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Optical lattices with micromechanical mirrors

K. Hammerer, K. Stannigel, C. Genes, and P. Zoller
Institute for Theoretical Physics, University of Innsbruck, and Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Technikerstrasse 25, 6020 Innsbruck, Austria

P. Treutlein,* S. Camerer, D. Hunger, and T. W. Hänsch
Max-Planck-Institute of Quantum Optics and Faculty of Physics, Ludwig-Maximilians-University Munich, Schellingstraße 4, D-80799 Munich, Germany

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We investigate a setup where a cloud of atoms is trapped in an optical lattice potential of a standing-wave laser field which is created by retroreflection on a micromembrane. The membrane vibrations itself realize a quantum mechanical degree of freedom. We show that the center-of-mass mode of atoms can be coupled to the vibrational mode of the membrane in free space. Via laser cooling of atoms a significant sympathetic cooling effect on the membrane vibrations can be achieved. Switching off laser cooling brings the system close to a regime of strong coherent coupling. This setup provides a controllable segregation between the cooling and coherent dynamics regimes, and allows one to keep the membrane in a cryogenic environment and atoms at a distance in a vacuum chamber.

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Optical lattices for ultracold atoms [1] are generated by standing laser light waves, obtained, for example, by retroreflecting a running-wave laser beam from a mirror. The intensity variation in the standing wave gives rise to a position-dependent ac Stark shift, thus providing a periodic array of optical microtraps for the atomic motion, which can realize a Hubbard model for bosonic or fermionic atoms. In such experiments great care is taken to avoid spurious effects from spontaneous emission into vacuum modes, back-action of the atomic motion on the laser fields, or thermal and other vibrations of the mirror reflecting the laser light.

Recent experiments on the other hand show that micrometer-sized mirrors [2] or reflecting dielectric membranes [3] can realize high-quality mechanical oscillators, which can be operated close to their quantum regime [4]. In light of this development a new generation of optical lattice experiments is in sight, where the mirror vibration provides an additional quantum degree of freedom. Here we analyze the feasibility and potential of a setup where a standing light wave is generated by reflection from a micromechanical mirror or membrane, which is thus coupled via the moving lattice to a *distant* ensemble of cold atoms (see Fig. 1). The back-action of atomic motion on the light fields [5], and thus on the membrane, can be substantial in a lattice with moderate detuning. Overall this results in a sizable coupling of atomic motion and mirror vibrations, mediated by the quantum fluctuations of the lattice laser light in *free space*.

We derive a full quantum model for this setup, in which this coupling emerges as part of the dynamics of *cascaded quantum systems* [6]. Our model fully encompasses quantum noise effects due to radiation pressure, consistently recovering previous results [7]. We show that via this coupling the mirror can be *sympathetically cooled* to the motional ground state by laser cooling the atoms. We envision that the proposed system presents only one instance where the well-developed tools for laser cooling of atoms are used to provide a mechanism for sympathetic cooling of polarizable particles (here the dielectric membrane).

Several experiments demonstrated the possibility of strong coherent coupling of the motion of atoms [8] or mechanical oscillators [4,9] to a *cavity mode*. Cavity-mediated coupling between atoms and mechanical oscillators, as suggested in [10], is very demanding, as it requires to combine ultrahigh vacuum (as required for experiments with cold atoms) with a cryogenic environment (as required for micromechanical systems) and a high-finesse optical cavity. Here we show that such a coupling can be obtained in free space, and that cryogenic and vacuum setups can be spatially separated.

