Universität Basel

Wirtschaftswissenschaftliche Fakultät

WWZ forum

January 2018

Global warming and technical change: Multiple steady-states and policy options

WWZ Working Paper 2018/03

Anton Bondarev, Alfred Greiner

A publication of the Center of Business and Economics (WWZ), University of Basel.

Universität Basel
Peter Merian-Weg 6
4052 Basel, Switzerland
wwz.unibas.ch

Corresponding Author:
Dr. Anton Bondarev
Tel: +41 (0) 61 207 33 81
Mail: anton.bondarev@unibas.ch

Global warming and technical change: Multiple steady-states and policy options

Anton Bondarev* Alfred Greiner†

Abstract

In this paper we develop an economic growth model that includes anthropogenic climate change. We include a publicly funded research sector that creates new technologies and simultaneously expands the productivities of existing technologies. The environment is affected by R&D activities both negatively, through the increase of output from productivity growth, as well as positively as new technologies are less harmful for the environment. We find that there may exist two different steady-states of the economy, depending on the amount of research spending: one with less new technologies being developed and the other with more technologies. Thus, a lock-in effect may arise that, however, can be overcome by raising R&D spending sufficiently such that the steady-state becomes unique. We derive the combinations of fiscal policy instruments for which that can be achieved and we study the implications for the economy and for the environment. In particular, the double dividend hypothesis may hold only under some specific conditions.

Keywords: Climate change, doubly-differentiated R&D, double dividend, fiscal policy instruments, technology lock-in

JEL classification: C61, C62, O38, O44, Q54, Q58

*Faculty of Business and Economics, University of Basel, Peter Merian-Weg 6, 4002 Basel, Switzerland, e-mail: anton.bondarev@unibas.ch

†Department of Business Administration and Economics, Bielefeld University, Universitätstraße 25, 33615 Bielefeld, Germany, e-mail: agreiner@wiwi.uni-bielefeld.de

Financial support from the Bundesministerium für Bildung und Forschung (BMBF) (grant 01LA1105C) and from Swiss Commission for Technology and Innovation (CTI) (contract KTI.2014.0114) is gratefully acknowledged. This research was part of the project 'Climate Policy and the Growth Pattern of Nations (CliPoN)'.

1 Introduction

Since the publication of the influential book (Meadows, Meadows, Randers, and Behrens 1972) by the Club of Rome, the role of the natural environment in the process of economic development has gained increased attention. Since then, numerous publications have been generated that raised our understanding of the interrelation between the environment and the economy. Even if some progress in the fight against the deterioration of the environment can be observed in a couple of areas, such as the better air quality in industrialized countries for example, the problem of environmental degradation is far from being resolved and strategies how to tackle that problem are needed.[1] In particular, global warming presents rather a severe threat because the global emissions of greenhouse gases (GHGs) as result of mankind burning fossil fuels are still far above the sustainable level, a problem already pointed out in (Meadows, Meadows, Randers, and Behrens 1972) (see Figure 15, p. 72).

There exist quite a many approaches dealing with climate change in the economics literature, starting with the seminal papers (Nordhaus 1992) and (Nordhaus 2007) which present an integrated assessment model that treats technical progress as an exogenous variable. In the middle- to long-run, however, neglecting the fact that technical change results from costly R&D can have significant implications. For example, (Popp 2004) shows that ignoring induced technical change overstates the welfare costs of an optimal carbon tax policy considerably. The importance of technical change has been pointed out in the paper (Acemoglu, Aghion, Bursztyn, and Hemous 2012), too. There the authors study the effects of endogenous directed technical change, where the final good is produced with dirty and clean input factors. The analysis of the model demonstrates that sustainable growth can be achieved with the help of taxes and subsidies that redirect innovations toward clean inputs.

Another problem in this respect arises when the long-run situation is characterized by multiple steady-states giving rise to a so-called lock-in phenomenon. Then, a situation can occur where an economy may be stuck in an equilibrium that yields a worse outcome

[1] see for example (OECD 2011) and (TheWorldBank 2012) that propose practical strategies to achieve green growth.

compared to the one associated with the other steady-state. For example, (Kalkuhl, Edenhofer, and Lessmann 2012) demonstrate that market imperfections can lead to a situation where an incumbant less efficient energy technology dominates a more efficient technology leading to welfare losses, in addition to those resulting from the market imperfections alone. The authors derive optimal policy measures to overcome this lock-in and show that they are welfare improving.

Multiple steady-states arise in the contributions by (Greiner and Semmler 2005) and (Greiner, Grüne, and Semmler 2010), too. There, it is the feedback effect of higher temperatures on Earth that generates a non-linearity in the climatic energy balance model which leads to multiple steady-states generating a lock-in effect. The main message of those papers is that delaying abatement activities that reduce greenhouse gas emissions can lead to a situation where the convergence to the equilibrium with a moderate temperature rise is not feasible any longer. Rather, the world converges to the second steady-state that is characterized by a higher greenhouse gas concentration and a higher surface temperature leading to larger damages.

In this paper we want to contribute to the environmental literature featuring multiple steady-states and lock-in effects, where we pay special attention to the role of technical progress. To do so, we develop a simple growth model of an economy which takes into account environmental damages resulting from global warming. The main focus of our paper is the effect of different forms of publicly funded technical change on the evolution of the economy and the environment. As regards technical change we allow for both horizontal and vertical innovations, i.e. we consider the improvement of existing technologies and the generation of newer ones, where we assume a lower emission intensity of newer technologies. Hence, the main departure from the majority of the literature on economic growth with environmental degradation concerns the way how the technological change is modelled. Further, we pay special attention to the role of fiscal policy instruments that can overcome the lock-in.

There are four main findings in the paper. First, the publicly funded R&D may lead to the multiplicity of (inefficient) equilibria and to a technology lock-in in the economy similar to (Acemoglu, Aghion, Bursztyn, and Hemous 2012). Second, this lock-in may be avoided by a sufficient increase in public R&D spending which leads to higher productivity and consumption. Third, the switch of the economy to the efficient path does not necessarily come along with a slowdown of environmental degradation. Nevertheless, situations are feasible such that both consumption and the state of the environment improve when the economy transits form the inefficient equilibrium to the efficient one. But, once the efficient allocation is realized, any further increase in consumption can be achieved only at the cost of a deteriorating environment. Thus, our findings are partially supporting the double dividend hypothesis. We further explore the available fiscal policy options of the government in reallocating the tax revenue between abatement activities and R&D spending. Our last finding is that there always exists a minimal tax rate which is necessary to realize the efficient R&D and environmental policies and not all combinations of environmental and fiscal policies enable the economy to avoid the technology lock-in.

The rest of the paper is organized as follows. The next section presents the structure of our model. Section 3 shows when multiple steady-states may emerge and section 4 analyzes the fiscal policy that guarantees a unique steady-state. Section 5, finally, concludes the paper.

2 The model

To start with, we describe the basic growth model featuring environmental degradation. The economy is populated by a continuum of homogeneous consumers normalized to one that can be represented by one consumer which maximizes the stream of discounted utilities from a consumption good over an infinite time horizon. Output can be consumed or invested and is a function of physical capital and of the stock (or state) of technology as production factors. The stock of technology is a purely public good that results from research and development (R&D) spending funded by the government. The stock of technology consists of a continuous range of single technologies that differ with respect to their productivities.[2] Finally, output is negatively affected by a stock of pollution that

[2]the stock of technology could alternatively be interpreted as a stock of public capital, such as telecommunication infrastructure, transportation infrastructure and health capital, for example.

equals accumulated weihgted past emissions. The emissions in the economy result from output as a negative externality and can be reduced by devoting resources to abatement activities.

