

How Well Do We Know The Supernova Equation of State?

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We give an overview about equations of state (EOS) which are currently available for simulations of core-collapse supernovae and neutron star mergers. A few selected important aspects of the EOS, such as the symmetry energy, the maximum mass of neutron stars, and cluster formation, are confronted with constraints from experiments and astrophysical observations. There are just very few models which are compatible even with this very restricted set of constraints. These remaining models illustrate the uncertainty of the uniform nuclear matter EOS at high densities. In addition, at finite temperatures the medium modifications of nuclear clusters represent a conceptual challenge. In conclusion, there has been significant development in the recent years, but there is still need for further improved general purpose EOS tables.

KEYWORDS: equation of state, core-collapse supernova, neutron star, symmetry energy

1. Introduction

In simulations of core-collapse supernovae (CCSNe) the equation of state (EOS) has to cover various different regimes: the progenitor star at the onset of collapse, the formation of the proto-neutron star (PNS), and the development of the explosion in the layers on top. The EOS has to be applicable at all these different evolutionary stages and spatial regions. It also should give a realistic description of the remaining cold NS. Such an EOS is equally applicable in simulations of NS mergers, where various different densities, temperatures, and electron fractions (or equivalently isospin asymmetries) are encountered, similarly as in a CCSN. A better name for the “supernova EOS” would thus be “general purpose EOS”.

CCSNe and NS mergers are usually modeled using sophisticated hydrodynamic simulations. Besides the neutrino-matter interactions, it is the EOS which provides the nuclear physics input. On the one hand, this happens via thermodynamic quantities such as pressure or energy density, on the other hand via the nuclear composition. The latter information is required for the neutrino interactions.

For cold NSs which are in beta-equilibrium without neutrinos, hundreds of different EOSs exist in the literature. Conversely, just few general purpose EOS are available, due to the huge parameter range of temperatures $0 \leq T \leq 100$ MeV, densities $10^4 \text{ g/cm}^3 \leq \rho \leq 10^{15} \text{ g/cm}^3$, and electron fractions $0 \leq Y_e \leq 0.6$, which has to be covered. General purpose EOSs are usually provided and used in tabular form. There are three crucial aspects of the general purpose EOS, which will be addressed briefly in the following: (1) some properties of uniform nuclear matter, (2) the formation of nuclei at low densities, and (3) additional non-nucleonic degrees of freedom at high densities and/or temperatures.

Table I. Based on [1]. Currently existing general purpose EOSs. Listed are the nuclear interaction model used, the maximum mass M_{\max} of cold NSs, and corresponding references.

EOSs containing nucleons and nuclei.				EOSs including additional degrees of freedom.			
Model	Nuclear Interaction	M_{\max} (M_{\odot})	References	Model	Nuclear Interaction	M_{\max} (M_{\odot})	References
H&W	SKa	2.21	[2, 3]	LS220 Λ	LS220	1.91	[15, 16]
LS180	LS180	1.84	[4]	LS220 π	LS220	1.95	[15, 17]
LS220	LS220	2.06	[4]	BHBA	DD2	1.96	[18]
LS375	LS375	2.72	[4]	BHBA ϕ	DD2	2.11	[18]
STOS	TM1	2.23	[5–7]	STOS Λ	TM1	1.90	[7]
FYSS	TM1	2.22	[8]	STOSYA30	TM1	1.59	[19]
HS(TM1)	TM1	2.21	[9, 10]	STOSYA30 π	TM1	1.62	[19]
HS(TMA)	TMA	2.02	[9]	STOSY0	TM1	1.64	[19]
HS(FSU)	FSUgold	1.74	[9, 10]	STOSY0 π	TM1	1.67	[19]
HS(NL3)	NL3	2.79	[9, 11]	STOSY30	TM1	1.65	[19]
HS(DD2)	DD2	2.42	[9, 11]	STOSY30 π	TM1	1.67	[19]
HS(IUFSU)	IUFSU	1.95	[9, 11]	STOSY90	TM1	1.65	[19]
SFHo	SFHo	2.06	[12]	STOSY90 π	TM1	1.67	[19]
SFHx	SFHx	2.13	[12]	STOS π	TM1	2.06	[20]
SHT(NL3)	NL3	2.78	[13]	STOSQ209n π	TM1	1.85	[20]
SHO(FSU)	FSUgold	1.75	[14]	STOSQ162n	TM1	1.54	[21]
SHO(FSU2.1)	FSUgold2.1	2.12	[14]	STOSQ184n	TM1	1.36	[21]
				STOSQ209n	TM1	1.81	[20, 21]
				STOSQ139s	TM1	2.08	[22]
				STOSQ145s	TM1	2.01	[23]
				STOSQ155s	TM1	1.70	[24]
				STOSQ162s	TM1	1.57	[25]
				STOSQ165s	TM1	1.51	[25]