Consider the setup shown in Fig. 1. The displacement $Z(t)$ of a micromechanical membrane (frequency ω_m, effective mass m_m, zero-point fluctuation $\ell_m = \sqrt{\hbar/\omega_m m_m}$) provides a time-dependent boundary condition for the electromagnetic (em) field, such that the standing-wave laser field is $E(z) \propto \sin[k_l(z - Z)]$. The corresponding lattice potential experienced by the atoms is proportional to the light intensity, such that $V(z_i) = V_0 \sin[k_l(z_i - Z)]^2 \simeq V_0 k_l^2(\delta z_i^2 - 2\delta z_i \delta Z)$, where V_0 is the potential depth. $\delta z_i = z_i - \bar{z}_i$ and $\delta Z = Z - \bar{Z}$ denote small fluctuations of atoms and membrane around their respective equilibrium positions $\bar{z}_i$ and $\bar{Z}$ with conjugate momenta δp_i and δP, respectively. In the expansion we have neglected the term of order δZ^2 without loss of generality, as it would give rise to a small shift in the mechanical resonance frequency ω_m only. The first term in $V(z_i)$ corresponds to a harmonic potential with trap frequency $\omega_{\rm at}^2 = 2V_0 k_l^2/m_{\rm at}$ (zero-point fluctuation $\ell_{\rm at} = \sqrt{\hbar/m_{\rm at}\omega_{\rm at}}$). The second term gives rise to a dynamical coupling between atoms and membrane. Consider the (dimensionless) center-of-mass position and momentum of the ensemble of N atoms, $x_{\rm at} = \sum_i \delta z_i/\ell_{\rm at}\sqrt{N}$ and $p_{\rm at} = \sum_i \delta p_i \ell_{\rm at}/\hbar\sqrt{N}$, and the (dimensionless) membrane position and momentum fluctuations, $x_m = \delta Z/\ell_m$ and $p_m = \delta P \ell_m/\hbar$. The potential $V(z_j)$

*Present address: Department of Physics, University of Basel, Switzerland; philipp.treutlein@unibas.ch

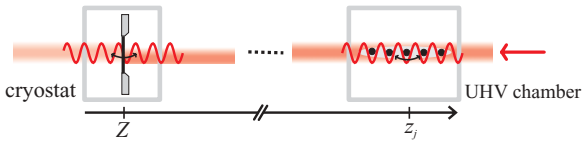

FIG. 1. (Color online) A laser field impinging from the right is partially reflected off a dielectric membrane and forms a standing-wave optical potential for an atomic ensemble. Vibrations of the membrane's fundamental mode will shift the standing-wave field, shaking atoms in the optical lattice. Conversely, oscillations of the atomic cloud (center of mass motion) will change the intensity of left- (right-) propagating field components, thus shaking the membrane via changing the radiation pressure on it. The membrane can be kept in a cryogenic environment, and the atoms at a distance in a vacuum chamber.

implies an equation of motion $\langle \dot{p}_{\rm at}\rangle = -\omega_{\rm at}\langle x_{\rm at}\rangle - g\langle x_m\rangle$, with an effective coupling strength $g = \omega_{\rm at}\sqrt{N m_{\rm at}/m_m}$, assuming resonance between the two systems $\omega_m = \omega_{\rm at}$. While the mass ratio will be exceedingly small under reasonable conditions, the coupling can still be significant due to the collective enhancement by the number of atoms. For example, for a megahertz trap frequency and a mass ratio $m_{\rm at}/m_m \simeq 10^{-14}$, a number of $N = 10^8$ atoms will give rise to a coupling g on the order of kilohertz.

The back-action of the atoms on the membrane can in fact be of the same order of magnitude, as can be understood from another instructive consideration: The restoring force $F = -m_{\rm at}\omega_{\rm at}^2 \sum_i \delta z_i$ experienced by the atoms is ultimately due to the transfer of photon momentum, and is thus accompanied by a change in power $\Delta P = Fc/2$ between the left- and right-propagating laser beams, as observed in [5] (c is the speed of light). This change in power will be experienced by the membrane as a change in radiation pressure of the reflected light. For a power reflectivity $\mathfrak{r}$ the change in the radiation pressure force on the membrane is $F_{\rm RP} = -2\mathfrak{r}\Delta P/c = -\mathfrak{r}F$. We thus arrive at an equation of motion $\langle \dot{p}_m\rangle = -\omega_m\langle x_m\rangle - \mathfrak{r}g\langle x_{\rm at}\rangle$ for the coupling of the membrane momentum p_m to the atomic center-of-mass position $x_{\rm at}$.