2.1 The household sector and the environment

The representative household maximizes the stream of discounted utility from the consumption good, C, subject its budget constraint. Neglecting labour as input factor[3] the optimisation problem can be written as,

$$J^H = \max_C \left\{ \int_0^\infty \mathrm{e}^{-\rho t} \left[\frac{C^{1-\gamma} - 1}{1-\gamma} \right] dt \right\} \tag{1}$$

with consumption being the single control variable subject to a flow budget constraint:

$$\dot{K} = (1-\tau)Y - C, \tag{2}$$

where τ is the income tax rate imposed by the government, $1/\gamma$ the intertemporal elasticity of substitution of consumption, Y denotes output and we neglect depreciation of the physical capital stock K. It should be noted that the household does not take into account neither the negative pollution externality of output nor the positive externalities of taxes on the stock of technology since these are external to him. Further, the outcome of this optimisation problem is equivalent to that of a decentralised economy with a competitive factor market such that the marginal product of capital equals the interest rate.

Output is a function of physical capital and of the stock of technology, A, and negatively affected by the stock of pollution, M.Thus, the output is given by

$$Y = \phi(M) A K^\alpha \tag{3}$$

with $0 < \phi(M) \leq 1$, $\phi(0) = 1$, $\phi'(.) < 0$, and $\alpha \leq 1$. The function $\phi(M)$ reflects the damages from environmental pollution where $M = 0$ denotes the unspoiled environment that does not go along with damages on output.

[3]we omit the time argument t whenever this does not lead to confusion.

Application of the standard Maximum Principle to the problem given by (1) and (2) leads to the following current-value Hamiltonian

$$\mathcal{H} = \frac{C^{1-\gamma} - 1}{1-\gamma} + \lambda_K \left((1-\tau)\phi(M)AK^{\alpha} - C\right), \tag{4}$$

which yields the necessary optimality conditions:

$$\frac{\partial \mathcal{H}}{\partial C} = C^{-\gamma} + \lambda_K = 0, \tag{5}$$

and co-state equation

$$\dot{\lambda}_K = \rho\lambda_K - (1-\tau)\alpha K^{\alpha-1}\phi(M)A. \tag{6}$$

In addition, the limiting transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K K = 0 \tag{7}$$

must hold.

The environment is affected by the economy through the usual transmission mechanism:

$$\dot{M} = -\nu M + (1-a)eY \tag{8}$$

where $a \in (0,1)$ is the rate of abatement activities financed by the government, e is the intensity of emissions from the output, being a function of the technology, and $\nu \in (0,1)$ is the rate at which the environment recovers. In the next subsection, we will show how the technology determines the emissions intensity. The emissions can be interpreted as greenhouse gas (GHG) emissions that raise the stock of GHGs in the atmosphere that negatively affects output in the economy.

2.2 The government and the R&D sector

The government collects taxes and distributes them between abatement efforts, a, and R&D spending R:

$$\begin{aligned} T &= \tau Y = aY + R, \\ R &= (\tau - a)Y \end{aligned} \tag{9}$$

The total R&D spending is next distributed between investments into the expansion of the variety of technologies, u, and investments into the increase of productivity of each individual technology, $g(i,t)$:

$$R = u + \int_0^{n(t)} g(i,t)di. \tag{10}$$

The R&D sector optimally allocates the R&D budget to vertical and horizontal investments into technologies. The results of R&D are free to use at the economy wide level. There are no patents and profits from R&D since the research is publicly funded. The alternative with market-funded R&D would yield equivalent results as long as one imposes free entry condition and if one assumes that the rate of return to R&D investments is equal to the discount rate of firms, r, see (Peretto and Connolly 2007) for example of such a treatment of R&D.

Formally, the R&D sector is described as a joint optimization problem of maximizing the output of innovations subject to the constraint (10):

$$J^{Tech} \stackrel{\text{def}}{=} \max_{u(\cdot),g(\cdot)} \int_0^{\infty} \mathrm{e}^{-rt} \left(\int_0^{n(t)} \left[q(i,t) - \frac{1}{2} g(i,t)^2 \right] di - \frac{1}{2} u(t)^2 \right) dt. \tag{11}$$

with:

- $u(t)$ - investments into variety expansion;
- $g(i,t)$ - investments into the productivity growth of technology i at time t.

The dynamics of variety expansion is linear in investments:

$$\dot{n} = \xi\, u, \tag{12}$$

with $\xi > 0$ a constant parameter reflecting the efficiency of investments. The dynamics of productivity of each technology increases as a result of investments and declines in the absence of such proportionally to the achieved level of productivity (reflecting the fact that the more refined a technology is, the more costly it is to support the necessary infrastructure):

$$\frac{\partial q(i,t)}{\partial t} = \psi(i) g(i,t) - \beta q(i,t), \tag{13}$$

where $\psi(i)$ is the efficiency of increasing productivity of technology i, different across technologies, and β is the rate of decay of productivity of technology i. The fact that the efficiency of investments differ across technologies is important since this is the main reason why a technology lock-in can arise in the economy. With equal efficiencies, $\psi(i) = \psi_c$ the dynamics of R&D is qualitatively the same as without a multiplicity of technologies (provided there is no out-dating).

We assume that each new technology has zero initial productivity upon the time of its invention,

$$\forall i \in [0; n] : \, q(i,t)_{i=n(t)} = 0. \tag{14}$$

The time of invention for the technology, $t_i(0)$, is the inverse function of the variety expansion process, $n(t)$:

$$t_i(0) = f^{-1}(n(t))|_{n(t)=i}. \tag{15}$$

The overall productivity of the economy is the total of productivities of individual technologies,

$$A = \int\limits_0^{n(t)} q(i,t)di, \tag{16}$$

All technologies differ from each other as concerns the environmental damage they cause, with newer technologies yielding lower damages. Each technology i has a cleanliness index, $\iota(i)$, which defines the extent to which the sector driven by technology i affects the environment.

Then, the overall intensity of emissions, $e(t)$, is an aggregate of the intensities of all the existing technologies at time t:

$$e(t) \stackrel{def}{=} (e_0/n(t)) \cdot \int_0^{n(t)} \iota(i)di, \tag{17}$$

with $e_0 > 0$ a constant.

The economy is affected by R&D through 3 different channels: first, overall R&D investments, R, are deduced from the income of the agent; second, the total productivity in the economy, A, grows as a result of R&D; third, the intensity of emissions, e, from output gradually changes due to the adoption of cleaner technologies.

2.3 The steady-state

In this subsection, we derive the steady-state for our economy. Noting the relationship

$$\frac{\dot{C}}{C} = -\frac{1}{\gamma}\frac{\dot{\lambda}_K}{\lambda_K} \tag{18}$$

yields the evolution of consumption as:

$$\dot{C} = \frac{C}{\gamma}\left((1-\tau)\alpha K^{\alpha-1}\phi(M)A - \rho\right). \tag{19}$$

The evolution of capital is given by (2) with output from (3):

$$\dot{K} = (1-\tau)\phi(M)AK^{\alpha} - C \tag{20}$$

Finally, the evolution of the environment, i.e. of the stock of pollution, is given by:

$$\dot{M} = -\nu M + (1-a)e\phi(M)AK^{\alpha} \tag{21}$$

The system (19), (20), (21) together with the transversality condition (7) fully describes the economic-environmental system apart from technology. The steady-state of this system is given by a rest point of the dynamic system described by the three differential equations (19), (20), (21).

Proposition 1 demonstrates that the steady-state is unique for a given value of the state of technology.

Proposition 1 (Uniqueness of steady-states)**.**
For a given stock of the technology, A, there exists a unique steady-state $\{\bar{C}, \bar{K}, \bar{M}\}$ of the economy described by the dynamic system (19), (20), (21).

