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Destructive interference of direct and crossed Andreev pairing in a system of two nanowires coupled via an *s*-wave superconductor

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We consider a system of two one-dimensional nanowires coupled via an s-wave superconducting strip, a geometry that is capable of supporting Kramers pairs of Majorana fermions. By performing an exact analytical diagonalization of a tunneling Hamiltonian describing the proximity effect (via a Bogoliubov transformation), we show that the excitation gap of the system varies periodically on the scale of the Fermi wavelength in the limit where the interwire separation is shorter than the superconducting coherence length. Comparing with the excitation gaps in similar geometries containing only direct pairing, where one wire is decoupled from the superconductor, or only crossed Andreev pairing, where each nanowire is considered as a spin-polarized edge of a quantum Hall state, we find that the gap is always reduced, by orders of magnitude in certain cases, when both types of pairing are present. Our analytical results are further supported by numerical calculations on a tight-binding lattice. Finally, we show that treating the proximity effect by integrating out the superconductor using the bulk Green's function does not reproduce the results of our exact diagonalization.

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Introduction. Topological superconductivity has garnered a great deal of attention in recent years [1–3] both theoretically and experimentally because the localized excitations of such systems, known as Majorana fermions, obey non-Abelian statistics and can be utilized for applications in quantum computing [4,5]. The proposals which have received the most attention to date involve engineering Majorana states in nanowires with Rashba spin-orbit coupling in the presence of a Zeeman field [6–17] or in ferromagnetic atomic chains [18–24]. In the absence of any Zeeman splitting, it is possible to generate an even more exotic time-reversal invariant topological superconducting phase which supports Kramers pairs of Majorana fermions [25–35]. One such proposal involves coupling two Rashba nanowires via an s-wave superconductor [31,32]. In this system, superconductivity is induced in the nanowires via *direct* Cooper pair tunneling, where both electrons of a Cooper pair tunnel into the same wire, and *crossed Andreev* tunneling, where one electron from a Cooper pair tunnels into each wire [36–41]. The topological phase can be realized if the strength of crossed Andreev pairing exceeds that of direct pairing. However, to date, the direct and crossed Andreev pairing strengths have been treated as theoretical parameters [31–34] and no rigorous treatment of the proximity effect in this system has been carried out.

In this paper, we study the interplay between direct and crossed Andreev pairing in a noninteracting double-nanowire system by calculating the proximity-induced excitation gap as a function of the interwire separation (d). We show that the two pairing types always interfere destructively. When the tunneling strengths into each nanowire are equal, the excitation gap in the presence of both types of pairing is simply the difference between the gap in the presence of only direct pairing and the gap in the presence of only crossed Andreev pairing, with the direct gap always being larger than the crossed Andreev gap. When the interwire separation is shorter than the superconducting coherence length (ξ_s), this destructive interference can lead to an order of magnitude reduction in the size of the excitation gap of the system.

Our results are based on an exact analytical diagonalization of the tunneling Hamiltonian via a Bogoliubov transformation. We derive a set of effective Bogoliubov-de Gennes (BdG) equations that we then solve to determine the excitation gap as a function of d. Additionally, we show that integrating out the superconducting degrees of freedom using the bulk superconducting Green's function, a common method for treating the proximity effect in low-dimensional systems [1,42–49], yields incorrect and qualitatively different results in our finite geometry.

