epsilon(i,s)$ *such that the choice function* $\Lambda(\Sigma_\epsilon(i,s))$ *changes its value at each of them.*

6. *This forms a sequence of robust policy schemes increasing in uncertainty tolerance level* $\Sigma^*(i,s) = \{\Sigma_1^{\epsilon_1^*}, .., \Sigma_K^{\epsilon_K^*}\}$ *where each next element is more robust and welfare optimal under given robustness level.*

Proof. Two first points are just reflecting the fact that with growing uncertainty intervals of definition for choice functions Ψ, Θ might shrink and at some point the previously chosen element of the given set can become non-optimal. Third point refers to Def. 3 and indeed, as soon as one of choice functions Ψ, Θ changes its value, given policy scheme either cannot perform the task of reaching the desired outcome of the game, either this regime is not sustainable (robust). Point 4 requires a non-empty set, which exists due to Corollary 2, otherwise there is no need for a subsidy. We associate to every policy scheme from this set a welfare function. Their comparison provides the choice criteria for Λ. Point 5 introduces the ordering in the set $\Sigma_\epsilon(i,s)$ in welfare terms. Then we can also construct an increasing sequence out of it. Point 6 follows from points 4 and 5. □

The (28) is another choice function, which selects the policy scheme among those social welfare improving and robust under ϵ. It selects the one which yields the highest welfare under regime s in worst case once policy scheme is applied.

The sequence of robust policy schemes is formed by increasing ϵ: once it crosses the threshold $\epsilon_k^*(i,s)$ the choice function changes and selects another scheme. It is important to note that these thresholds differ not only across schemes (which are independent of the regime) but also across the switches $\{i,s\}$: a given policy scheme may be more robust and/or welfare improving in one switching than in the other. These uncertainty thresholds may be found through application of choice function Λ to different values of ϵ. There is exactly the same number of thresholds as of welfare-improving robust policy schemes for switch $\{i,s\}$. Naturally for full certainty case the set of uncertainty thresholds is a singleton with $\epsilon_1^* = \infty$ and the policy scheme with the highest welfare is selected.

Corollary 3 (Optimal robustness level)**.**
Assume $\epsilon < \epsilon_s^W$. *The level of robustness* $\epsilon^{**}(i,s) = \epsilon_k^*(i,s)$ *is optimal for the switch* $\{i,s\}$*, if both*

$$
\begin{aligned}
&\min_{\epsilon_k^*}\{W_s^{\Sigma_k^{\epsilon_k^*}}\} - \min_{\epsilon_{k+1}^*}\{W_s^{\Sigma_{k+1}^{\epsilon_{k+1}^*}}\} + ||\min_{\epsilon_{k+1}^*}\{W_s^{\Sigma_k^{\epsilon_k^*}}\} - \min_{\epsilon_{k+1}^*}\{W_{m_k}^{\Sigma_k^{\epsilon_k^*}}\}|| \leq 0, \\
&\min_{\epsilon_{k-1}^*}\{W_s^{\Sigma_{k-1}^{\epsilon_{k-1}^*}}\} - \min_{\epsilon_k^*}\{W_s^{\Sigma_k^{\epsilon_k^*}}\} + ||\min_{\epsilon_k^*}\{W_s^{\Sigma_{k-1}^{\epsilon_{k-1}^*}}\} - \min_{\epsilon_k^*}\{W_{m_{k-1}}^{\Sigma_{k-1}^{\epsilon_{k-1}^*}}\}|| \geq 0, \\
&m_k = \Theta_{\epsilon_{k+1}^*}(M_{\Sigma_k^{\epsilon_k^*}})
\end{aligned}
\tag{29}
$$

It is unique for any $\{i,s\}$.

Proof. Both lines in (29) are sums of welfare loss and gain from increasing the robustness level by one threshold. The first term is the difference in worst-case welfare in regime s under two successive schemes from $\Sigma^*(i,s)$ under associated robustness thresholds. This is always non-positive, since at every threshold the maximum welfare is selected. The second term is the potential error from applying the preceding (less robust) scheme under higher robustness level. There are two kinds of potential errors: either this scheme is no longer worst-case welfare improving, or it is not robust. Both cases are described by the difference under modulo operation and differ only in sign. Taking absolute value gives positive value of error avoidance. Once the sum of those two terms is not increasing, there is no further gain in increasing robustness level for the planner. Since the sequences of thresholds and policies are increasing, this choice is always possible and unique one. □

4 Conclusions

In this paper we propose a fairly general algorithm for finding optimal robust policies aimed at support of newer technologies. The advantage of our approach in comparison with the earlier ones is that it does not require anything except the axiom of choice and the existence of the value for the underlying game to be formulated.