Proof. From (21) the $\dot{M} = 0$ isocline is obtained as:

$$K = c_1\left(\frac{M}{\phi(M)}\right)^{1/\alpha}, \; c_1 = \left(\nu/((1-a)eA)\right)^{1/\alpha}, \tag{22}$$

with $K = 0$ for $M = 0$ and

$$\frac{dK}{dM} = c_1(1/\alpha)\left(\frac{M}{\phi(M)}\right)^{-1+1/\alpha}\left(\frac{\phi(M) - M\phi'(M)}{\phi^2(M)}\right) > 0 \tag{23}$$

From (19) the $\dot{C} = 0$ isocline can be computed as:

$$K = c_2 \phi(.)^{1/(1-\alpha)}, \; c_2 = (A(1-\tau)\alpha/\rho)^{1/(1-\alpha)}, \tag{24}$$

with $K = c_2 > 0$ for $M = 0$ and

$$\frac{dK}{dM} = c_2(1/(1-\alpha))\phi(.)^{\alpha/(1-\alpha)}\phi'(.) < 0 \tag{25}$$

Hence, there exists a unique intersection point of the two isoclines giving a unique $\{\bar{K}, \bar{M}\}$. Inserting that in (20) yields the unique $\bar{C} = (1-\tau)\phi(\bar{M})A\bar{K}^{\alpha}$. □

Proposition 1 shows that the technological sector determines the long-run behaviour of the economy. Thus, if the technology has a unique steady-state the whole environmental-economic system is characterised by a unique steady-state. However, the R&D sector may exhibit a multiplicity of steady-states even for this simple setup, as we will demonstrate in the next subsection.

Further, it should be pointed out that for the case $\alpha = 1$, the steady-state can be explicitly computed as a function of the technology, A, from (19)-(21):

$$\begin{aligned} \bar{M} &= \phi^{-1}\left(\frac{\rho}{(1-\tau)A}\right), \\ \bar{K} &= \left(\frac{\nu \bar{M}}{(1-a)e\phi(\bar{M})A}\right), \\ \bar{C} &= (1-\tau)\phi(\bar{M})A\bar{K}. \end{aligned} \tag{26}$$

3 Multiplicity and uniqueness of steady-states

The budget constraint of the government limits R&D expenditures distributed to the R&D sector. When the constraint is binding, the problem of the R&D sector becomes an optimal control problem subject to a resource constraint, similar to the one studied in (Bondarev and Greiner 2017) for the case of a single firm. There, it is demonstrated that the R&D sector may possess one or two steady-states, depending on the value of R, with an associated variety of developed technologies and their productivities.

Denote by $\lambda_q(i,t)$ the shadow costs of investments into the productivity of technology i, $q(i,t)$, and by $\lambda_n(t)$ the shadow costs of investing into the increase of the variety of technologies, $n(t)$. To begin our analysis, we start with the definition of a steady-state for the R&D sector.

Definition 1 (R&D steady-state)**.**
The steady-state of the R&D sector is characterized by the following conditions:

$$\begin{aligned} \forall i \in [0;1] : \dot{q}(i,t) &= 0, \\ \dot{\lambda}_q(i,t) &= 0, \\ \dot{n}(t) &= 0, \\ \dot{\lambda}_n(t) &= 0. \end{aligned} \tag{27}$$

In order to illustrate the possibility of multiple steady-states and the technology lock-in phenomenon, we make use of a particular choice of the efficiency function $\psi(i)$ and of the emissions intensity function e described below in detail.

We assume the form of efficiency function $\psi(i)$ which reflects the increased difficulty of improving newer technologies:

$$\psi(i) = \psi_c \sqrt{1-i},\ \psi_c > 0. \tag{28}$$

Such a specific functional form is chosen for two reasons: first, it is a decreasing function and second it provides an opportunity for an analytical solution unlike most other decreasing monotonic functions. However it should be noted that results remain robust under the choice of this efficiency function as long as certain regularity conditions on $\psi(i)$ hold:

- It is decreasing in i,
- It is invertible,
- It is continuous but not necessarily continuously differentiable.

The emissions intensity e is defined in (17) and depends on the cleanliness index $\iota(i)$. This index is assumed to be a decreasing function of the technology:

$$\iota(i) = 1 - i. \tag{29}$$

Again, the function $\iota(i)$ is chosen to be linearly decreasing for simplicity. Because of the choice of the efficiency and of the emissions intensity functions, the variety of technologies is bounded from above:

$$n(t) \in [0; 1], \tag{30}$$

and emissions intensity may be expressed as

$$e(t) = \frac{e_0}{n(t)} \left(n(t) - \frac{1}{2} n^2(t) \right) \tag{31}$$

being a decreasing function of the available variety of technologies n.

Note that for any pair $n(t), \lambda_n(t)$ the dynamics of all technologies' productivities is fully defined and once the variety expansion reaches a steady-state, so does every individual technology state, $q(i, t)$ (see Appendix A). Thus, to fully characterize the steady-state of the R&D sector it suffices to characterize the $\lambda_n(t), n(t)$ dynamics.

Proposition 2 (Uniqueness and multiplicity of steady-states of the R&D sector)**.** *The dynamic system $\dot{\lambda}_n$, $\dot{n}$, given by* (A.15), (A.16)*, has at most two steady-states. Other things equal the R&D budget R defines the number of steady-states of the system. For $R < R^\star$ there exist two steady-states with a low and a high variety of technologies. With $R \geq R^\star$ there exists only one steady-state with $\bar{n} = 1$.*

Proof. See Appendix A. □

Figure 1 illustrates the case of a unique steady-state and of two steady-states for this system depending on the level of R. To draw Figure 1 we resorted to the parameter values given in table 1. The research budget is set to $R = 0.9$ for the multiple steady-states case and to $R = 4$ for the unique steady-state case.

The lower the level of the R&D budget is, the higher is the chance for a multiplicity of steady-states where one of the steady-states goes along with a lower level of variety of technologies and higher shadow costs of investments than the other. That holds because a decline in R&D shifts the $\dot{n}(t) = 0$ isocline upward and a sufficiently high R leads to the collapse of the two steady-states into a unique one for the system (A.15), (A.16). The unique steady-state implies $\bar{n} = 1$ and $\bar{\lambda}_n = 0$. Using this, one can easily compute the

Table 1: Parameters values used in Figure 1.

Parameter	Value
n_0	0
r	0.05
ψ_c	0.9
ξ	0.8
β	0.1

threshold level of the research budget, R^*, from $\dot{n}(t) = 0$ as:

$$R^* = \frac{2}{3}\frac{\psi_c}{r+\beta}. \tag{32}$$

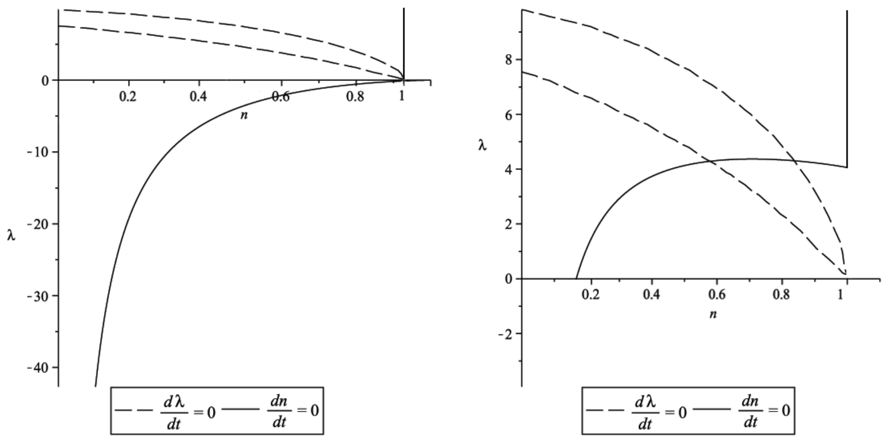
With the parameters given in table 1 one obtains $R^* = 4$.

It should be pointed that in the case of multiple steady-states both steady-states are inefficient in the sense that they do not allow the introduction of all potential technologies in the economy. Thus, we characterize the latter as an inefficient situation and we refer to the unique steady-state as the efficient one since it goes along with the maximum number of technologies.