These simple considerations indicate that an appreciable coupling between the two systems can be achieved *even in free space*. However, they also show that this coupling seemingly provides an *asymmetric* influence for a finite reflectivity $\mathfrak{r}$, such that a unique effective Hamiltonian cannot be inferred from the equations of motion. Moreover, the foregoing considerations do not provide any information about the noise processes, e.g., due to radiation pressure, against which we have to benchmark the coupling strength g.

In the following we therefore develop a fully quantum theoretical description of this setup. In particular, we will first derive a quantum stochastic Schrödinger equation (QSSE) and from this a Markovian master equation (MEQ). This MEQ first reveals that the present setup exhibits characteristics of a *cascaded quantum system* [6], which confirms and explains the asymmetric coupling, and second provides a consistent description of quantum noise in this system.

The Hamiltonian for the system is $H_{\rm full} = H + H_{\rm 3D}$, where H contains in a one-dimensional treatment the coupling of atoms and membrane vibrations to the paraxial field along the z direction as shown in Fig. 1,

$$H = \hbar\omega_m a_m^\dagger a_m + \sum_{j=1}^{N}\frac{p_j^2}{2m_{\rm at}} + \sum_{j=1}^{N}\frac{\mu^2}{\hbar\delta}E^-(z_j)E^+(z_j) + \epsilon_0 A(n^2-1)\left[E^-\left(\frac{l}{2}\right)E^+\left(\frac{l}{2}\right) - E^-\left(-\frac{l}{2}\right)E^+\left(-\frac{l}{2}\right)\right]\delta Z, \quad (1)$$

while coupling to all other field modes is subsumed in $H_{\rm 3D}$. The first two terms in Eq. (1) represent the energy of the fundamental vibrational mode of the membrane with annihilation operator a_m, and the kinetic energy of atoms with momentum p_j. The third term is the level shift operator [11] describing the dispersive interaction of the N two-level atoms (dipole matrix element μ) with off-resonant field modes detuned by $\delta = \omega_l - \omega_0$ from the atomic resonance ω_0 within a frequency band $\vartheta \ll |\delta|$ [12]. In this treatment electronically excited states are already adiabatically eliminated. The second line in Eq. (1) is the potential of the radiation pressure force on a slab of thickness l, cross section A, and index of refraction n. The radiation pressure force is proportional to the difference of the intensities on either side (at $\pm\frac{l}{2}$) of the membrane. The electric field is $E(z) = E_R(z) + E_L(z)$ and consists of the two one-dimensional (1D) continua of plane-wave modes impinging from the right (R) and left (L), respectively [cf. Fig. 2(a)]. The explicit mode functions can be found in [14], and will be denoted by $A_\sigma(k,z)(\sigma = L,R)$, consistent with the notation used there. Thus, $E_\sigma^+(z,t) = \mathcal{E}\int d\omega A_\sigma(k,z)b_{\omega,\sigma}e^{-i\omega t}$ with $[b_{\omega,\sigma}, b_{\bar{\omega},\bar{\sigma}}^\dagger] = \delta_{\sigma,\bar{\sigma}}\delta(\omega-\bar{\omega})$ and $k = \omega/c$. We are using a narrowband approximation $\mathcal{E} = \sqrt{\hbar\omega_l/4\pi c\epsilon_0 A}$, where ω_l is the laser frequency, and take the fields in an interaction picture with respect to the free-field Hamiltonian. The contribution of other than paraxial modes contained in $H_{\rm 3D}$ can be largely suppressed in the following. Their effect will be relevant and addressed separately in the context of momentum diffusion of atoms in the lattice. The driving laser of frequency ω_l is at this point included by assuming the field to be in a coherent state $|\alpha\rangle = D|{\rm vac}\rangle$ [with $D = \exp(\alpha^* b_{\omega_l,R} - {\rm H.c.})$], corresponding to a laser field of power $P = \hbar\omega_l\alpha^2/2\pi$ (with photon flux $\alpha^2/2\pi$) impinging from the right.