Model. The system we consider is displayed in Fig. 1. Two one-dimensional nanowires are coupled to a superconducting strip of finite width d, taken to occupy $0 < x < d$. The system is taken to be infinite in the y direction, allowing us to define a conserved momentum k_y. We consider a Hamiltonian of the form

$$H = H_{NW}^L + H_{NW}^R + H_{BCS} + H_t^L + H_t^R. \tag{1}$$

The nanowire Hamiltonian can be expressed generally as

$$H_{NW}^i = \sum_{\sigma,\sigma'} \int \frac{dk_y}{2\pi}\, \psi_{i\sigma}^\dagger(k_y)\mathcal{H}_{\sigma\sigma'}^i(k_y)\psi_{i\sigma'}(k_y), \tag{2}$$

where $\psi_{i\sigma}^\dagger(k_y)$ $[\psi_{i\sigma}(k_y)]$ creates (annihilates) an electron of spin σ and momentum k_y in nanowire i and the Hamiltonian density $\mathcal{H}_i(k_y)$ of each wire is left unspecified. We describe the superconductor by a BCS Hamiltonian,

$$H_{BCS} = \int \frac{dk_y}{2\pi} \int dx\, \Psi_s^\dagger(H_0\tau_z + \Delta\tau_x)\Psi_s, \tag{3}$$

where $\Psi_s = [\psi_{s\uparrow}(-k_y,x), \psi_{s\downarrow}^\dagger(k_y,x)]^T$, $\psi_{s\sigma}^\dagger(k_y,x)$ $[\psi_{s\sigma}(k_y,x)]$ creates (annihilates) an electron of spin σ and momentum k_y at position x inside the superconductor, $H_0 = -\partial_x^2/2m_s + k_y^2/2m_s - \mu_s$ (m_s is the effective mass and μ_s is the Fermi energy of the superconductor), Δ is the superconducting pairing potential, and $\tau_{x,y,z}$ are Pauli matrices acting in Nambu space. We also allow for electrons to tunnel between superconductor and wire, assuming that this process preserves

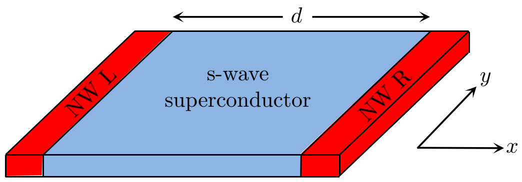

FIG. 1. Geometry of considered model. A 2D conventional s-wave superconductor of width d $(0 < x < d)$ separates two 1D nanowires. The system is infinite in the y direction.

both spin and momentum. Tunneling is described by

$$H_t^i = -t_i \sum_\sigma \int \frac{dk_y}{2\pi} [\psi^\dagger_{i\sigma}(k_y)\psi_{s\sigma}(k_y,x_i) + \text{H.c.}], \quad (4)$$

where t_i is a wire-dependent tunneling amplitude and x_i denotes the position of wire i.

Bogoliubov-de Gennes Equations. To solve the model under consideration, we perform an exact diagonalization of Hamiltonian (1) by introducing a transformation of the form

$$\psi^\dagger_{i\sigma}(k_y) = \sum_n [\gamma_n^\dagger u^*_{in\sigma}(k_y) + \gamma_n v_{in\sigma}(k_y)], \quad (5a)$$

$$\psi^\dagger_{s\sigma}(k_y,x) = \sum_n [\gamma_n^\dagger u^*_{sn\sigma}(k_y,x) + \gamma_n v_{sn\sigma}(k_y,x)], \quad (5b)$$

where γ_n describes the new quasiparticles of the system and $u(v)$ are electron (hole) wave functions. It is straightforward to show [50] that transformation (5) diagonalizes Hamiltonian (1) provided that the wave functions obey a set of BdG equations given by

$$Eu_i(k_y) = \mathcal{H}_i(k_y)u_i(k_y) - t_i u_s(k_y,x_i), \quad (6a)$$

$$-Ev_i(k_y) = \mathcal{H}_i^T(k_y)v_i(k_y) - t_i v_s(k_y,x_i), \quad (6b)$$

$$Eu_s(k_y,x) = H_0 u_s(k_y,x) + \Delta(i\sigma_y)v_s(-k_y,x) - \sum_i t_i\delta(x-x_i)u_i(k_y), \quad (6c)$$

$$-Ev_s(k_y,x) = H_0 v_s(k_y,x) + \Delta(i\sigma_y)u_s(-k_y,x) - \sum_i t_i\delta(x-x_i)v_i(k_y). \quad (6d)$$

In Eqs. (6), we introduce the spinor electron (hole) wave function $u(v)_j = [u(v)_{j\uparrow}, u(v)_{j\downarrow}]^T$ for $j = i,s$ and denote the Pauli matrix acting in spin space by $\sigma_{x,y,z}$.