Further, setting $R > R^\star$ implies a unique steady-state. However, that would be inefficient since the same result can be obtained for $R = R^\star$ because this gives the maximum value $\bar{n} = 1$. Any additional R&D spending is a waste of resources that reduces the steady-state capital stock and output and, thus, welfare. We state our result in the following proposition:

Proposition 3 (Technology lock-in with a constrained R&D sector).
In the R&D sector described by the system (A.13), (A.15), (A.16), *the budget constraint R is crucial as concerns the emergence of a technology lock-in.*

1. *For $R < \frac{2}{3}\frac{\psi_c}{r+\beta}$, the economy is characterized by a technology lock-in with two inefficient steady-states;*



(a) Unique steady-state, $R = 4$ (b) Multiple steady-states, $R = 0.9$

Figure 1: Multiplicity of R&D steady-states

2. *For* $R \geq \frac{2}{3}\frac{\psi_c}{r+\beta}$*, no lock-in effect arises and the variety of technologies reaches its maximum steady-state level* $\bar{n} = 1$*;*

3. *Efficient R&D expenditures are given by* $R^\star = \frac{2}{3}\frac{\psi_c}{r+\beta}$*.*

In the case of an efficient allocation, both the number of technologies and overall productivity are maximized. However, the effect on the environment is ambiguous because, on the one hand, a higher productivity implies higher output and, thus, more environmental degradation whereas, on the other hand, newer technologies go a long with a lower emissions intensity which can be seen from (31).

Now, recall that the total productivity of the economy is the integral over individual productivities:

$$A = \int_0^n q(i,t)di. \tag{33}$$

Differentiating and taking into account (14) one obtains the dynamics of overall productivity A (for the case $R \leq R^{\star}$),

$$\dot{A}^R(t) = \int_0^{n^R(t)} \frac{\partial q^R(i,t)}{\partial t} di =$$
$$= \int_0^{n^R(t)} \left\{ \psi(i) \left(\frac{\psi(i)}{r+\beta} - \frac{1}{1+n^R(t)} \left(\xi \lambda_n^R(t) - R + \frac{1}{r+\beta} \int_0^{n^R(t)} \psi(i) di \right) \right) - \beta q(i,t) \right\} di =$$
$$= \frac{R - \xi \lambda_n^R}{1+n^R(t)} \int_0^{n^R(t)} \psi(i) di + \frac{1}{r+\beta} \int_0^{n^R(t)} (\psi(i))^2 di - \frac{\left(\int_0^{n^R(t)} \psi(i) di \right)^2}{(r+\beta)(1+n^R(t))} - \beta A^R(t) \tag{34}$$

while in the unconstrained case it is simply,

$$\dot{A}^{\star}(t) = \frac{1}{r+\beta} \int_0^{n^{\star}(t)} (\psi(i))^2 di - \beta A^{\star}(t) \tag{35}$$

In the situation of constant steady-state research expenditures the total productivity will stay constant and its level is defined by the variety of technologies being developed, n, by its shadow price, λ, and by the efficiency function, $\psi(i)$, and by the level of R&D spending, R.

Next, we study how the total steady-state productivity affects the environment at the efficient steady-state. This is the contents of Corollary 1.

Corollary 1 (Overall productivity of the economy and the evironment)**.**
Assume that $\bar{A}_L < \bar{A}_H < \bar{A}^{\star}$ *and* $\bar{n}_L < \bar{n}_H < \bar{n}^{\star}$ *holds, where the subscript* L *(*H*) denotes the low (high) steady-state and* * *the efficient one.*

Then, $\bar{M}_L < \bar{M}_H < \bar{M}^{\star}$*, for* $\alpha = 1$*. For* $\alpha < 1$*,* $\bar{M}_L > \bar{M}_H > \bar{M}^{\star}$ *can hold only if* $\bar{K}_L < \bar{K}_H < \bar{K}^{\star}$*.*

Proof. This follows from $\dot{C} = 0 \leftrightarrow \rho = (1-\tau)\alpha K^{\alpha-1}\phi(M)A$ □

Corollary 1 states that a higher productivity always goes along with a higher stock of pollution at the steady-state for the case of a linear production function, i.e. for $\alpha = 1$. The reason for that outcome is that the marginal product of capital is constant in steady-state and independent of the capital stock with $\alpha = 1$. It must be pointed out that this is independent of the damages, determined by the function $\phi(M)$, and also independent of the emissions intensity $e(t)$. In the case of decreasing returns to capital, a higher steady-state total productivity may go along with a lower steady-state pollution if the steady-state capital stock rises. However, it must be pointed out that this is only a necessary condition and it is more likely that both total productivity, the capital stock and pollution increase in steady-state. Further, it must be noted that a lower steady-state pollution together with a higher total productivity can only occur if the reduction of the emissions intensity, as a result of introducing newer and cleaner technologies, is sufficiently high. This follows from $\dot{M} = 0 \leftrightarrow \nu \bar{M} = (1-a)e(\bar{n})\phi(\bar{M})\bar{A}\bar{K}^{\alpha}$. To get additional insight we will resort to numerical examples next.

Our analysis up to now has assumed that the R&D spending is exogenously given. This allowed us to draw clear-cut conclusions as regards the dynamics of the R&D sector. However, in equilibrium R&D spending is endogenously determined by the budget constraint of the government (9) and a steady-state of the economy is given by a rest point of the differential equations (19), (20), (21), (34), (A.15) and (A.16). But, since (A.15) is a polynomial of second order in λ_n^R, see Appendix A, multiple steady-states can occur in the economy in equilibrium, too. A formal analytical proof of that conjecture is not possible so that we confine our investigation to the analysis of a numerical example demonstrating that Propostion 2 remains valid. Further, we want to shed light on the question of how total steady-state productivity $\bar{A}$ varies with the variety of technologies at the steady-state, $\bar{n}$, and how this affects the economy and the environment.

To do so, we set $\alpha = 1$ so that the steady-state values $\{\bar{C}, \bar{K}, \bar{M}\}$ are given by (26). A rest point of (34), (A.15) and (A.16), with $R = R(\bar{K}, \bar{A}, \tau, a, \cdot)$ gives a steady-state, then. To analyze that system, we choose the parameter values assumed in table 1 and, in addition, we set $\tau = 0.08$, $a = 0.02$, $e_0 = 0.9$, $\nu = 0.1$ and $\rho = 0.05$. The function ϕ is specified as $\phi = 1 - a_1 M$, with $a_1 = 0.01$. This is our benchmark example.

Then, the first steady-state with the lower number of technologies is given by:

$$\{\bar{Y}_L = 15.6, \bar{C}_L = 14.35, \bar{K}_L = 287.05, \bar{M}_L = 99.76, \bar{A}_L = 24.48, \bar{n}_L = 0.55, \bar{\lambda}_n^R = 4.22\}$$

and the second steady-state is:

$$\{\bar{Y}_H = 19.38, \bar{C}_H = 17.83, \bar{K}_H = 356.65, \bar{M}_H = 99.83, \bar{A}_H = 31.76, \bar{n}_H = 0.83, \bar{\lambda}_n^R = 3.85\}$$

The efficient steady-state is obtained for $\tau = 0.1966$ and the steady-state values are:

$$\{\bar{Y}^\star = 22.65, \bar{C}^\star = 18.2, \bar{K}^\star = 363.9, \bar{M}^\star = 99.88, \bar{A}^\star = 51, \bar{n}^\star = 1, \bar{\lambda}_n^R = 0\}$$

This example shows that a higher variety of steady-state technologies goes along with a higher total steady-state productivity and more output and consumption. However, it is also associated with a higher steady-state pollution as predicted by Corollary 1.

According to Corollary 1 this could only change in the case of decreasing returns to capital. Therefore, we next chose $\alpha = 0.3$ and again computed steady-state values. It turned out that the qualitative results remain unchanged, that is a higher number of steady-state technologies goes along with a higher steady-state productivity, a rise in output and a higher stock of pollution. Further, setting e_0 to a higher value, $e_0 = 2$, in the emissions intensity function does not change the qualitative result, just as choosing the different emissions intensity function $e(t) = e_0/(1 + a_2 n)$, with $e_0 = 0.9$ and $a_2 = 1$ and, alternatively, $a_2 = 2$.