We are interested in treating those processes whose amplitudes are enhanced by the laser amplitude α in either first or second order. In order to identify those we apply a Mollow transformation [15] and move to a displaced picture, where the field is effectively in vacuum $|{\rm vac}\rangle = D^\dagger|\alpha\rangle$ and the new Hamiltonian is accordingly $H' = D^\dagger H D$. In H' the second-order terms in α provide the lattice potential (with trap depth $V_0 = 4\mu^2\mathcal{E}^2\alpha^2\sqrt{\mathfrak{r}}/\hbar|\delta|$), and an unimportant mean radiation pressure force on the membrane. The terms of first order in α, which contain the coupling of the motion of membrane and atoms to em field modes initially in vacuum, are treated in a Lamb-Dicke expansion in $(k_l\ell_{\rm at})$. The result can be interpreted as a (Stratonovich) quantum stochastic

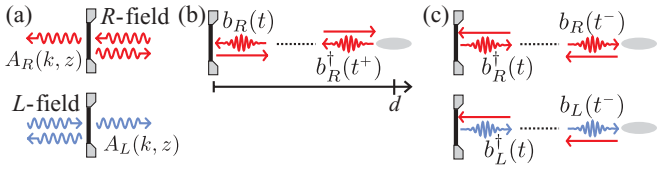

FIG. 2. (Color online) (a) Schematic illustration of mode functions $A_\sigma(k,z)$ of R and L modes for fields impinging from the right and left, respectively. All modes are in vacuum but a single one in the R field at frequency ω_l which is driven by a laser field. (b) Mediated action of atomic COM fluctuations on the membrane: At the advanced time t^+ atoms absorb a photon from the right-propagating component of the laser field (with amplitude $\sqrt{\mathfrak{t}}\alpha$, straight arrow) and reemit a sideband photon to the R field [$b_R^\dagger(t^+)$, wavy arrow], receiving a momentum kick of $2\hbar k_l$ [cf. the term in Eq. (2a)]. At a later time t the sideband photon is annihilated [] and transferred back to the laser field, giving a momentum kick to the membrane [cf. first term in Eq. (2b)]. (c) Reverse action: Via corresponding emission and reabsorption processes as given in Eqs. (2b) and (2c), membrane vibrations affect atomic COM fluctuations. In these processes both the R (upper panel) and the L field (lower panel) contribute, effectively mediating a stronger action than the process shown in (b).

Schrödinger equation [12] with *time delays,*

$$\frac{d}{dt}|\Psi\rangle = -iH(t,t^-,t^+)|\Psi\rangle$$

$$= \{-iH_{\rm sys} - x_{\rm at}[\sqrt{g_{{\rm at},R}}b_R(t^+) - {\rm H.c.}] \quad (2a)$$

$$+x_m[i\sqrt{g_{m,R}}b_R(t) - \sqrt{g_{m,L}}b_L(t) - {\rm H.c.}] \quad (2b)$$

$$+x_{\rm at}[\sqrt{g_{{\rm at},R}}b_R(t^-) + i\sqrt{g_{{\rm at},L}}b_L(t^-) - {\rm H.c.}]\}|\Psi\rangle, \quad (2c)$$