While Eqs. (6) were derived without making any assumptions about the nanowire Hamiltonian, for the remainder of the paper we focus on the simple case where each nanowire is a normal conductor that can be described by $\mathcal{H}_i(k_y) = \xi_k$, with $\xi_k = k_y^2/2m_n - \mu_n$ (m_n and μ_n are the effective mass and Fermi energy of the nanowires). With this simple choice for the nanowire Hamiltonian, we are able to eliminate the trivial spin sector from the BdG equations; essentially, we can reduce Eqs. (6) from matrix equations to scalar equations. Equations (6a) and (6b) form an independent algebraic system that yields the solutions

$$u_{i\uparrow}[v_{i\downarrow}](k_y) = \frac{t_i}{\xi_k \mp E} u_{s\uparrow}[v_{s\downarrow}](k_y,x_i). \quad (7)$$

Substituting Eq. (7), we can decouple Eqs. (6c) and (6d) to obtain a system of differential equations describing the wave functions in the superconductor,

$$\left(\pm H_0 + \frac{t_L^2\delta(x-x_L)}{E\mp\xi_k} + \frac{t_R^2\delta(x-x_R)}{E\mp\xi_k}\right)u_{s\uparrow}[v_{s\downarrow}](k_y,x) + \Delta v_{s\downarrow}[u_{s\uparrow}](-k_y,x) = Eu_{s\uparrow}[v_{s\downarrow}](k_y,x). \quad (8)$$

The solution to Eq. (8) within the left l $(0 < x < x_L)$, middle m $(x_L < x < x_R)$, and right r $(x_R < x < d)$ regions of the superconductor is

$$\psi_l(k_y,x) = c_1\begin{pmatrix}u_0\\v_0\end{pmatrix}\sin(p_+x) + c_2\begin{pmatrix}v_0\\u_0\end{pmatrix}\sin(p_-x),$$

$$\psi_m(k_y,x) = c_3\begin{pmatrix}u_0\\v_0\end{pmatrix}e^{ip_+x} + c_4\begin{pmatrix}u_0\\v_0\end{pmatrix}e^{-ip_+x} + c_5\begin{pmatrix}v_0\\u_0\end{pmatrix}e^{ip_-x} + c_6\begin{pmatrix}v_0\\u_0\end{pmatrix}e^{-ip_-x}, \quad (9)$$

$$\psi_r(k_y,x) = c_7\begin{pmatrix}u_0\\v_0\end{pmatrix}\sin[p_+(d-x)] + c_8\begin{pmatrix}v_0\\u_0\end{pmatrix}\sin[p_-(d-x)],$$

where $\psi(k_y,x) = [u_{s\uparrow}(k_y,x), v_{s\downarrow}(-k_y,x)]^T$ is a spinor wave function, $p_\pm^2 = 2m_s(\mu_s \pm i\Omega) - k_y^2$, $\Omega^2 = \Delta^2 - E^2$, and $u_0^2(v_0^2) = (1 \pm i\Omega/E)/2$. To determine the eight unknown coefficients c_{1-8}, we must impose appropriate boundary conditions at $x = x_L$ and $x = x_R$ (note that a vanishing boundary condition at $x = 0$ and $x = d$ has already been imposed). In addition to continuity of the wave function, the boundary conditions on the derivatives of the wave functions are determined by the delta-function terms of Eq. (8) and are obtained by direct integration:

$$\partial_x u_{s\uparrow}[v_{s\downarrow}](k_y,x_L) = \pm\frac{2k_F\gamma_L}{E\mp\xi_k}u_{s\uparrow}[v_{s\downarrow}](k_y,x_L), \quad (10a)$$

$$\partial_x u_{s\uparrow}[v_{s\downarrow}](k_y,x_R) = \mp\frac{2k_F\gamma_R}{E\mp\xi_k}u_{s\uparrow}[v_{s\downarrow}](k_y,x_R). \quad (10b)$$