The same holds if we choose a lower value for e_0. Setting $e_0 = 0.05$, in the example presented above, leads to the same qualitative outcome with $a_1 = 0.25$. It must be pointed out that the parameter reflecting the damages, a_1, must be set to a higher value than in the example above, once e_0 falls short of a certain threshold. Otherwise, a steady-state does not exist. However, if a steady-state exists, for a lower e_0 together with a higher a_1, the qualitative properties are identical to those in the example presented above.

This example demonstrates that the economy can escape the lock-in by devoting more resources to R&D leading to a higher total productivity, to a larger variety of technologies and to a higher production in steady-state. However, that goes at the expense of the environment since the positive effect of newer, less polluting technologies on the environment is dominated by the negative effect of a higher productivity generating more output and pollution.

But, the government can combine policy instruments in order to achieve an efficient steady-state that keeps the state of the environment at the same level as in the inefficient situation. In this case, a certain part of the gain in output, compared to the inefficient situation, must be used for abatament, at the expense of consumption. Thus, a second efficient steady-state is obtained for $\tau = 0.5902$ and by simultaneously raising the abatement share to $a = 0.5$. The corresponding steady-state values are now:[4]

$$\{\bar{Y}^{\star} = 44.34, \bar{C}^{\star} = 18.17, \bar{K}^{\star} = 363.38, \bar{M}^{\star} = 99.76, \bar{A}^{\star} = 51, \bar{n}^{\star} = 1, \bar{\lambda}_n^R = 0\}$$

One realizes that this second steady-state gives the same steady-state level of pollution as the low steady-state in the inefficient economy, whereas consumption is larger. Consumption is also larger than consumption at the high steady-state of the inefficient economy.

Comparing the two efficient steady-states, one realizes that the first, with $\tau = 0.1966$ and $a = 0.02$, yields a slightly higher consumption level and a larger stock of pollution compared to the second one with $\tau = 0.5902$ and $a = 0.5$.

If we increase the tax rate and the abatement share further and set $a = 0.6$ and $\tau = 0.6722$ we obtain the following steady-state values:

$$\{\bar{Y}^{\star} = 55.39, \bar{C}^{\star} = 18.16, \bar{K}^{\star} = 363.12, \bar{M}^{\star} = 99.70, \bar{A}^{\star} = 51, \bar{n}^{\star} = 1, \bar{\lambda}_n^R = 0\}$$

Now, consumption is higher than in both inefficient steady-states and the stock of pollution is lower so that the efficient steady-state is clearly preferable to the inefficient steady-states. However, comparing the efficient steady-states we realize that there is again a trade-off: when both the abatement share and the tax rate rise, the state of the environment improves, but, that goes at the expense of consumption which declines.

We should like to point out that this result holds in the case of decreasing returns to capital, too. Setting $\alpha = 0.3$, as above, and $a = 0.1$, $\tau = 0.282$ and $a = 0.2$, $\tau = 0.3641$ and $a = 0.5$, $\tau = 0.6107$ shows that there is a trade-off between consumption and pollution along different efficient steady-states. Further, an inefficient steady-state can be overcome by an appropriate choice of the income tax rate and the abatement share.

Thus, we can state that a double dividend can occur when economic policy moves the economy from an inefficient situation to an efficient one with a unique steady-state. But, it must be stressed that this is only possible by a combination of fiscal policies, i.e. by raising both the tax rate and the abatement share. To achieve an efficient situation, the policy maker can choose between different combinations of τ and a that all avoid the lock-in and that give rise to a unique steady-state. However, once an efficient situation

[4] the strong rise in $\bar{Y}^{\star}$ compared to the first efficient steady-state is due to the increase of $\phi(M^{\star})$ by about 96 percent.

has been reached, there is a trade-off between the level of consumption and environmental quality: a higher level of consumption implies a higher stock of pollution and vice versa.

The question of feasibility of efficient research spending and abatement policy is studied in the next section.

4 Feasible R&D and efficient policies

The government in our economy has three policy instruments: the overall income tax rate, τ, the abatement rate, a, and the research budget, R, that are interrelated by the government budget constraint (9).

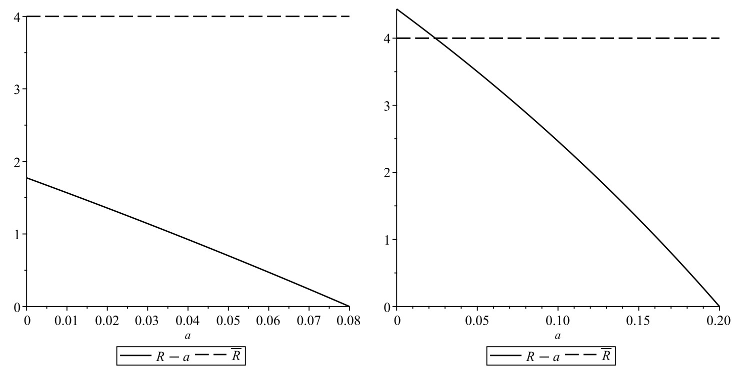
At the steady-state the economy only depends on the variety of technologies, $\bar{n}$. We first express the policy equation, relating R&D spending to the tax rate and to the abatement rate, in terms of this variety by substituting $\bar{A}^R$, obtained from $\dot{A}^R(t) = 0$, into (26) and the resulting output level into (9), where we confine ourselves to the case $\alpha = 1$. We then express the R&D expenditures R as a function of the tax rate τ, of the abatement rate a and of the steady-state variety of technologies n, where we set $\bar{\lambda}_n^R = 0$ which must hold for the efficient steady-state. This gives:

$$R = 2\frac{(\tau - a)\nu}{e_0(1-a)(2-n)} - \frac{1}{2}\frac{X(a,n,\tau)}{Y(a,n,R)} \qquad (36)$$

where $X(a,n,\tau), Y(a,n,R)$ are polynomials (B.1), provided in the Appendix B.

Following the discussion from the previous section we know that the only efficient steady-state for the variety n is $\bar{n} = 1$. This can be achieved only if $R \geq R^*$. Therefore, we set n in (36) equal to $\bar{n} = 1$. This leads to $R - a$ combinations as an implicit function of τ. We next plot this $R - a$ combination for different values of τ against the R^* line using the parameters from our benchmark example presented in the previous section.

Figure 2 illustrates the fact that it is not possible for any tax rate to find a combination of abatement and research expenditures granting convergence of the economy to the efficient steady-state. In particular, for the chosen parameter values of our benchmark example, a tax rate of 8 percent will inevitably lead to the inefficient equilibrium implying a lock-in: the R^* line does not intersect the $R-a$ line for any positive $a \leq \tau$ as illustrated by Figure 2a. Once the tax rate is higher, there is a range of a values for which R^* is

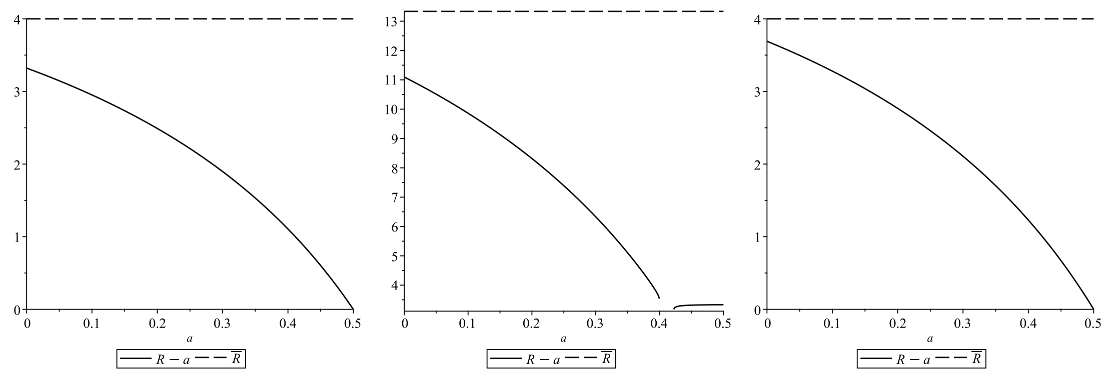


(a) $\tau = 0.08$ (b) $\tau = 0.2$ (c) $\tau = 0.5$

Figure 2: Research and abatement expenditures for different tax regimes

feasible. For $\tau = 0.2$ this is the range of a below about 0.05 as illustrated by the Figure 2b and for $\tau = 0.5$ the feasible abatement rate is bounded by about 40 percent, see Figure 2c.