Where $H_{\rm sys} = \omega_m a_m^\dagger a_m + \omega_{\rm at} a_{\rm at}^\dagger a_{\rm at}$ denotes the system Hamiltonian for the free evolution of the membrane and the atomic center of mass (COM) mode, with annihilation operator $a_{\rm at}$ for the latter. The remaining terms denote the coupling of position fluctuations $x_{m({\rm at})} = (a_{m({\rm at})} + {\rm H.c.})/\sqrt{2}$ of these modes to vacuum fluctuations of the em field described by slowly varying field operators $b_\sigma(t) = \int [d\omega/\sqrt{2\pi}b_\sigma(\omega)]\exp[-i(\omega-\omega_l)t]$ $(\sigma = R,L)$, evaluated at time t as well as at *advanced* and *retarded* times $t^\pm = t \pm d/c$, where d is the distance between the atomic ensemble and the mirror [cf. Fig. 2(b)]. In this Hamiltonian the linear extension of the ensemble is neglected and we used the approximation $b(t \pm z_j/c) \simeq b(t \pm d/c)$ for all atoms.[1] The coupling constants of membrane vibrations and atomic COM position fluctuations to the em field are given by $g_{m,R} = 2(\alpha k_l \ell_m)^2\mathfrak{r}^2/\pi$ and $g_{{\rm at},R} = 2\pi N(\omega_{\rm at}/4\alpha k_l \ell_{\rm at})^2$, where $g_{m({\rm at}),L} = \frac{\mathfrak{t}}{\mathfrak{r}}g_{m({\rm at}),R}$ with the membrane transmittivity $\mathfrak{t} = 1 - \mathfrak{r}$. In Figs. 2(b) and 2(c) we provide an intuitive interpretation of each term in the QSSE (2a) as an (anti-)Stokes backscattering process, where a laser photon is absorbed or emitted together with, respectively, an emission or absorption of a sideband photon at frequency $\omega_l \pm \omega_{m({\rm at})}$ in one of the initially empty modes of the R or L field, along with a momentum transfer of $2\hbar k_l$ to either the membrane or the atomic COM mode. The effective mediated coupling between atoms and membrane, as well as radiation pressure noise on each system, will eventually result from second-order processes, where sideband photons are emitted to and reabsorbed from the R or L field [Figs. 2(b) and 2(c)].

The corresponding effects of these second-order processes can be seen explicitly when Eq. (2a) is transformed into an equivalent master equation (MEQ) *without* time delays in the limit $t^\pm \to t$. This assumes the standard Born-Markov approximation for the em field in vacuum, and neglects retardation effects assuming distances $d \ll c/\omega_m$. The technicalities and assumptions of this procedure, which requires some care regarding correct time ordering, are outlined in the Appendix below. The result is a master equation for the density operator ρ of the membrane and the COM motion of the form

$$\dot\rho = -i(H_{\rm sys} - g x_{\rm at}x_m, \rho) + C\rho + L_m\rho + L_{\rm at}\rho. \quad (3)$$

This MEQ contains, first, an *effective atom-membrane interaction* with a coupling $g = 2(\sqrt{g_{m,R}g_{{\rm at},R}} + \sqrt{g_{m,L}g_{{\rm at},L}}) = \omega_{\rm at}\sqrt{Nm_{\rm at}/m_m}$, where we assume $\omega_m \simeq \omega_{\rm at}$. Second, we find a contribution

$$C\rho = -\frac{i\mathfrak{t}g}{2}([x_m, x_{\rm at}\rho] - [\rho x_{\rm at}, x_m]),$$

which is proportional to the membrane transmittivity $\mathfrak{t}$. The peculiar form of this term is in fact generic for cascaded quantum systems [6,12]. Its effect becomes clear, for example, when looking at the evolution of mean values according to Eq. (3), $\langle \dot p_{\rm at}\rangle = -g\langle x_m\rangle + \cdots$, while $\langle \dot p_m\rangle = -g\mathfrak{r}\langle x_{\rm at}\rangle + \cdots$, where the ellipses stand for contributions of $H_{\rm sys}, L_m$, and $L_{\rm at}$. Thus, the action of atoms on the membrane is reduced by a factor $\mathfrak{r}$ as compared to the action of membrane fluctuations on atoms, correctly reproducing the introductory qualitative considerations.[2] The present description reveals this to be a consequence of the fact that the action of the membrane on atoms is mediated through both fields, R and L, while only the R field contributes to the action of atoms on the membrane (see Fig. 2).

The strength of coherent coupling has to be compared to various rates of decoherence, described in Eq. (3) by a number of Lindblad terms $L_m, L_{\rm at}$. One channel of decoherence is caused by coupling to the em vacuum field, which is contained in the QSSE in Eq. (2a) and results in radiation-pressure-induced momentum diffusion described by Lindblad terms $L^{\rm diff}_{{\rm at}(m)}\rho = \frac{1}{2}\gamma^{\rm diff}_{{\rm at}(m)}D[x_{{\rm at}(m)}]\rho$, where $D[a]\rho = 2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a$. The atomic momentum diffusion rate is $\gamma^{\rm diff}_{\rm at} = (k_l\ell_{\rm at})^2\gamma_{se}V_0/\hbar|\delta|$, where γ_{se} is the natural linewidth of the transition. Note that the momentum diffusion rate $\gamma^{\rm diff}_{\rm at}$ for the atomic COM motion is equal to the well-known single-atom diffusion rate [13] only if all interactions between atoms are neglected. In the 1D treatment of the QSSE (2a) these interactions are in fact overestimated, as they do not fall off with the distance between atoms. Only a more careful 3D treatment, taking into account all terms neglected in $H_{\rm 3D}$, reveals the correct distance dependence of dipole-dipole