2. Properties of Uniform Nuclear Matter

At present, there are 17 general purpose EOS tables available which consider nucleons and nuclei as particles degrees of freedom, as listed in the left part of Table I. There are much more which are still in preparation, and other, equally important works about the EOS in NS (mergers) and CCSNe, which just concentrate on parts of the parameter space and do not aim at providing a full table. All of the 17 available “nucleonic” general purpose EOSs employ mean-field interactions of the nucleons and a phenomenological description of nuclei. Thus it is important to confront these models with experimental data and more solid theoretical nuclear matter calculations. Some of the 17 EOS tables share the same nucleon interactions. In total, only 13 different interactions are used, as can be seen in Table I. Here we neglect small differences, such as the usage of different nucleon masses or minor changes at low densities, as employed, e.g., in [10, 26]. For further details, see [1].

2.1 Symmetry Energy

The symmetry energy describes how matter behaves when going to neutron-rich conditions, and thus is quite important for NSs and CCSNe. Many experimental constraints for the symmetry energy are available in the literature. In a careful analysis of several experimental probes, Lattimer and Lim [27] derive $J = (29.0 - 32.7)$ MeV and $L = (40.5 - 61.9)$ MeV for the value of the symmetry energy J and the slope parameter L , both at saturation density n_B^0 . Using this constraint, only the general purpose EOSs based on IUFSU, SFHo, DD2, FSUgold, and FSUgold2.1 remain as viable models as

can be seen in Fig. 1. Note that FSUgold2.1 has the same values of J and L as FSUgold, and the values of the three LS versions are all the same.

2.2 Neutron Star Maximum Mass

The recent observations of two NSs with masses around $2 M_{\odot}$ [28–30] put an important constraint on the high-density part of the EOS. Four of the 17 nucleonic general purpose EOSs are not able to reach the lower 1-sigma limit of [28] of $1.97 M_{\odot}$. These are LS180, HS(IUFSU), HS(FSU), and SHO(FSU). The maximum mass of HS(IUFSU) with $1.95 M_{\odot}$ is just slightly too low. The last two models are both based on the FSUgold interactions, and thus have a very similar mass-radius curve and a low maximum mass around $1.74 M_{\odot}$. Note that the FSUgold2.1 interaction used in SHO(FSU2.1) is a modification of FSUgold, where an additional pressure contribution has been added at high densities to increase the maximum mass sufficiently.

2.3 Viable Models

Even if we use just the two constraints for the symmetry energy (J and L), and the maximum mass, most of the general purpose EOSs are ruled out. Only three of 17 pass these two tests: HS(DD2), SFHo, and SHO(FSU2.1). One always has to be careful with such a simple pass-fail classification. Obviously, the relevance of any constraint depends on the context where the EOS is applied. For example in a CCSN of a very light progenitor, the mass of the PNS stays much below $2 M_{\odot}$. Thus for this particular scenario even an EOS with a too low maximum mass might still be acceptable. On the other hand, it is clear that such an EOS would not be the most realistic one for a general context.