In Eqs. (10) we introduce an energy scale associated with tunneling which is proportional to the square of the tunneling amplitude, $\gamma_i = t_i^2/v_F$, where $v_F = k_F/m_s$ is the Fermi velocity of the superconductor. Assuming that the Fermi momentum of the superconductor greatly exceeds that of the nanowires ($k_F \gg k_{Fn}$) allows us to approximate $p_\pm = k_F \pm i\Omega/v_F$ (because k_y is conserved, typical values take $k_y \lesssim k_{Fn} \ll k_F$; we also expand in the limit $\mu_s \gg \Delta$). However, even by making these simplifications the solvability condition of Eqs. (10) cannot be solved besides numerically for an arbitrary parameter set [50].

In order to proceed analytically, we assume that the superconductor is only weakly coupled to each nanowire, so that $\gamma_i \ll \Delta$. In this limit, the relevant pairing energies in the nanowires are small and we can focus our attention on energies $E \ll \Delta$. We also assume that the nanowires are (symmetrically) located near the ends of the superconductor, such that $x_L = x_w$ and $x_R = d - x_w$ with $x_w \ll d$. The solvability condition in this limit can be expressed as $a(\xi_k,d)E^4 - b(\xi_k,d)E^2 + c(\xi_k,d) = 0$, with the complicated

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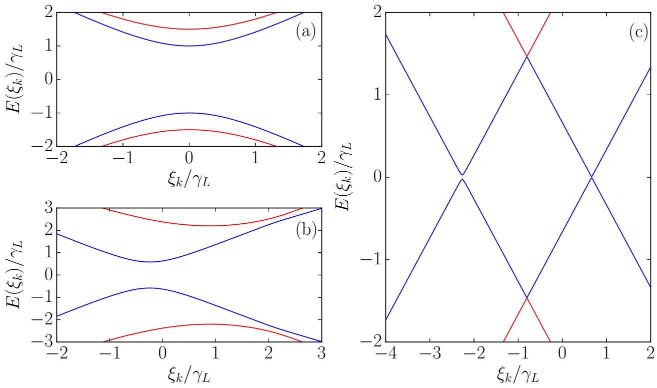

FIG. 2. Excitation spectra for (a) $d = 100\xi_s$, (b) $d = \xi_s$, and (c) $d = 0.01\xi_s$. For all plots, we choose $\gamma_R/\gamma_L = 1.5$ and $k_F\xi_s = 100$.

expressions for the coefficients given in the Supplemental Material [50]. This equation can be solved exactly for the energy spectrum $E(\xi_k)$, which is plotted for several values of d in Fig. 2. When $d \gg \xi_s$ [Fig. 2(a)], the spectrum consists of two parabolic bands and has a gap given by $\min\{\gamma_L,\gamma_R\}$; this represents the decoupling of two nanowires with a large spatial separation. When the wires are brought closer together [Figs. 2(b) and 2(c)], crossed Andreev pairing reduces the size of the gap while single-particle couplings induced by tunneling effectively shift the chemical potentials of each band [33,50].

Excitation gap in the weak-coupling limit. Our goal is to calculate the excitation gap (the global minimum of the spectrum) as a function of d. Although we are able to solve for the spectrum exactly, it is still quite cumbersome to determine the excitation gap for all d when tunneling is asymmetric ($\gamma_L \neq \gamma_R$).