[1]This is well justified if the "sideband wavelength" $c/\omega_{m({\rm at})}$ is smaller than the linear extension of the atomic ensemble.

[2]Note that the coupling strength g itself tends to zero for $\mathfrak{r} \to 0$, as the lattice depth and therefore also $\omega_{\rm at}$ vanish in this limit and $g \propto \omega_{\rm at}$.

interactions and allows these effects to be ignored for a dilute sample. The diffusion rate of the membrane mode due to radiation pressure is $\gamma_m^{\rm diff} = (4\mathfrak{r}P/m_m c^2)(\omega_l/\omega_m)$. This last result is consistent with calculations in [7] on the effects of radiation pressure on a mirror in free space.

On top of these decoherence channels there will be thermal heating of the membrane due to clamping losses and laser absorption, which drives the vibrational mode under consideration to thermal equilibrium at temperature T, i.e., $L_m^{\rm th}\rho = \frac{\gamma_m}{2}(\bar{n}+1)D[a_m]\rho + \frac{\gamma_m}{2}\bar{n}D[a_m^\dagger]\rho$, with thermal occupation $\bar{n} = k_B T/\hbar\omega_m$, and $\gamma_m = \omega_m/Q_m$ for a mechanical quality factor Q_m. For atoms in turn, it is important to note that the dissipation processes $L_{\rm at}$ can to a large extent be *engineered*. In particular, it is possible to use Raman sideband laser cooling to cool atoms individually close to their ground states in the optical lattice potential [16]. In the master equation we account for this by adding a Lindblad term for the COM mode, $L_{\rm at}^{\rm cool}\rho = \frac{1}{2}\gamma_{\rm at}^{\rm cool} D[a_{\rm at}]\rho$, with a Raman-cooling rate $\gamma_{\rm at}^{\rm cool}$. If laser cooling is switched off, only the background decoherence at a rate $\gamma_{\rm at}^{\rm diff}$ remains. The dynamics of the master equation (3) thus provides two interesting regimes: with efficient laser cooling of atoms it is possible to *sympathetically cool* the mirror, while for $\gamma_{\rm at}^{\rm diff} \ll g$ a regime of *coherent evolution* becomes accessible, limited only by the decoherence rate of the membrane $\gamma_m^{\rm th} = \gamma_m\bar{n}$.

We estimate the magnitude of the relevant processes as follows: For a small SiN membrane of dimensions 150 μm $\times$ 150 μm $\times$ 50 nm exhibiting a mechanical resonance at $\omega_m = 2\pi \times 0.86$ MHz with an effective mass of $m_m = 8\times 10^{-13}$ kg and a high mechanical quality of $Q_m = 10^7$ at $T \lesssim 2$ K, a power reflectivity $\mathfrak{r} = 0.31$ can be achieved for the wavelength of $\lambda = 780$ nm relevant for ^{87}Rb [3]. A lattice beam with power $P = 7$ mW and a waist of 230 μm at the position of atoms, detuned by $\delta = -2\pi \times 1$ GHz, can provide a sufficient lattice depth such that the longitudinal trap frequency $\omega_{\rm at} \simeq \omega_m$. For an ensemble of $N = 3\times 10^8$ atoms [16], a strength of coherent coupling $g = 40$ kHz can be achieved. For the decoherence due to radiation pressure noise one finds a momentum diffusion at rate $\gamma_m^{\rm diff} = 52$ Hz for the membrane, and an atomic momentum diffusion in the lattice at rate $\gamma_{\rm at}^{\rm diff} = 16$ kHz. For the membrane the dominating dissipative effect will clearly be thermal decoherence at rate $\gamma_m^{\rm th} = 4$ MHz (24 kHz) at room temperature (2 K). On the other hand, Raman sideband cooling of atoms at a fast rate $\gamma_{\rm at}^{\rm cool} \simeq 20$ kHz is possible [16]. For these parameters, a regime where $\omega_m = \omega_{\rm at} \gg g \gtrsim \gamma_{\rm at}^{\rm cool} \simeq \gamma_m^{\rm th} \gtrsim \gamma_{\rm at}^{\rm diff} \gg \gamma_m^{\rm diff}$ is accessible. To suppress trap loss due to light-assisted collisions, blue detuning of the lattice laser is advantageous [16]. Transversally, the atoms can be confined in a far-detuned 2D lattice, so that atomic densities of order 10^{12} atoms/cm^3 are realistic [17]. A concern is the spread of vibrational frequencies $\Delta\omega_{\rm at} \simeq 2\pi \times 24$ kHz across the ensemble due to the Gaussian intensity profile of the laser. It can be reduced by using a beam with a flat top profile. Due to its modularity, the presented setup is very flexible, for example, several atomic ensembles could be trapped in multiple foci of the same lattice laser, thereby enhancing N and thus g.