Here we have chosen only the constraints for the symmetry energy and the maximum mass, because they belong to the most important for the physics of compact stars and they are well studied and are among the most robust. Of course there are much more constraints available, see also [31,32]. If one would apply all existing constraints blindly, probably all EOSs would be excluded, as different constraints sometimes contradict each other. But even considering just a few more constraints could be problematic. For example DD2 does not fulfill the so-called flow-constraint of [33], and SFHo shows some minor deviations for the neutron matter EOS compared to theoretical constraints from the Chiral EFT calculations of [34], see [11].

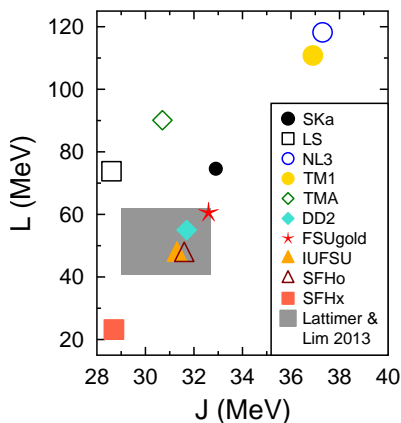


Fig. 1. Symmetry energy at saturation density J and slope parameter L for the interactions of the general purpose EOSs in comparison with the constraint of [27].

3. Description of Nuclei

The different EOS models make different simplifications regarding the nuclear degrees of freedom at low densities. For light nuclei, often only the alpha particle is considered (H&W, LS, STOS, SHT(NL3), SHO(FSU), SHO(FSU2.1)). For heavy nuclei, in some models the single nucleus approximation is used (LS, STOS), where the thermal distribution of different nuclear species is replaced by a single representative heavy nucleus.

3.1 Light Nuclei

It has been found in a number of works that light clusters, such as deuterons or tritons, can be more abundant than free protons in the shock-heated matter of a CCSN, see, e.g., [10, 11, 35]. High abundances of light nuclei are found in the vicinity of the neutrino spheres, and thus one can expect

that they might have an impact on the neutrino luminosities and mean energies. It has been found that they can modify the neutrino-driven wind [36,37], which is important for nucleosynthesis. Their effect on the supernova dynamics and neutrino quantities is still not completely settled, but probably it is only moderate, see [38,39].

Nuclear clusters are also formed in heavy-ion collisions, under very similar thermodynamic conditions as in a CCSN. Qin et al. [40] measured charged particle and neutron yields at Texas A&M and also extracted temperatures and densities at different stages of the collision from this data. They compared the experimental data with predictions of some of the general purpose EOSs. However, they did not take into account the differences between matter in the experiment and the astrophysical environment, such as charge neutrality and Coulomb interactions in supernova matter or the limited number of nucleons involved in the heavy-ion collision.

Using the same experimental data of [40], these aspects were considered in [41]. Also some errors in the theoretical EOS data shown in [40] were corrected. From the comparison it was concluded that the ideal gas behavior is clearly ruled out at similar densities as they occur around the neutrinospheres in a CCSN. To obtain a good agreement with the experimental data, three ingredients seem to be necessary: (i) inclusion of all relevant particle degrees of freedom, (2) mean-field interactions of unbound nucleons, (3) a suppression mechanism of nuclei at high densities (e.g., Pauli-blocking or excluded volume). The largest deviations were found for the LS EOSs which show a notable underproduction of alpha particles and/or too many nucleons, and for the SHT(NL3) and SHO(FSU2.1) EOSs, with too many alpha particles and/or too little nucleons predicted. The current experimental data is not accurate enough to distinguish details of the medium modifications of the light clusters.

It has to be noted that this experimental constraint does not have the same significance and robustness as, e.g., the study of [27] or the observed pulsar masses, as some model assumptions are involved in the extraction of the data. Nevertheless, the obtained conclusions appear to be reasonable. Found deviations can be linked conclusively to deficits or missing aspects in the theoretical models, and for the most advanced models regarding medium modifications of light clusters [42,43], good agreement with the experimental data is found.