Let us consider the symmetric-tunneling case $\gamma_L = \gamma_R$, leaving the asymmetric case for the Supplemental Material [50]. Under the assumption of symmetric tunneling, it is quite straightforward to solve for the gap for any value of d [50]. Assuming that $d/\xi_s \gg \gamma/\Delta$, the gap is

$$E_g(d) = \frac{\gamma \sinh(d/\xi_s)}{\cosh(d/\xi_s) + |\cos(k_F d)|}. \tag{11}$$

[The gap in principle also depends on the wire position x_w through an additional factor $\sin^2(k_F x_w)$; because this is a rather arbitrary quantity, we simply replace it by its mean value $\langle \sin^2(k_F x_w)\rangle = 1/2$ throughout.] When the superconductor is very wide ($d \gg \xi_s$), the gap approaches $E_g = \gamma$. When the superconductor is very narrow ($d \ll \xi_s$), the gap oscillates on the scale of $1/k_F$ between its maximum value $E_g^{\max} = \gamma d/\xi_s$, attained for $k_F d = \pi(n + 1/2)$ ($n \in \mathbb{Z}$), and its minimum value $E_g^{\min} = \gamma d/2\xi_s$, attained for $k_F d = n\pi$.

Note that because we chose $\gamma_L = \gamma_R$, we are unable to distinguish between direct and crossed Andreev pairing in our result for the gap [Eq. (11)]. We again must stress that we are not solving an effective model, so the direct and crossed Andreev pairing functions are not parameters of our theory as in Refs. [31–34]. Instead, we identify direct terms as being proportional to t_i^2 (γ_i) and crossed Andreev terms as being proportional or $t_L t_R$ ($\sqrt{\gamma_L\gamma_R}$). In an attempt to differentiate between the two contributions, we compare the gap in the presence of both pairing types to that of similar systems containing only one type of pairing.

First, we isolate direct pairing in our system by decoupling one of the wires from the superconductor. Setting $t_L = 0$ in Eq. (8), we find a direct gap of the form [50]

$$E_g^D(d) = \frac{\gamma \sinh(2d/\xi_s)}{\cosh(2d/\xi_s) - \cos(2k_F d)}. \tag{12}$$

If the superconductor is very wide, the gap approaches $E_g = \gamma$ as before. If the superconductor is very narrow, the gap is $E_g = (2\gamma d/\xi_s)/[1 - \cos(2k_F d) + 2d^2/\xi_s^2]$. The gap is sharply peaked near $k_F d = n\pi$ and has a maximum value $E_g^{D,\max} = \gamma\xi_s/d$. The gap is minimized near $k_F d = \pi(n + 1/2)$ and takes the value $E_g^{D,\min} = \gamma d/\xi_s$.

To isolate crossed Andreev pairing in our system, we consider a situation where both nanowires are spin polarized and have opposite spin; i.e., they are edge states of two quantum Hall systems with opposite chirality. In this case, we introduce a spin dependence to the tunneling amplitudes, $t_i \to t_{i\sigma}$. Assuming for example that $t_{L\uparrow} = t_{R\downarrow} \neq 0$ while $t_{L\downarrow} = t_{R\uparrow} = 0$, we set $\gamma_R = 0$ in the equation for the electron wave function (which has spin-up) and $\gamma_L = 0$ in the equation for the hole wave function (which has spin-down) to find a crossed Andreev gap given by [50]

$$E_g^C(d) = \frac{2\gamma \sinh(d/\xi_s)}{\cosh(2d/\xi_s) - \cos(2k_F d)}|\cos(k_F d)|. \tag{13}$$