In order to evaluate the corresponding efficiency of sympathetic cooling of the membrane via laser cooling of atoms, we solve the master equation (3) for the steady state occupation

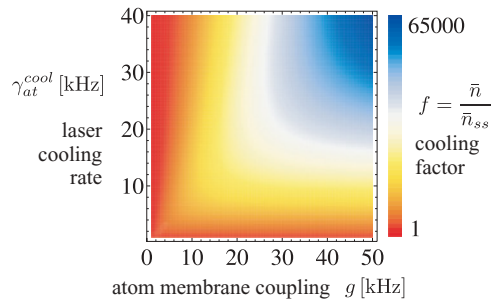

FIG. 3. (Color online) Cooling factor $f = \bar{n}/\bar{n}_{\rm SS}$ versus effective coupling g between the atomic COM motion and the membrane vibrations and rate of Raman sideband laser cooling $\gamma_{\rm at}^{\rm cool}$, as determined from an exact numerical solution of the effective master equation (3) for parameters as given in the case study. A cooling factor of 10^4 yields a steady state occupation $\bar{n}_{\rm SS}$ below 1 if the system is precooled to below 500 mK. At room temperature the sympathetic cooling effect will be still clearly observable.

$\bar{n}_{\rm SS} = \langle a_m^\dagger a_m\rangle_{\rm SS}$. For the parameters given above one finds a cooling factor $f = \bar{n}/\bar{n}_{\rm SS} \simeq 2\times 10^4$, which is sufficient for *ground state cooling* ($\bar{n}_{\rm SS} \simeq 0.8$) of the membrane when starting from 500 mK (see Fig. 3). In order to get some more insight into the cooling efficiency we consider the weak-coupling limit $\gamma_{\rm at}^{\rm cool} \gg g$. In this case we can eliminate the atomic COM mode adiabatically along the lines of the treatments of the equivalent problem of optomechanical laser cooling [4]. For the mean occupation one finds a rate equation $\frac{d}{dt}\langle a_m^\dagger a_m\rangle = -\Gamma_m(\langle a_m^\dagger a_m\rangle - \bar{n}_{\rm SS})$ with an effective cooling rate $\Gamma_m = \gamma_m + \mathfrak{r}g^2/2\gamma_{\rm at}^{\rm cool}$ and a final occupation $\bar{n}_{\rm SS} \simeq (\gamma_m/\Gamma_m)\bar{n} + \left(\gamma_{\rm at}^{\rm cool}/4\omega_m\right)^2$. For large enough cooling rate, the thermal contribution can be suppressed and the limitation is due only to Stokes-scattering processes. As in laser cooling of ions or optomechanics, this can be suppressed in the resolved sideband limit, which amounts here to having $\gamma_{\rm at}^{\rm cool} \ll \omega_m$. Under this condition, ground state cooling is possible.