It has to be noted, that the result is not independent of the choice of parameters. For any tax rate, there are parameter ranges such that the efficient research budget is infeasible, as illustrated by the Figure 3.



(a) $e_0 = 3$ (b) $\psi_c = 3$ (c) $a_1 = 0.03$

Figure 3: Infeasible efficient research for $\tau = 0.5$

In this figure we have selected three key parameters and multiplied them by three, keeping all other parameters as in our benchmark example except the tax rate that is set to $\tau = 0.5$. It can be seen that in all these simulations the efficient allocation is infeasible and a higher tax rate is necessary to reach the efficient size of the research budget. There could be situations when there does not exist any tax rate $\tau < 1$ such that R^* becomes feasible. For example, letting the damage parameter a_1 increase above $a_1 = 0.046$ makes the efficient research budget infeasible for any tax rate (with the other parameters as in the benchmark example).

On the other hand, setting R equal to the efficient value R^* we obtain the geometric place of points in the $a - \tau$ plane, which gives the efficient R^*: for any abatement rate higher than this, the efficient R&D expenditure level is infeasible and any abatement rate lower than this value is sub-optimal, since no further increase in R above R^* may provide a rise in the variety of technologies. The value for $a^\star$ can be computed as:

$$a^* = \frac{6\,\tau\ \left(\left(\tau/2 - 1/2\right)\psi + \rho\,\beta\ (r+\beta)\right)(r+\beta)\,\nu - \psi^2 e_0\ (\tau - 1)\ a_1}{6\ \left(\left(\tau/2 - 1/2\right)\psi + \rho\,\beta\ (r+\beta)\right)(r+\beta)\,\nu - \psi^2 e_0\ (\tau - 1)\ a_1} \tag{37}$$

This $a^* - \tau$ curve is illustrated in Figure 4 for different structural parameters with respect to the environment and with respect to its impact on the economy: the regeneration rate ν (Figure 4b), the initial emissions intensity e_0 (Figure 4c) and the damage intensity a_1 (Figure 4a).

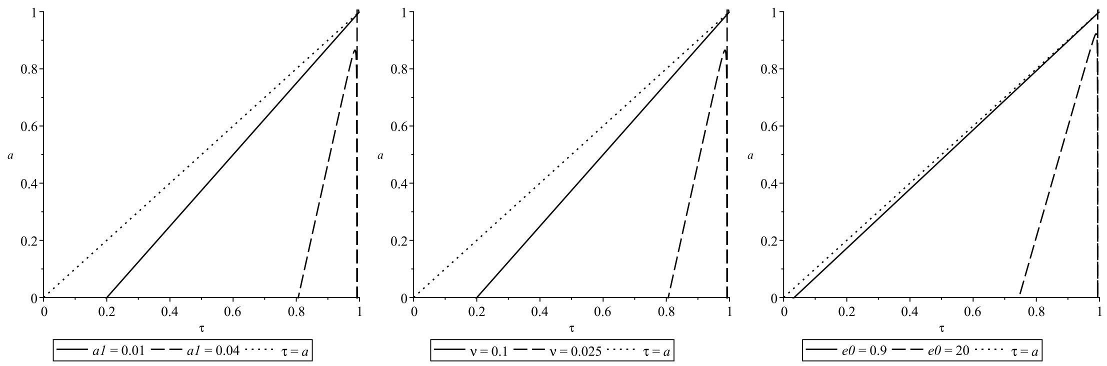


Figure 4: Efficient abatement rate as function of the tax rate

This Figure illustrates the fact that the stronger is the problem of environmental pollution (as measured by this triple of parameters), the higher the tax rate and the abatement rate should be because it is more difficult to escape the technology lock-in in this case. Therefore, we define the environmental stress of the economy as a norm of the vector $\mathbf{S} := ||a_1, 1-\nu, e_0||_1$: the higher is this norm, the more resources are to be devoted to fight off environmental degradation.

It must also be noted that different efficient combinations of τ and $a^\star$ imply different steady-state consumption levels and different levels of environmental pollution. Our numerical example form the previous section has shown that low values of τ and a^* generate a high level of consumption and a high stock of environmental pollution, whereas high values of τ and $a^\star$ lead to a small level of consumption with a low stock of pollution. Thus, there is a trade-off between the level of consumption and a clean environment at the efficient steady-state.

We summarize those observations in the following Proposition 4.

Proposition 4 (Characteristics of an efficient policy and environmental stress)**.**
A) Given the efficient abatement rate a^* via (37)*, the following holds:*

1. *For every set of parameters there exists a minimal and a maximal tax rate, $0 \leq \tau_{min}(\mathbf{S}) \leq \tau_{max}(\mathbf{S}) \leq 1$ such that the efficient research budget is feasible only for $\tau \in (\tau_{min}, \tau_{max})$*

2. *The smaller is the environmental stress of the economy, the larger is the admissible range of the tax rates $\tau_{min}(\mathbf{S}), \tau_{max}(\mathbf{S})$.*

B) In the efficient state of the economy, there exists a trade-off between consumption and the environment: higher consumption goes along with a larger stock of pollution.

Proof. The first claim of A) follows from the observation that the roots of (37) are defined by the numerator which is a second degree polynomial in τ so that there exist up to 2 intersections with the $a = 0$ line, which we denote by τ_{min}, τ_{max}. The second claim of A) follows from the observation that $\partial\tau_{min}/\partial\mathbf{S} < 0$ holds, since a_1, e_0 affect the roots negatively and ν positively.

To prove B) we note that (26) implies for the efficient steady-state:

$$\bar{C} = \left(\frac{1-\tau}{1-a^\star}\right)\frac{\nu\bar{M}}{e_0/2}$$

The budget constraint of the government (9) gives, $\tau = a^\star + R^\star/(\phi(\bar{M})\bar{A}\bar{K})$. Using this to substitute for τ in $\bar{C}$ leads to

$$\bar{C} = \frac{\nu \bar{M}}{e_0/2} - R^\star,$$

showing that a higher $\bar{C}$ goes along with a higher $\bar{M}$. □

We have seen that the economy may experience a technology lock-in for a given state of the environment that can be avoided by increasing the research budget sufficiently. When we treat the environmental pressure as a parameter, it turns out that the higher this pressure is, the more difficult it is to escape the technology lock-in, since more and more resources have to be devoted to fight off climate changes. Finally, when the efficient steady-state of the economy is reached, there is a trade-off between consumption and the environment: a higher level of consumption always goes along with a more polluted environment and vice versa.

5 Conclusion

In this paper we have developed a model of economic growth that takes into account climate change, where simultaneous vertical and horizontal innovations are decisive as regards the economic outcome and as regards the state of the environment. The key feature of these innovations is that all of the new technologies are different from each other as concerns their productivities and with respect to their GHG emissions.

We have seen that a constraint on the R&D budget can give rise to multiple steady-states. This multiplicity is the result of an uneven distribution of investments between the introduction of new technologies and the development of older ones. With limited research expenditures it is likely that the majority of resources will be spent on the development of existing technologies, rather than on the introduction of newer ones. This will lead to the technology lock-in phenomenon, as described in the literature, when newer technologies are underdeveloped or even non-existent. However, due to the structure of the R&D process considered in our paper, this lock-in may be overcome by an increase of R&D spending above a certain threshold which depends on the structure of the R&D process.