If the superconductor is very wide, the gap oscillates on the scale $1/k_F$ and decays on the scale ξ_s, $E_g = 2\gamma|\cos(k_F d)|e^{-d/\xi_s}$. If the superconductor is very narrow, we expand to find $E_g = (2\gamma d/\xi_s)|\cos(k_F d)|/(1 - \cos(2k_F d) + 2d^2/\xi_s^2)$. Similarly to the direct pairing case, the gap is sharply peaked near $k_F d = n\pi$, having a maximum value $E_g^{C,\max} = \gamma\xi_s/d$. The crossed Andreev gap is minimized near $k_F d = \pi(n + 1/2)$, where it vanishes. The vanishing of the gap indicates a change in sign of the crossed Andreev pairing function (see also Ref. [50]). Therefore, it should be possible to form a π junction by coupling two systems of different d. Such π phase shifts are crucial for engineering Majorana fermions in similar setups [28,29].

The three gaps that we have calculated are plotted in Fig. 3(a). The gaps are related through the remarkably simple expression

$$E_g(d) = E_g^D(d) - E_g^C(d), \tag{14}$$

indicating that direct and crossed Andreev pairing interfere with one another destructively. This effect is maximized when the superconductor is very narrow, as crossed Andreev reflection is not significantly suppressed by the interwire separation. Quite interestingly, because the direct and crossed Andreev gaps attain their maxima at the same thickness ($k_F d = n\pi$), the gap E_g is minimized when pairing is maximized. Furthermore, destructive interference between the two pairing processes at these points leads to an order of magnitude reduction of the gap [specifically, a reduction by a factor of order $\mathcal{O}(\xi_s^2/d^2)$].

We also support our analytical results with a standard tight-binding calculation in the geometry of Fig. 1 [50]. Results of the tight-binding calculation are plotted in Fig. 3(b), showing very good qualitative agreement with Fig. 3(a). We also plot the

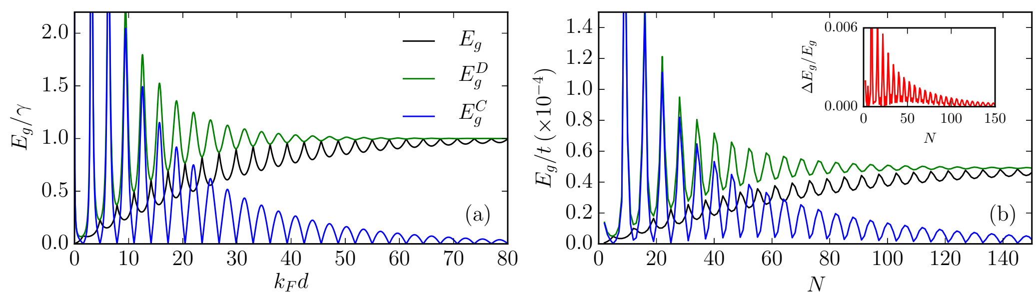

FIG. 3. Proximity-induced gaps plotted as a function of superconductor width. (a) Analytical results for $k_F\xi_s = 20$. Black curve corresponds to symmetric tunneling ($\gamma_L = \gamma_R$) [Eq. (11)], green curve corresponds to case of a single wire ($\gamma_L = 0$) [Eq. (12)], and blue curve corresponds to quantum Hall regime ($\gamma_{L\uparrow} = \gamma_{R\downarrow} \neq 0$ and $\gamma_{L\downarrow} = \gamma_{R\uparrow} = 0$) [Eq. (13)]. (b) Numerical results for $\Delta = 0.02t$, $\mu_s = 0.3t$, $\mu_n = 0.03t$, and $t_L = t_R = 0.01t$, where t is the hopping parameter in the superconductor. Inset: plot of $\Delta E_g \equiv |E_g - (E_g^D - E_g^C)|$, showing very good quantitative agreement with Eq. (14).

quantity $\Delta E_g \equiv |E_g - (E_g^D - E_g^C)|$ in the inset of Fig. 3(b), showing very good quantitative agreement with Eq. (14).