If laser cooling of atoms is switched off ($\gamma_{\rm at}^{\rm cool} = 0$), a regime of coherent coupling is accessible, at least for cryogenic temperatures, where $g \gtrsim \gamma_m^{\rm th}, \gamma_{\rm at}^{\rm diff}$. As compared to the usual optomechanical setup, where the equivalent parameter to $\gamma_{\rm at}^{\rm cool}$ is the cavity decay rate that is a fixed parameter, this is a qualitatively new feature of the setup considered here. It is well known, and has been extensively studied in various contexts [18], that the given coupling Hamiltonian $\sim x_{\rm at}x_m$—which becomes $\sim (a_m a_{\rm at}^\dagger + a_m^\dagger a_{\rm at})$ in the rotating wave approximation ($\omega_m \gg g$)—allows for a coherent state exchange of the two systems.

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APPENDIX: CONVERSION OF THE QSSE FROM STRATONOVICH TO ITO FORM

We provide here the details for how to transform the QSSE with time delays, Eq. (A2) of the main text, to the Markovian master equation (3). We transform the Stratonovich QSSE to

an Ito QSSE by first integrating Eq. (A2) up to a time $t+\Delta t$ (such that $\Delta t \gg d/c \gg 1/\vartheta$, where ϑ is the bandwidth of modes) and then expand to second order,

$$U(\Delta t)|\Psi(t)\rangle = \left(1 - i\int_t^{t+\Delta t} dt_1 H(t_1, t_1^-, t_1^+) \right. \tag{A1}$$
$$\left. - \int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 H(t_1, t_1^-, t_1^+) H(t_2, t_2^-, t_2^+)\right)|\Psi(t)\rangle$$
$$= \left\{1 - i\left[H_{\rm sys} - (2\sqrt{g_{m,R}g_{{\rm at},R}} + \sqrt{g_{m,L}g_{{\rm at},L}})x_m x_{\rm at}\right]\Delta t \right.$$
$$+ i\sqrt{g_{m,R}}x_m \Delta B_R^\dagger + (\sqrt{g_{m,L}}x_m + i\sqrt{g_{{\rm at},L}}x_{\rm at})\Delta B_L^\dagger$$
$$\left. -\frac{1}{2}(g_{m,R} + g_{m,L})x_m^2 \Delta t - \frac{1}{2}g_{{\rm at},L}x_{\rm at}^2 \Delta t\right\}|\Psi(t)\rangle. \tag{A2}$$

We use here the Ito increments $\Delta B_\sigma(t) = \int_t^{t+\Delta t} dt' b_\sigma(t')$ and their property $\Delta B_\sigma(t)|\Psi\rangle \equiv 0$ for all times. In the double time integral care needs to be taken in dealing with the time delays. It is easy to check that $\int_t^{t+\Delta t} dt_1 \int_t^{t_1} dt_2 b_\sigma(t_1) b_{\bar\sigma}^\dagger(t_2+\delta) = \theta(\delta)\delta_{\sigma,\bar\sigma}\Delta t$, where $\theta(\delta) = 0, \frac{1}{2}, 1$ for $\delta \lessgtr 0$ is the step function. After applying this rule it is possible to take the delay to zero, $t^\pm \to t$, as has been done in the second step of the last equation. This limit requires that carrier and sideband photons are still in phase after propagating a distance d, which will be the case for $d\omega_m/c \ll 1$. The same assumption also guarantees that the free evolution due to $H_{\rm sys}$ can be neglected on the time scale of retardation. The result can then be interpreted as an Ito QSSE, which can in turn be converted to the master equation (3) following standard procedures [12]. Note that this procedure results in a seemingly *collectively enhanced* atomic momentum diffusion, which is an artifact of the 1D model adopted in Eq. (1) of the main text. A more careful treatment based on a 3D model for the coupling of atoms to the em field correctly yields the well-known individual momentum diffusion of atoms in an optical lattice at the rate given in Eq. (3), along with dipolar interactions which are suppressed at low densities. This complication arises from the fact that a 1D treatment misses the fall-off of interatomic interactions with the cubed distance, giving rise to an entirely collective diffusion.

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