When the lock-in is eliminated the economic performance in terms of total steady-state productivity, in terms of the variety of technologies and in terms of steady-state output improves. However, that goes at the expense of the environment that is characterized by a higher steady-state pollution if this is achieved through a single fiscal policy, i.e. by a higher income tax rate for example. It is true that newer technologies have a lower emissions intensity and, therefore, are less polluting, but, this positive effect on the environment is compensated by the increase in total productivity that raises output and pollution.

But, it must be noted that it is also possible to escape the lock-in and to reach an efficient unique steady-state by a combination of fiscal policies, such as jointly varying the tax rate and the abatement share. Then, a certain part of the gain in output is used for abatement to reduce the environmental pollution. In this case, consumption can rise without raising environmental pollution, when moving from the inefficient economy to the efficient one. Hence, there is a sort of double dividend in the sense that it is possible to raise consumption without damaging the environment. It must be underlined that this can only be achieved by a combination of fiscal policy measures.

Further, such a double dividend can occur only when moving from the inefficient state of the economy to the efficient one. Once the efficient steady-state is reached, economic policy cannot further raise output and consumption without damaging the environment. Instead, there is a trade-off: either raise output further and accept a more polluted environment or vice versa.

As regards an efficient economic policy that eliminates the lock-in, we could show that it is excluded for low values of the income tax rate, irrespective of the abatement share. In addition, we could demonstrate that the higher is the environmental stress, defined by the structural parameters of the environmental-economic system, the smaller is the range of combinations of the tax rate and of the abatement share that allow an efficient economic policy. Finally, we found that an efficient policy is excluded if the environmental stress exceeds a certain critical value. In that case, the lock-in cannot be overcome.

Hence, whenever the lock-in can be eliminated, one policy option for the government is to escape the lock-in and to raise economic efficiency and, thus, output and consumption, which leads to a deterioration of the environment. Another option is to increase economic efficiency and to use a certain part of the gain in GDP for abatement in order to reduce the resulting higher environmental pollution. Thus, the policy maker can keep the environmental pollution at the low level of the inefficient situation and simultaneously raise consumption.

A Derivation for the R&D sector

With a constraint on research expenditures R the problem of R&D is a resource-constrained distributed optimal control problem, given by the objective functional (11), dynamic constraints (12) and (13) and resource constraint (10) plus static constraint (14). The associated augmented Hamiltonian is constructed by adjoining a resource constraint to the standard Hamiltonian function:

$$\mathcal{H}^R = \int_0^{n(t)} \Big[q(i,t) - \frac{1}{2}g(i,t)^2\Big] di - \frac{1}{2}u(t)^2 + \lambda_n(t) \cdot \xi u(t) +$$
$$\int_0^{n(t)} \lambda_q(i,t) \cdot (\psi(i)g(i,t) - \beta q(i,t)) + l(t) \cdot \Big(R - u(t) - \int_0^{n(t)} g(i,t)di\Big), \tag{A.1}$$

with the resource constraint being the complementary slackness condition, i.e. if $R > u(t) + \int_0^{n(t)} g(i,t)di$ then $l(t) = 0$. We denote this threshold by $R^\star$.

First-order conditions for controls are

$$u(t) = \xi\lambda_n(t) - l(t); \tag{A.2}$$
$$g(i,t) = \psi(i)\lambda_q(i,t) - l(t); \tag{A.3}$$
$$R - u(t) - \int_0^{n(t)} g(i,t)di = 0, \tag{A.4}$$

where (A.4) holds only if $R < R^\star$. Otherwise $l(t) = 0$ and the model of R&D fully resembles the unconstrained one as in (Bondarev 2012). Indeed, investments $u(t), g(i,t)$ increase up to the steady-state and are maximal there. Then, it follows that, once the allocated budget exceeds this maximal overall R&D investments level $R^\star$, complementary slackness requires $l(t) = 0$. Moreover, once $R = const$ and $u(t), g(i,t)$ monotonically increase, it follows $l(R,t)$ has only one switching point in R and is always either zero or given by (A.4).

The co-state system is:

$$\dot{\lambda}_n(t) = r\lambda_n(t) - \frac{\partial \mathcal{H}^R}{\partial n} = r\lambda_n(t) - q(i,t)|_{i=n(t)} + \frac{1}{2}g^2(i,t)|_{i=n(t)} - \\ - \frac{1}{r+\beta}\left(\psi(n(t))g(i,t)|_{i=n(t)} - \beta q(i,t)|_{i=n(t)}\right) + l(t)g(i,t)|_{i=n(t)}, \tag{A.5}$$

$$\forall i \leq n(t) : \dot{\lambda}_q(i,t) = (r+\beta)\lambda_q(i,t) - 1, \tag{A.6}$$

giving immediately with the usual transversality condition $\lim_{t\to\infty} \mathrm{e}^{(r+\beta)t}\lambda_q(i,t) = 0$ constant shadow costs for productivity investments:

$$\lambda_q(i,t) = \frac{1}{r+\beta}. \tag{A.7}$$

First, we make use of the necessary conditions to obtain expression for the resource Lagrange multiplier $l(t)$:

$$l(t) = \begin{cases} \frac{1}{1+n(t)}\left(\xi\lambda_n(t) - R + \frac{1}{r+\beta}\int_0^{n(t)}\psi(i)di\right), & R \leq R^\star \\ 0, & R > R^\star \end{cases} \tag{A.8}$$

We derive results for the case $R < R^\star$ first (marked by superscript R) and obtain the unconstrained case by letting $l(t) = 0$ in every case for completeness (marked with $\star$).

The constrained control functions of research expenditures are given by:

$$u^R(t) = \lambda_n^R(t) - \frac{1}{1+n^R(t)}\left(\xi\lambda_n^R(t) - R + \frac{1}{r+\beta}\int_0^{n^R(t)}\psi(i)di\right) \tag{A.9}$$

$$g^R(i,t) = \frac{\psi(i)}{r+\beta} - \frac{1}{1+n^R(t)}\left(\xi\lambda_n^R(t) - R + \frac{1}{r+\beta}\int_0^{n^R(t)}\psi(i)di\right) \tag{A.10}$$

with unconstrained counterparts

$$u^\star(t) = \lambda_n^\star(t) \tag{A.11}$$

$$g^\star(i,t) = \frac{\psi(i)}{r+\beta}. \tag{A.12}$$

The dynamics of individual productivities are given by

$$\frac{\partial}{\partial t}q^R(i,t) = \psi(i)\left(\frac{\psi(i)}{r+\beta} - \frac{1}{1+n^R(t)}\left(\xi\lambda_n^R(t) - R + \frac{1}{r+\beta}\int_0^{n^R(t)}\psi(i)di\right)\right) - \beta q^R(i,t) \tag{A.13}$$

for the constrained case and simply by

$$\frac{\partial}{\partial t}q^\star(i,t) = \frac{\psi^2(i)}{r+\beta} - \beta q^\star(i,t) \tag{A.14}$$

for the unconstrained case.

