**On fast boundary element methods for parametric surfaces**

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In many situations, practical problems arising from science and engineering can be formulated in terms of differential equations for an unknown function. If a Green's function of the underlying differential operator is known, they may be reformulated by means of a boundary integral equation

$$(\mathcal{A}u)(\mathbf{x}) = \int_\Gamma G(\mathbf{x},\mathbf{y})u(\mathbf{y})\,\mathrm{d}o_{\mathbf{x}} = f(\mathbf{x}), \quad \mathbf{x}\in\Gamma.$$

The Green's function $G(\cdot,\cdot)$ is, for instance, known in case of the Laplace equation, the Helmholtz equation, and the heat equation. The major advantage of considering boundary integral equations is the reduction of the problem's dimensionality, which especially gives the possibility for easily treating also exterior boundary value problems.

In general, boundary integral equations are solved by the boundary element method. However, due to the non-locality of the integral operator $\mathcal{A}$, one usually ends up with large and densely populated system matrices. Thus, the numerical solution of such problems is rather challenging. Different approaches have been proposed to overcome this obstruction, such as the fast multipole method, adaptive cross approximation, or wavelet matrix compression.

This talk intends to give an overview on fast boundary element methods which are tailored to the context of parametric surfaces. This means, the surface $\Gamma$ is subdivided into several smooth *patches*

$$\Gamma = \bigcup_{i=1}^{M} \Gamma_i$$

such that the intersection $\Gamma_i \cap \Gamma_{i'}$ consists at most of a common vertex or a common edge for $i \neq i'$. Moreover, for each patch, there exists a smooth diffeomorphism

$$\boldsymbol{\gamma}_i : \square \to \Gamma_i \quad \text{with} \quad \Gamma_i = \boldsymbol{\gamma}_i(\square) \quad \text{for} \quad i = 1,2,\ldots,M,$$

where $\square := [0,1]^2$ denotes the unit square. This surface representation is in contrast to the common approximation of surfaces by flat panels. Nonetheless, parametric surface representations are easily accessible from Computer Aided Design (CAD) and are the topic of recent studies in isogeometric analysis.

We address different issues of fast boundary element methods for the solution of boundary integral equations on parametric surfaces as considered in [1]–[5]. This includes storage requirements, adaptivity, and higher-order ansatz functions. In particular, it turns out that the additional information, which is imposed by the parametric surface representation, enables numerous simplifications and optimizations of the underlying data structures and algorithms. Several numerical examples are provided in order to quantify and qualify the proposed methods.

## References

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