3.2 Heavy Nuclei

Heavy nuclei dominate the composition in a CCSN during the early stage of the collapse of the iron core. Later, they are present in the matter which is accreted onto the PNS from the outer layers of the progenitor. They are especially relevant for the electron-captures during the collapse, which affects, e.g., the mass of the core at bounce. In [44], three statistical models, HS, FYSS, and SMSM [45], were compared, which all go beyond the single nucleus approximation for heavy nuclei, i.e., which contain an ensemble of various different species. On the one hand, overall similar trends are found for various densities, temperatures, and asymmetries. On the other hand, the limitations and simplifications used in each of the models are visible in the details of the nuclear composition, especially when going to more extreme conditions.

4. Additional Degrees of Freedom

At high densities or temperatures additional degrees of freedom such as pions, hyperons, or quarks can appear. Currently 23 general purpose EOSs exist which consider such additional degrees of freedom. They are listed in the right half of Table I. Most of them employ the TM1 interactions for the nucleons, whose symmetry energy is problematic as the values of J and L are much higher than in experimental constraints, see Fig. 1. Models which do not employ TM1 are LS220 Λ and LS220 π , representing extensions of LS220, and BHBA and BHBA ϕ , representing extensions of HS(DD2). As the symmetry energy of LS220 is also slightly too high in comparison with the constraints from [27], only BHBA and BHBA ϕ remain as directly compatible models.

Additional degrees of freedom often lead to a substantial reduction of the maximum mass. Even if one uses only the maximum mass constraint, just few EOSs remain as viable models: only STOSQ139s, STOSQ145s, STOS π , and BHBA ϕ are in direct agreement with the constraint from [28]. The maximum masses of BHBA and LS220 π are slightly too low. All others have maximum masses around or below $1.9 M_{\odot}$. Using the two constraints for the symmetry energy and the maximum mass together, BHBA ϕ is the only general purpose EOS which fulfills both of them. In this model only the Lambda of all possible hyperons has been added to the HS(DD2) EOS.

5. Summary and Conclusions

The supernova or general purpose EOS has to cover a huge parameter space in density, temperature, and electron fraction. It remains a conceptual and numerical challenge to develop an EOS model and to calculate an EOS table covering all the different thermodynamic regimes. As a consequence, only a very limited number of general purpose EOSs is currently available. All of them are to a large extent based on a phenomenological description. Advances in nuclear experiments, astrophysical observations, but also in theoretical ab-initio calculations allow to significantly constrain the general purpose EOS. Here we discussed constraints for the symmetry energy and its slope parameter, the neutron star maximum mass, and cluster yields from heavy-ion collisions.

Regarding the nuclear composition, even though the models give overall similar trends for varying temperature, density and asymmetry, there is still some significant model dependency remaining. Further improvements of the general purpose EOS regarding the description of heavy and light nuclei and their medium modifications, within a statistical distribution, is clearly demanded. One of the most advanced models in this respect is FYSS, which implements aspects of the more systematic and microscopic approaches of [42, 43], but also in this EOS still a lot of phenomenological modeling is involved.

Even if one considers only the maximum mass and the symmetry energy constraint, just three nucleonic general purpose EOS remain as viable models: HS(DD2), SFHo, and SHO(FSU2.1). These remaining three models can be taken as a representative sample illustrating the current uncertainties of the nucleon interactions at high densities. It has to be noted that all of them have aspects which can be improved further.

Regarding general purpose EOSs which address the appearance of additional degrees of freedom at high densities and/or temperatures (pions, hyperons, and/or quarks) the situation is even more severe. Just four EOSs exist which have a sufficiently high maximum mass. Only one EOS, BHBA ϕ , an extension of HS(DD2) where lambda hyperons have been added, passes both constraints. This means at present there is no general purpose EOS which has a good behavior of the symmetry energy, a maximum mass above $2 M_{\odot}$, and which considers pions or quarks. This situation is certainly not satisfactory and should be improved. From our perspective it is not justified to simply ignore these additional degrees of freedom. New, further improved general purpose EOSs should be developed in the future, also exploring these additional degrees of freedom.

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