Using the fact $q(i,t)|_{i=n(t)} = 0$, the expressions for the controls, (A.3), and for the Lagrange multiplier, (A.8), and efficiency of investments into the boundary technology, $\psi(n) = \psi_c\sqrt{1-n(t)}$, one obtains the explicit expression for the evolution of the co-state variable as a function of $n(t)$ and of the R&D budget R as:

$$\begin{aligned}
\dot{\lambda}_n^R(t) = &\left(-\frac{\beta\,\xi^2 r}{\left(n^R(t)+1\right)^2(r+\beta)^2} - 1/2\,\frac{r^2\xi^2}{\left(n^R(t)+1\right)^2(r+\beta)^2} - 1/2\,\frac{\beta^2\xi^2}{\left(n^R(t)+1\right)^2(r+\beta)^2}\right)\cdot \\
&\cdot\left(\lambda_n^R(t)\right)^2 + \frac{r^2\xi\,R - 2/3\,\psi_c\,r\xi + \beta^2\xi\,R + 2\,\beta\,\xi\,rR - 2/3\,\psi_c\,\beta\,\xi + 2/3\,\psi_c\,(1-n(t))^{3/2}\,\beta\,\xi}{(n(t)+1)^2(r+\beta)^2}\cdot \\
&\cdot\lambda_n(t) + \left(\frac{\psi_c\sqrt{1-n(t)}\beta\,\xi + \psi_c\sqrt{1-n^R(t)}r\xi}{(r+\beta)^2\left(n^R(t)+1\right)} + r + 2/3\,\frac{\psi_c\left(1-n^R(t)\right)^{3/2}r\xi}{(n(t)+1)^2(r+\beta)^2}\right)\lambda_n^R(t) - \\
&-7/6\,\frac{{\psi_c}^2(n(t))^2}{\left(n^R(t)+1\right)^2(r+\beta)^2} - \frac{5}{18}\,\frac{{\psi_c}^2\left(n^R(t)\right)^3}{\left(n^R(t)+1\right)^2(r+\beta)^2} - 2/3\,\frac{\psi_c\,(1-n(t))^{3/2}\,\beta\,R}{\left(n^R(t)+1\right)^2(r+\beta)^2} - \\
&-2/3\,\frac{\psi_c\left(1-n^R(t)\right)^{3/2}rR}{\left(n^R(t)+1\right)^2(r+\beta)^2} + 1/3\,\frac{{\psi_c}^2\left(n^R(t)\right)^2}{(r+\beta)^2(n(t)+1)} + 2/3\,\frac{{\psi_c}^2\sqrt{1-n(t)}}{(r+\beta)^2\left(n^R(t)+1\right)} - \\
&-1/2\,\frac{R^2\beta^2}{\left(n^R(t)+1\right)^2(r+\beta)^2} - \frac{\psi_c\sqrt{1-n^R(t)}\beta\,R}{(r+\beta)^2\left(n^R(t)+1\right)} - \frac{\psi_c\sqrt{1-n(t)}rR}{(r+\beta)^2\left(n^R(t)+1\right)} + \\
&+2/3\,\frac{\psi_c\,Rr}{(n(t)+1)^2(r+\beta)^2} - \frac{R^2\beta\,r}{\left(n^R(t)+1\right)^2(r+\beta)^2} - 1/2\,\frac{R^2r^2}{\left(n^R(t)+1\right)^2(r+\beta)^2} + \\
&+7/6\,\frac{{\psi_c}^2 n(t)}{(n(t)+1)^2(r+\beta)^2} - 5/3\,\frac{{\psi_c}^2}{(r+\beta)^2\left(n^R(t)+1\right)} + 4/9\,\frac{{\psi_c}^2(1-n(t))^{3/2}}{\left(n^R(t)+1\right)^2(r+\beta)^2} + \\
&+4/3\,\frac{{\psi_c}^2 n^R(t)}{(r+\beta)^2\left(n^R(t)+1\right)} + 2/3\,\frac{\psi_c\,R\beta}{\left(n^R(t)+1\right)^2(r+\beta)^2} + 1/18\,\frac{{\psi_c}^2}{\left(n^R(t)+1\right)^2(r+\beta)^2}
\end{aligned} \tag{A.15}$$

Substitution of (A.2) with $l(t)$ defined by (A.8) into the dynamics of variety expansion from (12) yields the following dynamics of $n(t)$:

$$\dot{n}^R(t) = \xi^2 \lambda_n^R(t) \frac{n^R(t)}{1+n^R(t)} - \left(\frac{1}{3}\right) \frac{\xi\left(2\psi_c - 2\psi_c(1-n^R(t))^{3/2} - 3R(r+\beta)\right)}{(r+\beta)(1+n^R(t))}, \qquad \text{(A.16)}$$

The inspection of the equation (A.15) shows that the steady-state condition for the shadow price λ_n^R is a polynomial of second order in this variable. From fundamental algebra we know that such a polynomial has exactly two roots[5]. Thus, for every value of $n^R(t)$ there are two steady-state values of the shadow price. At the same time, the equation (A.16) is linear in the shadow price so that there is only one steady-state of $n^R(t)$ for every value of λ_n^R. These considerations demonstrate that the system (A.15), (A.16) can have at most two different steady-states. The isocline $\dot{\lambda}_n^R = 0$ generates two lines with one origin at $n^R \leq 1$, and the isocline $\dot{n}^R(t) = 0$ is an initially rising concave function that becomes vertical at $n = 1$. Two steady-states arise when the $\dot{n}^R(t) = 0$ isocline intersects the $\dot{\lambda}_n^R = 0$ isocline for values of $n < 1$ and a unique steady-state is obtained when the isoclines intersect at $n = 1$.

The unique steady-state implies $\bar{n} = 1$ and $\bar{\lambda}_n = 0$. Given this, the threshold level of the research budget, $R^\star$, is easily obtained from $\dot{n}^R(t) = 0$ as $R^\star = 2\psi_c/(3(r+\beta))$.

B Policy tools derivations

Function for the research expenditures equation (36):

$$Y(a,n,R) = (1-\tau)a1\,e(1-a)\psi\left((-4/3\,\psi + R(r+\beta))(-1+n)\sqrt{1-n} + (2/3\,\psi - 3/4)n^3 + \right.$$
$$\left. + (-2\,\psi + 3/4)n^2 + (2\,\psi + 3/2)n - 4/3\,\psi + R(r+\beta)\right),$$
$$X(a,n,\tau) = -3\nu\,\rho\,(\tau - a)(1+n)(r+\beta)\beta \qquad \text{(B.1)}$$

[5] we do not distinguish between complex and real roots, since this affects only stability and not the existence of steady states.

References

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012). The environment and directed technical change. *American Economic Review 102*(1), 131–66.

Bondarev, A. (2012). The long run dynamics of heterogeneous product and process innovations for a multi product monopolist. *Economics of Innovation and New technology 21*(8), 775–799.

Bondarev, A. and A. Greiner (2017). Technology lock-in with horizontal and vertical innovations through limited r&d spending. *Quaterly Journal of Operations Research Online first publication*, doi: 10.1007/s10288–017–0348–0.

Greiner, A., L. Grüne, and W. Semmler (2010). Growth and climate change: Threshold and multiple equilibria. In J. Crespo Cuaresma, T. Palokangas, A. Tarasyev, S. Mittnik, and W. Semmler (Eds.), *Dynamic Systems, Economic Growth, and the Environment*, Volume 12 of *Dynamic Modeling and Econometrics in Economics and Finance*, pp. 63–78. Springer Berlin Heidelberg.

Greiner, A. and W. Semmler (2005). Economic growth and global warming: A model of multiple equilibria and thresholds. *Journal of Economic Behavior and Organization 57*(4), 430–447.

Kalkuhl, M., O. Edenhofer, and K. Lessmann (2012). Learning or lock-in: Optimal technology policies to support mitigation. *Resource and Energy Economics 34*(1), 1–23.

Meadows, D., D. Meadows, J. Randers, and W. Behrens (1972). *The Limits to Growth*. Universe Books, New York.

Nordhaus, W. D. (1992). The "dice" model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming. *Cowles Foundation discussion paper no. 1009*.

Nordhaus, W. D. (2007). *The challenge of global warming: economic models and environmental policy*, Volume 4. Yale University, Yale.

OECD (2011). *Towards Green Growth*. OECD Publishing. http://dx.doi.org/10.1787/9789264111318-en.

Peretto, P. and M. Connolly (2007). The manhattan metaphor. *Journal of Economic Growth 12*(4), 329–350.

Popp, D. (2004). Entice: Endogenous technological change in the dice model of global warming. *Journal of Environmental Economics and Management 48*(1), 742–768.

TheWorldBank (2012). *Inclusive Green Growth. The Pathway to Sustainable Development*. The World Bank, Washington, D.C.