Starting with an arbitrary dynamic game with finite number of states measuring the technologies' development, we formulate the ordering of individually and socially preferred outcomes based on the abstract notion of the choice function. The advantage of this abstract approach is that a choice function for a finite collection of sets exists always once axiom of choice is assumed.

Based solely on this notion we are able to find the ordering of robust policy schemes derive the criterion for the selection of the most optimal one and formulate the concept of the optimal robustness level relative to the uncertainty.

The approach proposed here can be applied to any dynamic game with finite number of players and asymmetric uncertainty. In particular it may be applied to questions of subsidizing green technologies, which are frequently characterized by the uncertain potential and relative disadvantage in comparison with existing older technologies.

References

Arrow, Kenneth J. (1962) The economic implications of learning by doing. *The Review of Economic Studies* 29 (3): 155–173.

Arthur, W. Brian. (1989) Competing technologies, increasing returns, and lock-in by historical events. *The Economic Journal* 99 (394): 116–131.

D'Aspremont, C. and Jacquemin, A. (1988). Cooperative and noncooperative r & d in duopoly with spillovers. *The American Economic Review*, 78(5):1133–1137.

Athanassoglou, S. and Xepapadeas, A. (2012) Pollution control with uncertain stock dynamics: When, and how, to be precautious. *Journal of Environmental Economics and Management* 63 (3): 304–320.

Babonneau, F., Haurie, A., and Vielle, M. (2013). A robust meta-game for climate negotiations. *Computational Management Science*, 10(4):299–329.

Ben-Tal, A., El Ghaoui, L. and Nemirovski, A. (2009) Robust Optimization. Princeton University Press, Princeton.

Ben-Youssef, S. and Zaccour, G. (2014). Absorptive Capacity, R&D Spillovers, Emissions Taxes and R&D Subsidies. *Strategic Behavior and the Environment*, 4(1):41–58.

Bondarev, A. (2014). Endogenous specialization of heterogeneous innovative activities of firms under the technological spillovers. *Journal of Economic Dynamics and Control*, 38(0):235 – 249.

Bondarev, A. and Greiner, A. (2017) Catching-up and falling behind: Effects of learning in an R&D differential game with spillovers. *Journal of Economic Dynamics and Control, In press online version,* doi: 10.1016/j.jedc.2017.10.001.

Brock, W., Xepapadeas, A., and Yannacopoulos, A. (2014). Robust Control and Hot Spots in Spatiotemporal Economic Systems. *Dynamic Games and Applications*, 4(3):257–289.

Dawid, H., Greiner, A., and Zou, B. (2010). Optimal foreign investment dynamics in the presence of technological spillovers. *Journal of Economic Dynamics and Control*, 34(3):296 – 313.

Hinloopen, J., Smrkolj, G., and Wagener, F. (2013). From mind to market: A global, dynamic analysis of R&D. *Journal of Economic Dynamics and Control*, 37(12):2729–2754.

Krysiak, F. (2011) Environmental regulation, technological diversity, and the dynamics of technological change. *Journal of Economic Dynamics and Control* 35 (4): 528–544.

Todorov, E. (2009). Efficient computation of optimal actions. *Proceedings of the national academy of sciences*, 106(28):11478–11483.

Unruh, G. (2000) Understanding carbon lock-in. *Energy Policy* 28 (12): 817–830.

Unruh, G. (2002) Escaping carbon lock-in. *Energy Policy* 30 (4): 317–325.

Vardas, G. and Xepapadeas, A. (2010). Model Uncertainty, Ambiguity and the Precautionary Principle: Implications for Biodiversity Management. *Environmental & Resource Economics*, 45(3):379–404.

Vardas, G. and Xepapadeas, A. (2015). Uncertainty aversion, robust control and asset holdings. *Quantitative Finance*, 15(3):477–491.