Integrating out superconductor. Finally, we show that integrating out the superconducting degrees of freedom from Eq. (1) using the bulk superconducting Green's function does not reproduce the results of our exact diagonalization in a finite geometry. Assuming that tunneling is weak and symmetric ($\gamma_L = \gamma_R \ll \Delta$) and that the superconductor is very narrow ($d \ll \xi_s$), we integrate out the superconductor to yield an effective Hamiltonian describing superconductivity induced in the nanowires [50]. This effective Hamiltonian yields a low-energy spectrum $E_\pm^2(k) = \delta^2\gamma_c^2 + (\beta\gamma_d + \xi_k \pm \eta\gamma_c)^2$, where $\gamma_{d(c)}$ differentiate one-wire (two-wire) tunneling processes, $\beta = \cot(k_F d/2)$, $\delta = -\cos(k_F d/2)$, and $\eta = \cos(k_F d/2)\cot(k_F d/2)$. In obtaining the low-energy spectrum, we expanded the effective Hamiltonian to order $\mathcal{O}[(d/\xi_s)^0]$. The minimum excitation gap of the spectrum is $E_g = \gamma_c|\cos(k_F d/2)|$. Therefore, if the superconductor is integrated out using the bulk Green's function, one would incorrectly find that crossed Andreev pairing always dominates over direct pairing in the limit $d \ll \xi_s$ [note that direct pairing shows up in the effective Hamiltonian only at order $\mathcal{O}(d/\xi_s)$].

Physically, this procedure gives a false result because it fails to properly account for the boundary conditions that must be imposed when evaluating the Gaussian path integral. When the width of the superconductor is small compared to the coherence length, these boundary effects cannot be neglected. We find that integrating out the superconductor using the bulk Green's function reproduces the correct spectrum only in the limit $d \gg \xi_s$, when the boundary effects can be safely neglected [50]. We also discuss in the Supplemental Material how one can properly account for the boundary effects when integrating out [50].

Conclusions. We have shown that direct and crossed Andreev pairing interfere destructively in a system of two nanowires coupled via an s-wave superconducting strip. When the interwire separation d is shorter than the coherence length ξ_s, this destructive interference can lead to an order of magnitude reduction in the size of the excitation gap when compared to similar systems containing only a single type of pairing. Our analytical solution is based on an exact treatment of the proximity effect through the diagonalization of the tunneling Hamiltonian (via a Bogoliubov transformation) and is supported by numerical tight-binding calculations. Additionally, we have explicitly shown that integrating out the superconductor using the bulk Green's function does not reproduce the results of our exact diagonalization.

The interference effects discussed in this paper, which are manifested through oscillations of the excitation gap on the scale of the Fermi wavelength $1/k_F$, can most easily be observed when the interwire separation is smaller than the coherence length ξ_s. If the superconductor is metallic, observing these oscillations is not feasible. However, proximity-inducing superconductivity in a low-density semiconducting two-dimensional electron gas (2DEG) such as InGaAs/InAs (as in Ref. [51]), has several advantages. Inducing superconductivity by the proximity effect will make both ξ_s and $1/k_F$ larger and will allow the density of the 2DEG to be tuned with a gate voltage (so that $k_F d$ can be varied using a single sample). Due to our assumption of translational invariance along the y direction, the interface between superconductor and nanowire must be made smooth (on the scale of ξ_s) and d must be made uniform.

Finally, we note that crossed Andreev pairing is always weaker than direct pairing in the absence of interactions. Therefore, intrawire repulsive electron-electron interactions are needed to stabilize the time-reversal invariant topological phase in the double-nanowire system, as they can significantly reduce direct pairing while leaving crossed Andreev pairing unaffected [52,53]. In this case, the nanowires support Kramers pairs of Majorana fermions and parafermions [31]. However, the destructive interference between direct and crossed Andreev pairing in the double-wire setup allows for a conventional topological superconducting phase to form at significantly reduced magnetic field strengths compared to the case of a single wire with only direct pairing Ref. [54].

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