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## **The Relationship between Risk-Premium and Convenience-Yield Models**

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CHAPTER XYZ

THE RELATIONSHIP BETWEEN RISK-PREMIUM AND  
CONVENIENCE-YIELD MODELS

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## SUMMARY

Arbitrage pricing cannot be applied to commodity futures because the physical commodity does not represent a pure asset: Since consumption and processing of the commodity can drive down inventories to zero, it is not always possible to construct a replicating portfolio for the futures contract, and commodity spot prices do not (fully) reflect price expectations and risk premiums.

The two alternative valuation principles for commodity futures are risk premium model (RPM) and convenience-yield model (CYM). Risk-premium models derive futures prices from expected commodity spot prices at maturity, and convenience-yield models derive futures prices from the current commodity spot price.

The chapter shows that the two valuation principles are mutually consistent if convenience yields are regarded as the deviation of the commodity spot price from its asset value (the present value of the expected commodity spot price at maturity). By combining risk-premium models and convenience-yield models, it can be shown that convenience yields reflect the proportion of the expected change in commodity spot prices which is not attributable to the risk premium and the riskfree rate (i.e. to the quasi asset value of the commodity).

All relationships are summarized in Exhibit 7.

The relationship between futures returns and convenience yields, or the term structure respectively, is of particular interest. Can futures returns be predicted based on the term structure or convenience yields? At first sight, equations (12a) and (12b) seem to suggest such a relationship. But it has been shown that convenience yields only reflect the temporary "consumption value" of the commodity and are, in general, independent from the expected risk premium. Again, convenience yields reflect the expected change in commodity spot prices which is not driven by the risk premium or the cost of carry. The same interpretation should be applied to roll returns, because they differ from (storage cost adjusted) convenience yields by the riskfree rate. Therefore, average roll yields reflect the expected deviation of the spot price change from the risk premium.

Our empirical illustrations confirm this view: The futures term structure, convenience yields and roll returns largely anticipate subsequent spot price changes. However, a small portion of roll returns, is not compensated by subsequent spot price changes, and could be explained time-varying risk premiums conditional on roll returns and expected spot price changes. This requires more detailed analysis.

## Exhibit 7 Summary

	Term Structure	Futures Returns
	$\ln \frac{F_{t,T_2}}{F_{t,T_1}}$	$\tilde{r}_{F,t,t+1,T}$
Risk Premium Model	$\hat{\alpha}_S^C(t, T_1, T_2) - rp(T_2 - T_1)$	$rp + \tilde{\Delta} \ln E_{t,t+1,T}$
Convenience Yield Model	$(r + m - cy)(T_2 - T_1)$	$\underbrace{-[r + m - cy]}_{\text{roll } y} + \tilde{r}_{S,t,t+1}^C$
Relationship	$\hat{\alpha}_S^C(t, T_1, T_2) = [r + rp + m - cy](T_2 - T_1)$	$\tilde{\Delta} \ln E_{t,t+1,T} = -[r + rp + m - cy] + \tilde{r}_{S,t,t+1}^C$
		$E_t [\tilde{\Delta} \ln E_{t,t+1,T}] = -[r + rp + m - cy] + E_t (\tilde{r}_{S,t,t+1}^C)$ $= -\frac{1}{2} \sigma^2$
	For $T_1 = t + 1, T_2 = T$ :	
	$\hat{\alpha}_S^C(t, t + 1, T) = \left( E_t [\tilde{r}_{S,t,t+1}^C] + \frac{1}{2} \sigma^2 \right) (T - t - 1)$	